National Conference with International Participation
ENGINEERING MECHANICS 2007
Svratka, Czech Republic, May 14 – 17, 2007

SIMULATION MODELING OF MOBILE ROBOT UNDERCARRIAGE EQUIPPED WITH OMNIDIRECTIONAL WHEELS

T. Kubela*, A. Pochylý*

Summary: *This contribution summarizes the results of work carried out during the proposal and simulation modeling of the mobile robot undercarriage equipped with omnidirectional wheels.*

In order to make an accurate design and appropriate dimensioning of driving units, there was carried out a simulation of dynamic properties of the undercarriage with omnidirectional wheels in Mathworks MATLAB 7.

There is described a kinematical and a dynamical model of the whole robot platform; incorporating an idea of skew wheel angles modeling having a significant impact on the real robot trajectory. By using this functionality we can relatively overcome the problem of miss-alignments of the wheels during the assembly. Results of the simulation were summarized in form of a state space model of the whole robot platform.

This work was done under the terms of the research plan "Simulative modeling of mechatronic systems" on the Institute of Production Machines, Systems and Robotics; FME, BUT.

1. Introduction

The main advantage of mobile robot undercarriages with omnidirectional wheels deals mainly with very good maneuverability. Using omnidirectional wheels was the basic and initial assumption for the project. Particularly for the reason of very good maneuverability, it can be considered as an ideal tool for verification of various types of algorithms determined to local navigation, path planning, mapping and further development with respect to university-indoor environment and robotics classes.

At the Institute of Production Machines, Systems and Robotics (IPMSaR) has been approached to a decision to design a mobile robot with this type of undercarriage. However, each design process should be preceded by optimally chosen parameters that can influence the resulting behavior of the whole platform; mechatronic and systemic approach. The simplest form how these parameters can be achieved deals with a description of a complex simulation

* Ing. Tomáš Kubela, Ing. Aleš Pochylý: Institute of Production machines, Systems and Robotics, FME Brno University of Technology; Technická 2/2896; 616 69 Brno; tel.: +420.541 142 448, fax: +420. 541 142 449; e-mail: kubela@fme.vutbr.cz, pochyly.a@fme.vutbr.cz

model which results in an assessment of the final platform behavior with these parameters. The purpose of this contribution is to describe extended kinematical and dynamical model of the mobile robot with a possibility to implement skew wheel angles overcoming the problem of miss-alignments of the wheels having impact on the real robot's trajectory.

Further there is shortly described the design of the robot, selected wheels, driving units, gear boxes and incremental encoders and a simulation of power demands with respect to specified conditions and limitations.

2. Kinematical model of the robot

Provided that the robot is moving only within 2D environment, its absolute position in a global coordinate system is defined by the vector $[\xi, \psi, \theta]$ as shown in Fig. 1.

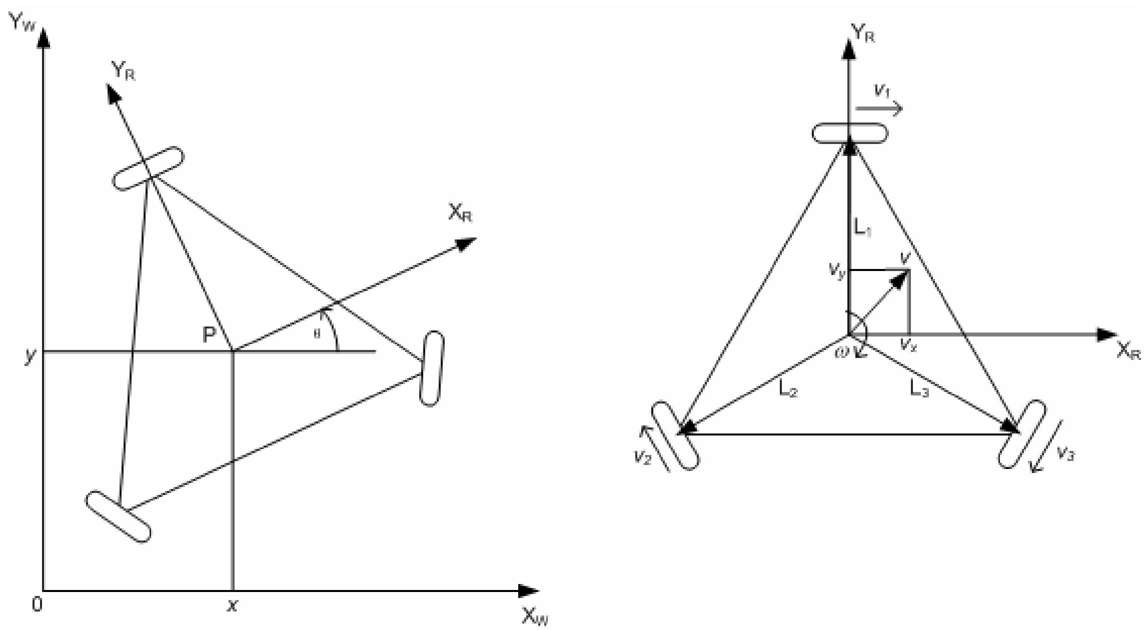

Fig. 1. Kinematical scheme of the omnidirectional mobile robot

The inverse kinematics equations are given by

$$(\omega_1 \quad \omega_2 \quad \omega_3)^T = \frac{1}{r} \cdot A \cdot R(\theta) \cdot (\dot{x} \quad \dot{y} \quad \dot{\theta})^T \tag{1}$$

where ω_i, i = 1, 2, 3, is the angular velocity of each wheel of the robot, r is the wheel radius,

$$A = \begin{pmatrix} 1 & 0 & L_1 \\ -1/2 & \sqrt{3}/2 & L_2 \\ -1/2 & -\sqrt{3}/2 & L_3 \end{pmatrix} \tag{2}$$

$$R(\theta) = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{3}$$

where L_i is the radius of the robot platform. By means of this equation (1) we can compute angular velocities of all wheels that are defined by desired trajectory of motion.

3. Robot dynamics modeling

Forces that are caused by the motion of the whole platform can be described as follows:

$$\begin{aligned} (m\ddot{x} \quad m\ddot{y} \quad J_R\ddot{\theta})^T &= A^T \cdot (f_{R1} \quad f_{R2} \quad f_{R3})^T \\ (f_{R1} \quad f_{R2} \quad f_{R3})^T &= (A^{-1})^T (m\ddot{x} \quad m\ddot{y} \quad J_R\ddot{\theta})^T \end{aligned} \qquad (4),(5)$$

where m is the robot mass, J_R is the robot moment inertia, f_{Ri} is the traction force of each wheel.

Dynamic model for each wheel:

$$J_w\dot{\omega}_i + c\omega = nM - rf_{wi} \qquad (6)$$

where J_w is the inertial moment of the wheel, c is the viscous friction factor of the omniwheel, M is the driving input torque, n is the gear ratio and f_{wi} is the driving force due to each wheel.

The dynamics of each DC motor can be described using the following equations:

$$\begin{aligned} L\frac{di}{dt} + Ri + k_1\omega_m &= u \\ J_m\dot{\omega}_m + b\omega_m + M_{ext} &= k_2 i \end{aligned} \qquad (7),(8)$$

where u is the applied voltage, i is the current, L is the inductance, R is the resistance, k_1 is the emf constant, k_2 is the motor torque constant, J_m is the inertial moment of the motor, b is the vicious friction coefficient, M_{ext} is the moment of an external load and ω_m is the angular speed of the motor.

By merging equations (2)-(8) we obtain a mathematical model of each essential dynamic properties of mobile robot undercarriage.

4. Robot model

The trajectory of motion is described by a list of points, each with four important parameters *[x, y, v, ω]*. From these points obtained needed vector of velocities *[v_x, v_y, ω]* by the Trajectory controller module. Inverse kinematics is used to translate this vector into individual theoretically required velocities of wheels. Dynamics module is then used to compute inertial forces and actual velocities of wheels. By means of Direct Kinematics module, these velocities are re-translated into the final vector of velocities of the whole platform. Simulator module obtains the actual performed path of the robot by their integration.

The whole model [Kubela, Pochylý, Knoflíček, 2006] was designed as a basis for modeling of mobile robot motions in order to analyze the impact of each constructional parameter on its behavior. For this reason, there is not used any feedback to control the motion on a desired trajectory. This approach allows choosing key parameters more precisely for better constructional design.

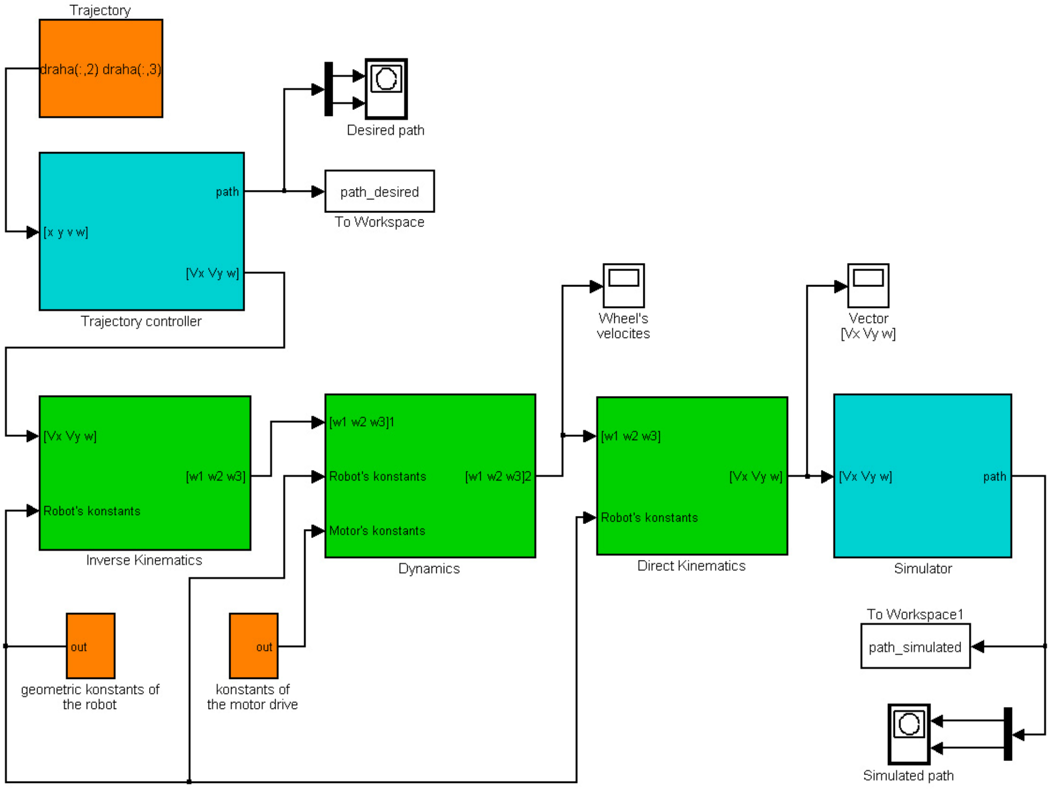

Fig. 2. Robot model in Matlab Simulink

In order to create this simulation model there was used software MATLAB Simulink 7.0.1. The emphasis was laid particularly on its schematic clearness and good encapsulation of each individual module. It has an important impact on an extensibility of the model in the future in order to create and analyzed other function blocks.

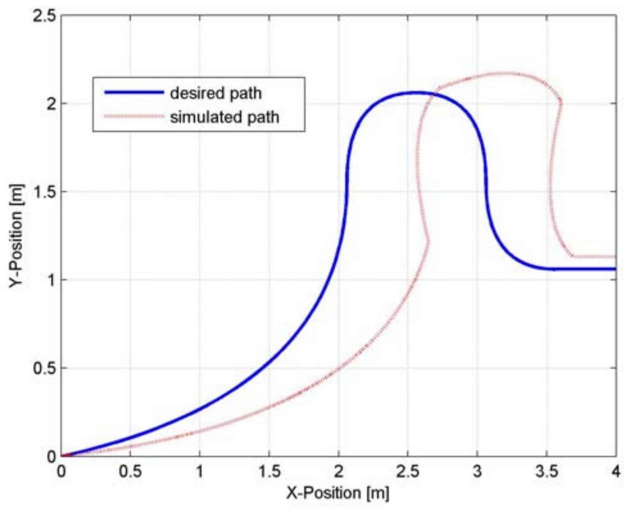

Fig. 3. Example of the impact of dynamic properties on a performed path of the robot

4.1 State-space model

State model of the whole platform are described by the folowing equations:

$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{A}\cdot\mathbf{x}+\mathbf{B}\cdot\mathbf{u}\\ \mathbf{y} &= \mathbf{C}\cdot\mathbf{x}\end{aligned} \quad (9)$$

$$\mathbf{x}=\begin{bmatrix}x_w & y_w & \theta & \dot{x}_w & \dot{y}_w & \dot{\theta}\end{bmatrix}^T \quad (10)$$

$$\mathbf{u}=\begin{bmatrix}\tau_1 & \tau_2 & \tau_3\end{bmatrix}^T \quad (11)$$

$$\mathbf{y}=\begin{bmatrix}\dot{x}_w & \dot{y}_w & \theta\end{bmatrix}^T \quad (12)$$

where **x** are vector of the state variables, **u** are the torques acting on the wheel axis generated by the motors, input variables and **y** are vector of the output variables.

Matrix of states are described:

$$\mathbf{A}=\begin{bmatrix}0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 1\\ 0 & 0 & 0 & -\frac{3c}{3I_w+2mr^2} & -\frac{3I_w}{3I_w+2mr^2}\dot{\theta} & 0\\ 0 & 0 & 0 & \frac{3I_w}{3I_w+2mr^2}\dot{\theta} & -\frac{3c}{3I_w+2mr^2} & 0\\ 0 & 0 & 0 & 0 & 0 & -\frac{3cL^2}{3I_wL^2+I_vr^2}\end{bmatrix} \quad (13)$$

the matrix of inputs:

$$\mathbf{B}=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\\ \frac{k_r}{3I_w+2mr^2}\beta_1 & \frac{k_r}{3I_w+2mr^2}\beta_2 & \frac{2k_r}{3I_w+2mr^2}\cos\theta\\ \frac{k_r}{3I_w+2mr^2}\beta_3 & \frac{k_r}{3I_w+2mr^2}\beta_4 & \frac{2k_r}{3I_w+2mr^2}\cos\theta\\ \frac{k_rL}{3I_w+2mr^2} & \frac{k_rL}{3I_w+2mr^2} & \frac{k_rL}{3I_w+2mr^2}\end{bmatrix} \quad (14)$$

where

$\beta_1=-\sqrt{3}\sin\theta-\cos\theta$, $\beta_2=\sqrt{3}\sin\theta-\cos\theta$

$\beta_3=\sqrt{3}\cos\theta-\sin\theta$, $\beta_4=-\sqrt{3}\cos\theta-\sin\theta$

and the matrix of outputs:

$$\mathbf{C} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \quad (15)$$

In order to create faster and more advanced simulation model, key parts of the model were transformed to state space. These parts are not supposed to be further improved what is not the case of other parts in the simulation model - dynamics of gearing mechanism including stiffness of belts, as well as the model of the robot as a whole in order to provide state feed-back control.

Idea of skew angles modeling and robot calibration

The robot model in the previous chapter can be extended including so-called skew wheel angles (Fig. 4). Using this functionality we can overcome the problem of miss-alignments of the wheels that have undoubtedly an impact on the real robot trajectory. The idea of skew angles modeling is based on [Rodrigues, Brandao, Lobo, Rocha, Dias, 2005].

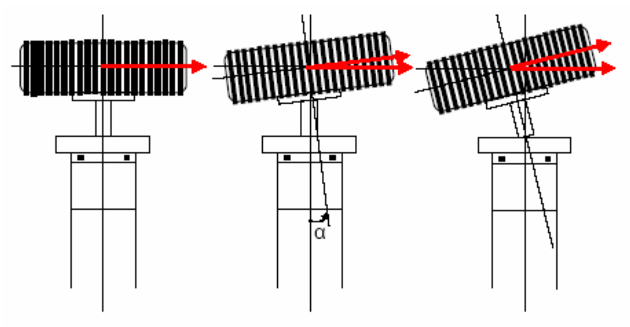

Fig. 4. Skew wheel angle

The purpose of skew wheel angles modeling and consequently a calibration procedure of the robot deals with reliable control based on odometry.

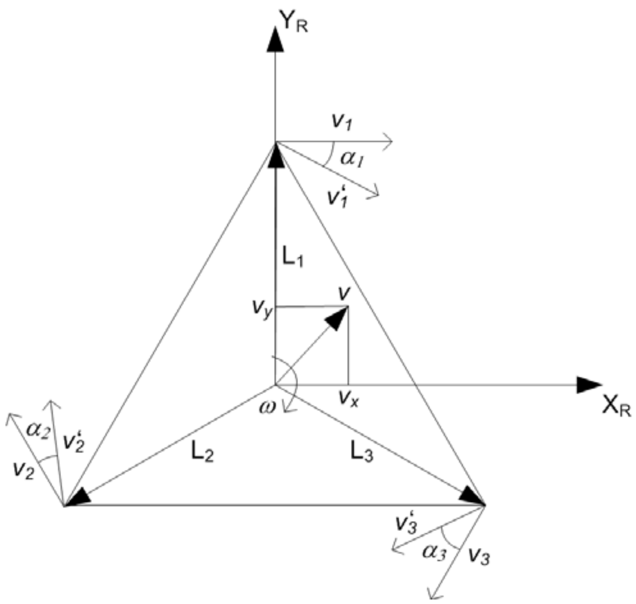

Fig. 5. Kinematical model including defined skew angles - α_1, α_2, α_3

Skew wheel angles modeling can be implemented using the following equations:

$$(\omega_1 \quad \omega_2 \quad \omega_3)^T = \frac{1}{r} \cdot S \cdot (\dot{x} \quad \dot{y} \quad \dot{\theta})^T \tag{16}$$

where

$$S = \begin{bmatrix} \cos(\alpha_1) & -\sin(\alpha_1) & -\cos(\alpha_1)r \\ -\cos\left(\frac{\pi}{3}+\alpha_2\right) & \sin\left(\frac{\pi}{3}+\alpha_2\right) & -\cos(\alpha_2)r \\ -\cos\left(\frac{\pi}{3}-\alpha_3\right) & -\sin\left(\frac{\pi}{3}-\alpha_3\right) & -\cos(\alpha_3)r \end{bmatrix} \tag{17}$$

5. Design of the mobile robot

There was chosen a symmetric undercarriage with omnidirectional wheels in order to simulate the behaviors, properties and characteristics of the mobile robot. It was the starting point from the previous year. Concerning the design of the mobile robot fulfilling the following conditions and limitations:

- Technologically simple construction,
- Modular, symmetrical platform,
- Maximal diameter of 550 mm,
- Maximal velocity of 10 kmph,
- Maximal acceleration of 2 m/s^2
- Maximal weight of approximately 30 kg.

In order to select suitable driving units for the mobile robot, there was carried out a simulation of power demands (Fig. 6) with respect to conditions and limitations stated above. The very dark tones represent a critical area of inadmissible power loading.

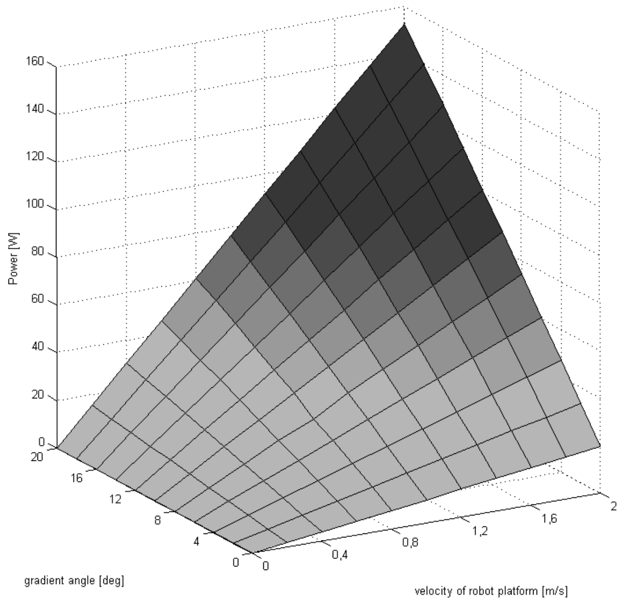

Fig. 6. Simulation of power demands in Matlab

There were selected and purchased the following key components (with its respective parameters) for the mechanical part in order to build the mobile robot:

- Driving unit: Maxon RE 30 (Ø30 mm, graphite brushes, 60 Watt, Nominal Voltage - 36 V, No load speed - 8590rpm, Nominal speed - 7810rpm, Nominal torque - 83.4mNm),
- Gear box: GP32C (Ø32 mm, ceramic version, ball bearing, Max. input speed - <8000rpm, Max. continuous torque - 3Nm, Max. efficiency - 75%, Gearing ratio - 14:1),
- Type of Encoder: Encoder MR (Type L, Counts per turn - 500, 3 channels, with Line Driver).

There were selected and purchased available omnidirectional wheels with the diameter of 80 mm from Interroll (http://www.interroll.com) (Fig. 7).

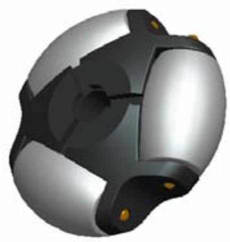

Fig. 7. Used Interrol omnidirectional wheel

At present is the mobile robot (OMR IV) in phase of construction work and all partial constructional units are prepared for manufacturing and assembly - there is little late regarding building the robot, caused mainly by delays in terms of key elements delivery. However, there is created complete drawing documentation and design of the robot – the planned term of robot assembly is to be fulfilled.

5.1 Design of the whole system - OMR IV

The design of the robot is characterized on the following figures. The base is composed of lightweight sheet-metal parts. The power from the driving units to omnidirectional wheels is led via a toothed belt drives. The shaft is mounted in two bearing units and there are used standard KM nuts and MB lock washers to fix the position of wheels and belt pulley on the shaft.

Fig. 8. OMR IV design – view on the whole system

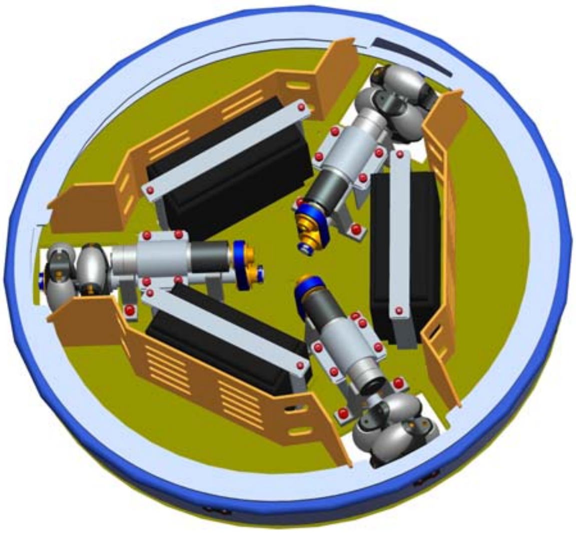

Fig. 9. OMR IV design - view on locomotion subsystem

6. Conclusion

This contribution summarizes a kinematical and dynamical model of the mobile robot without a feedback (open-loop) simulated in Matlab Simulink environment. The whole model was designed as a basis for motions modelling of a mobile robot undercarriage in order to analyze its key factors that influence the final motion and allow an optimal choice of these parameters which are required for a constructional proposal.

7. Acknowledgement

Published results were acquired using the subsidization of the Ministry of Education Youth and Sports of the Czech Republic research plan MSM 0021630518 „Simulation modelling of mechatronic systems".

8. References

Kubela, T., Pochylý A., Knoflíček R. (2006) Dynamic properties modeling of mobile robot undercarriage with omnidirectional wheels: *Proceedings of International Conference PhD2006, Pilsen, pp 45-46.*

Rodrigues, J., Brandao, S., Lobo, J., Rocha, R., Dias, J. (2005) RAC Robotic Soccer small-size team: Omnidirectional Drive Modelling and Robot construction, *Robótica 2005 – Actas do Encontro Científico, pp 130-135.*

Yong, L., Xiaofei, W., Jim, Z., Jae, L. (2003) Omni-Directional Mobile Robot Controller Design by Trajectory Linearization.