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INVESTIGATION OF DRAG FORCE AND DRAG MOMENT OF ROTATING TRANSLATIONALLY MOVING SPHERE IN CALM WATER

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Summary: *The paper describes results of experiments with rotating spherical particles moving quasi-steadily in the calm water. The motion of the particles was recorded by a digital video camera. The Cartesian coordinates and the angle of rotation of the particles were determined from the record of the particles motion. The dimensionless drag coefficient, drag moment coefficient and translational and rotational Reynolds numbers were calculated from the time series of the particles coordinates and angle of rotation for each recorded frame. The effect of the particles translational motion on the drag moment and the effect of the particles rotation on the drag force were evaluated from the experimental data.*

1. Introduction

The description of forces acting on a spherical particle moving in the fluid is important for many engineering disciplines, for instance in the river engineering, water treatment and gas-particle or liquid-particle flow. The slowdown of the spherical particle rotation in a liquid depends on a viscous hydrodynamic moment or so called drag moment. Similarly, the decreasing of the particle translational velocity is caused by the drag force. The drag force acting on a sphere moving translationally without rotation in a fluid is fairly well investigated; see e. g. *Vakhrushev (1965)*. *Sawatzki (1970)* described the drag moment of a rotating sphere, which does not move translationally. The influence of rotation on drag force was studied by *Maccoll (1928)*, *Barkla & Auchterlonie (1971)*, and *Tanaka et al. (1990)*. As far as the aware of the authors the effect of translational motion on drag moment was not almost investigated. The paper deals with the experimental investigation of the drag force and drag moment of the spherical particle moving translationally with simultaneous rotation in a fluid.

1. 1. Drag force

For a very slow translational motion, where Reynolds number $Re = d_p\, u_r / \nu << 1$ (d_p is spherical particle diameter, u_r is particle relative velocity, ν is fluid kinematic viscosity), the drag force of non-rotating spherical particle is given by well known Stokes law (*Landau & Lifsic, 1988*):

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$$\vec{F}_d = -6\pi\mu\, r_p \vec{u}_r\,, \tag{1}$$

where $\mu = \nu / \rho_f$ is the dynamic viscosity of fluid, ρ_f is the fluid density, r_p is the particle radius.

The value of the drag force is generally given by the following formula:

$$\vec{F}_d = -C_d \frac{\pi d_p^2}{4} \frac{\rho_f \vec{u}_r |\vec{u}_r|}{2}\,, \tag{2}$$

where C_d is a drag force coefficient. *Vakhrushev (1965)* recommends for $0 < Re < 5\text{x}10^5$ the following relationship:

$$\begin{aligned} C_d = &\left(\frac{25}{Re} - \frac{0.491}{\sqrt{Re}} + \frac{4.565}{\sqrt[3]{Re}}\right) \times \left(1 - \text{th}\, 0.00025 Re\right) + \\ &+ 0.42\, \text{th}\, 0.00025 Re + 0.02\, \text{th}\, 0.0001 Re \end{aligned} \tag{3}$$

Since for $Re << 1$ the Eqs. (2) and (3) give the Eq. (1), the Eq. (3) may be generally used for calculating the drag force F_d acting on a non rotating spherical particle for Reynolds number varying from values of creeping flow up to so called "drag crisis", when drag force coefficient decreases abruptly due to transformation of laminar boundary layer into the turbulent one, *Nigmatulin (1987)*.

If the particle moves translationally and simultaneously rotates the drag coefficient C_d depends not only on particle Reynolds number *Re,* but also on the rotational Reynolds number $Re_\omega = (\omega\, r_p^2)/\nu$, where ω is the angular velocity of the particle. The influence of particle rotation on the drag coefficient C_d was suggested by *Lukerchenko et al. (2005)* as $C_d = C_{d0}\,(1 + 0.065\, Re_\omega^{0.3})$ for the Reynolds number range $300 \le Re \le 4\text{x}10^4$ and $200 \le Re_\omega \le 4\text{x}10^4$, where C_{d0} is the drag coefficient for the particle movement without rotation, i.e. for $Re_\omega = 0$.

Tanaka et al. (1990) measured the forces acting on a rotating sphere in a wind tunnel. The experiments were conducted for the following range of parameters: $6.08\text{x}10^4 \le Re \le 1.4\text{x}10^5$, $0 \le \Gamma \le 1.3$, where $\Gamma = r_p\, \omega / u_r$ is the ratio of the surface velocity of the spherical particle to the relative particle-liquid velocity, $\Gamma = 2\, Re_\omega / Re$.

For illustration, the map of coordinates Re_ω vs. Re of the conducted experiments dealing with the drag force is presented in Figure 3.

1. 2. Drag moment

The drag moment acting on a rotating sphere in the infinite calm fluid for $Re_\omega << 1$ is given according to *Landau & Lifsic (1988)* as

$$\vec{M} = -8\pi\mu\, r_p^3 \vec{\omega}\,, \tag{4}$$

Similarly as for the drag force the value of drag moment can be generally given as

$$\vec{M} = -C_S \frac{\rho_f}{2} \vec{\omega} |\vec{\omega}| r_p^5\,, \tag{5}$$

where C_S is the dimensionless drag moment coefficient. *Sawatzki (1970)* determined experimentally the values of drag moment coefficient for a range of angular velocities given by $1 < Re_\omega < 5x10^6$, i.e. for the flow from creeping flow ($Re_\omega \rightarrow 0$) to the laminar boundary layer, through the laminar-turbulent transition regime up to the fully developed turbulent boundary layer. The results are presented in graphical form for the dimensionless coefficient of the drag moment C_S, defined by Eq. (5). The data of Sawatzki can be read from the graphs with accuracy about 3% and they are valid only for the sphere that does not move translationally. In the case of translational and rotational motion the drag moment coefficient C_S depends also on both Reynolds numbers, Re_ω and Re, *Lukerchenko et al. (2005)*.

2. Experimental procedure

The experiments were carried out in a rectangular glass vessel, which was 786 mm long, 602 mm wide and 990 mm high. The water depth was kept on the level 812 mm. The rubber, silicone, and glass spherical balls were used as model particles, see Table 1.

Table 1. The spherical particle parameters

Sphere	Mass, (±0.01g)	Volume, cm^3	Calculated diameter, cm	Density, g/cm^3	Number of trajectories
Rubber 1	10.50	9.67±0.03	2.64	1.09	8
Rubber 2	25.70	25.2±0.1	3.64	1.02	3
Silicone (Stomaflex® Solid)	70.09	36.6±0.1	4.12	1.92	10
Glass	18.72	7.58±0.03	2.44	2.47	3

The hairlines were drawn on the balls along two perimeters with the angle of 90° between them to make possible to visualise the particle rotation. Each measured particle was speeded up in a special chute ensuring the required particle rotational and translational velocity. The different initial heights of the particles at the chute were used to provide the required values of the initial translational and angular velocities of the individual particle.

The motion of the particles was recorded by a digital video camera. Video recording rate was 25 frames per second. The dimension of obtained frames was 720x576 pixels. One pixel equalled approximately 2 mm in the plane of the particle motion, the error of coordinate determination was one pixel.

During measurement of the light particles from 100 to 200 images were recorded for one trajectory. The trajectory of the dense particles consists from 10 to 20 images. From the images the Cartesian coordinates $x\,(t)$, $y\,(t)$ of the particle centre and the angle of particle rotation $\varphi(t)$ were read using the software Graph2Digit. To evaluate the particle coordinates and the angle of revolution only particle trajectory segments close to straight lines were used; the non-steady process of the particle entry into the water was rejected.

3. Numerical method

For the quasi-steady process of 2D particle motion in fluid a steady approximation of drag force and drag moment acting on a spherical particle was considered. In the equations of motion we take into account the known unsteady forces, i.e. the history force and force of added mass, which are supposed to be negligible. Under such assumption the flow around the particle and hence the forces and the moment are completely determined by the following set of parameters: ρ_f, μ, d_p, ω, u_r. Two dimensionless numbers, Re and Re_ω, can be determined from the above mentioned parameters; and both dimensionless coefficients, drag coefficient C_d and drag moment coefficient C_S depend on these two dimensionless numbers: $C_d = C_d(Re, Re_\omega)$ and $C_S = C_S(Re_\omega, Re)$, respectively

According to *Lukerchenko et al. (2005*) the equations of the particle motion are

$$\Omega\rho \frac{d\vec{u}_r}{dt} = \vec{F}_d + \vec{F}_g + \vec{F}_M + \vec{F}_B + \vec{F}_m, \tag{6}$$

$$J \frac{d\vec{\omega}}{dt} = \vec{M}, \tag{7}$$

where $J = \rho\pi\, d_p^5/60$ is the particle moment of inertia, Ω is the particle volume, ρ is the particle density, and

$$\vec{F}_g = \Omega\left(\rho - \rho_f\right)\vec{g}, \tag{8}$$

$$\vec{F}_M = C_M \Omega \rho_f \left[\vec{\omega} \times \vec{u}_r\right], \tag{9}$$

$$\vec{F}_B = -\frac{3}{2} d_p^2 \rho_f \sqrt{\pi\nu} \int_0^t \frac{d\vec{u}_r}{d\tau} \frac{d\tau}{\sqrt{t-\tau}}, \tag{10}$$

$$\vec{F}_m = -C_m \Omega \rho_f \frac{d\vec{u}_r}{dt}, \tag{11}$$

where F_g, F_M, F_B, and F_m are the gravitational submerged force, the lateral Magnus force, the Basset history force (*Mei et al., 1991*), the added mass force, respectively; $\vec{g}$ is the gravity acceleration vector, C_M is the Magnus force coefficient and $C_m = 0.5$ is the dimensionless added mass coefficient.

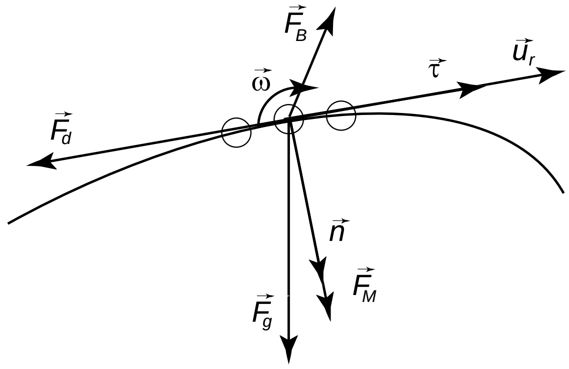

Figure 1 The forces acting on the rotating particle moving translationally in calm water.

The forces acting on a particle and their orientation are shown in Figure 1. $\vec{F}_d$ is directed oppositely to the curve tangent unit vector $\vec{\tau}$, $\vec{F}_M$ is parallel to the curve normal unit vector $\vec{n}$. The values of the coefficients C_d, and C_S can be calculated independently.

Since the scalar product of the unit vector $\vec{\tau}$ (tangential to the particle trajectory) and the Magnus force $\vec{F}_M$ equals zero, the unknown Magnus force cancels and the drag coefficient C_d can be expressed as

$$C_d = \frac{\left\{ \vec{F}_g + \vec{F}_B - \Omega\left(\rho + C_m \rho_f\right) \frac{d\vec{u}_r}{dt} \right\} \vec{\tau}}{\frac{\pi d_p^2}{4} \rho_f \frac{u_r^2}{2}} . \tag{12}$$

From Eq. (7) the drag moment coefficient C_S can be expressed:

$$C_S = -\frac{J \frac{d\omega_z}{dt}}{\frac{\rho_f}{2} \omega_z \left|\vec{\omega}\right| r_p^5} , \tag{13}$$

where ω_z is the projection of $\vec{\omega}$ on axis perpendicular to the plane of motion.

For the numerical calculation of the Basset force integral an approximate method proposed by *Brush et al. (1964)* was used:

$$\int_0^t \frac{d\vec{u}_r}{d\tau} \frac{d\tau}{\sqrt{t-\tau}} = \int_0^{t-\Delta t} \frac{d\vec{u}_r}{d\tau} \frac{d\tau}{\sqrt{t-\tau}} + \int_{t-\Delta t}^{t} \frac{d\vec{u}_r}{d\tau} \frac{d\tau}{\sqrt{t-\tau}} \approx \int_0^{t-\Delta t} \frac{d\vec{u}_r}{d\tau} \frac{d\tau}{\sqrt{t-\tau}} + 2 \frac{d\vec{u}_r}{d\tau} \sqrt{\Delta t} . \tag{14}$$

Equations (12) and (13) allow the calculation of the dimensionless coefficients for each point of recorded particle trajectory, provided that the first and the second time derivatives of the particle coordinates and of the angle of rotation are known. The experimental data *x (t)*, *y (t)* were fitted using the least square method with polynomial functions up to the third power of *t*, *φ (t)* was fitted with rational function *(a + t) / (b + ct)* and the first and the second derivatives were calculated.

The drag coefficient and drag moment coefficient were calculated numerically for each frame of the particle trajectory, except for the first two and the last two frames for which the second derivatives were not available. The corresponding values of the Reynolds number *Re* and the rotational Reynolds number Re_ω were also calculated for each frame of the relevant particle record.

The following procedure was applied to average the experimental data. The experimental area *Re* vs. Re_ω ($4.0 \times 10^3 < Re < 4.0 \times 10^4$ and $2.0 \times 10^3 < Re_\omega < 4.0 \times 10^4$) was split into 30x30 cells, whose dimensions grow according to the geometric series. The use of geometric series for the length and width of a cell along *Re* and Re_ω axes makes the cells look uniform in logarithmic coordinates. For a cell, where at least four data points exist, C_d and C_S were calculated as an arithmetic mean of all data points in the cell. In most cases a cell comprised points from more than one trajectory. The positions of the individual cells were represented by the value of *Re* and Re_ω, which are the geometric mean of the values on the responsible boundary. The experimental data and the cells are illustrated in Figure 2.

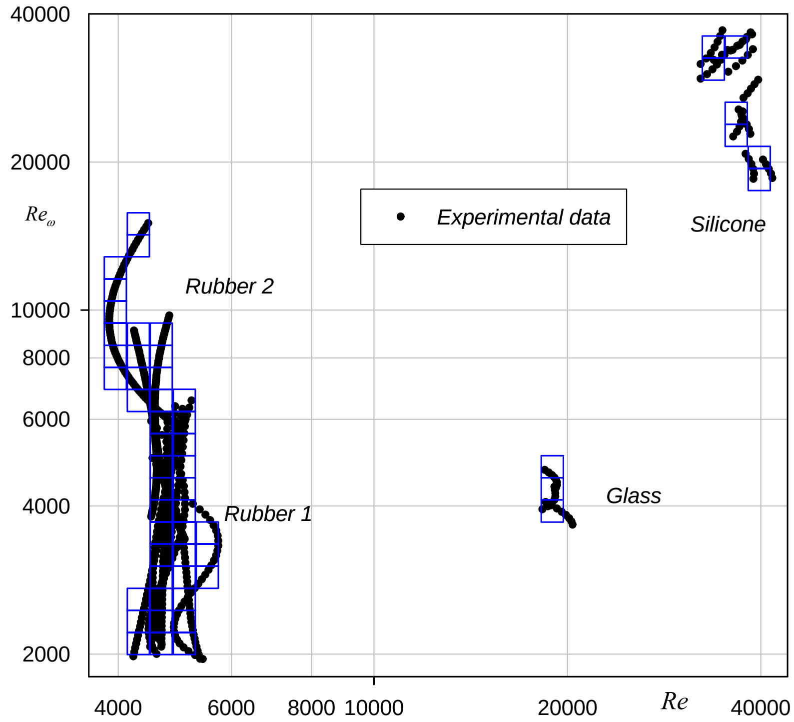

Figure 2 The Re_ω vs. Re map of the experimental data and the cells.

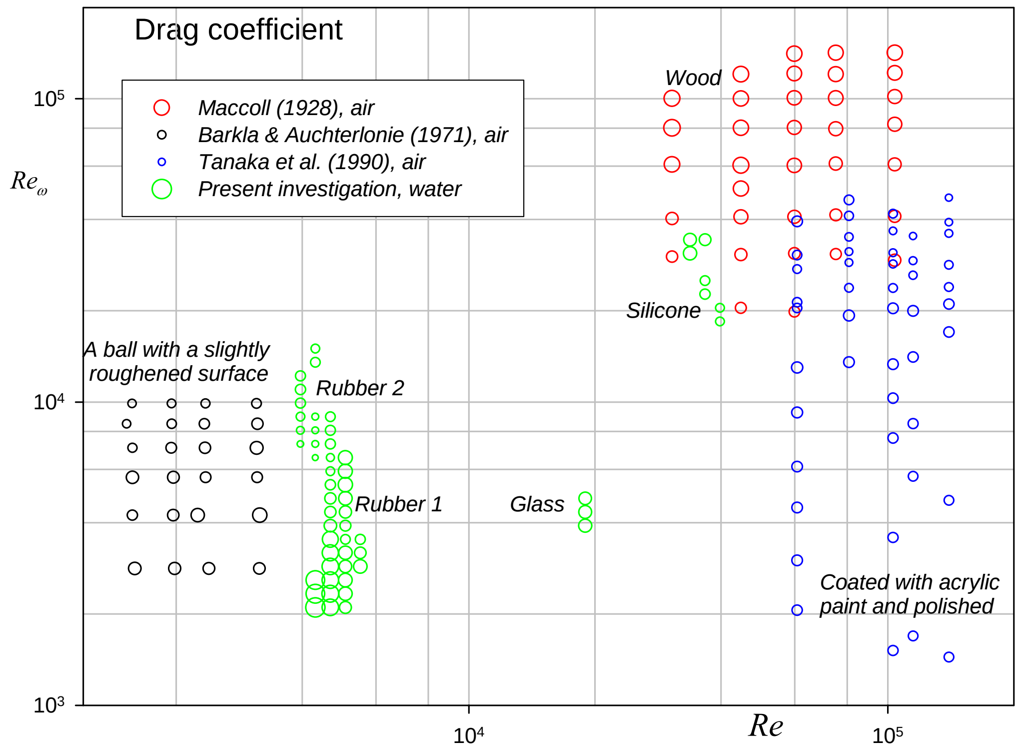

Figure 3 The bubble plot of drag force coefficient. The radius of a bubble is proportional to drag coefficient.

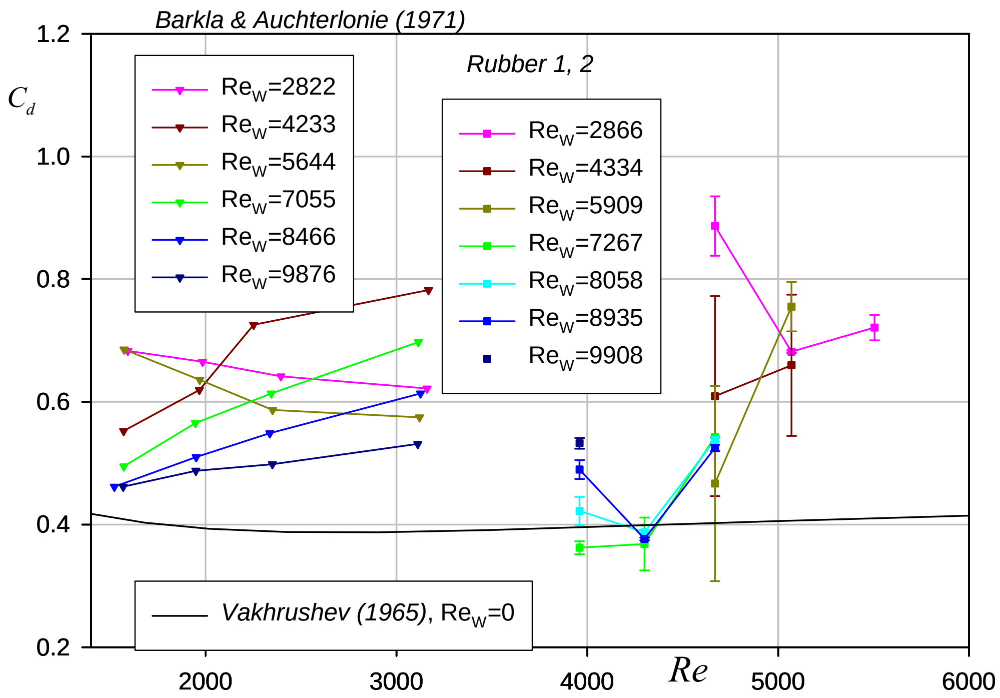

Figure 4 The comparison of the calculated drag coefficient with *Barkla & Auchterlonie (1971)* data.

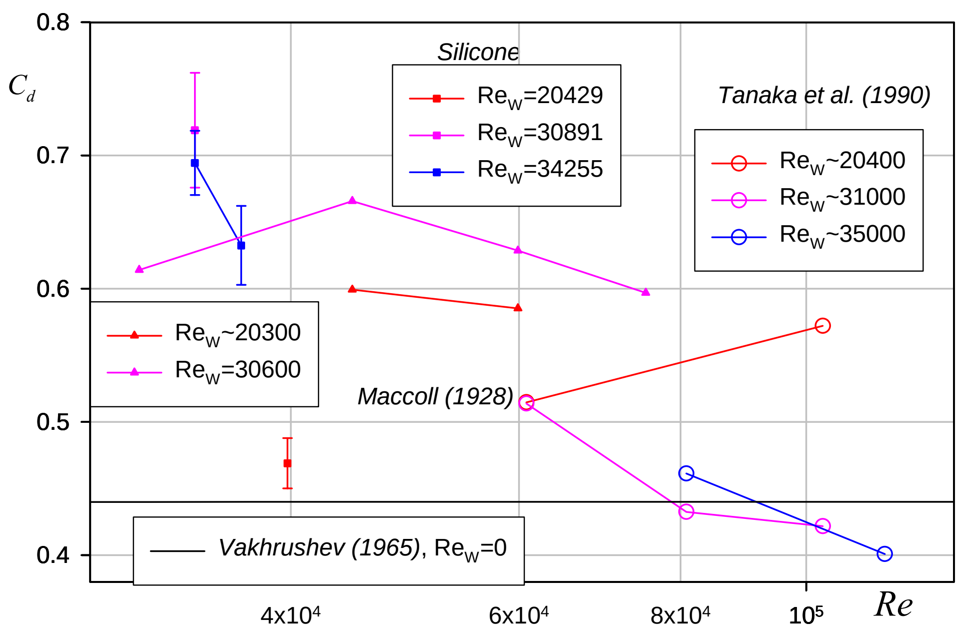

Figure 5 The comparison of the calculated drag coefficient with *Maccoll (1928)* and *Tanaka et al. (1990)* data.

4. Results and discussion

4. 1. Drag coefficient

The bubble plot of the drag coefficient in the area Re_ω vs. Re is presented in Figure 3. The comparison of the chosen drag coefficient data of Rubber 1 and Rubber 2 particles with the data of *Barkla & Auchterlonie (1971)* is presented in Figure 4. Only those points are plotted in the Figure 4, which have approximately the same value of Re_ω and can mutually be compared. The comparison of the drag coefficient data of silicone ball with the data of *Maccoll (1928)* and *Tanaka et al. (1990)* is presented in Figure 5.

The rotation of the particle increases the drag coefficient. However, some results of *Tanaka et al. (1990)* show a drag coefficient lower than that without rotation, especially at the high rotational and translational Reynolds numbers. Several points of our results also show the drag coefficient lower than that without rotation; it does not comply well with adjacent points of *Barkla & Auchterlonie (1971)*.

Influence of the rotation movement on a drag coefficient is presented in Figure 6. The relationship $C_d = C_d\,(Re, Re_\omega)$ does not allow a separation of the individual variables Re and Re_ω effect, thus the Figure 6 shows only the relationship $C_d / C_{d0} = f\,(Re_\omega)$, which is only a rough approximation.

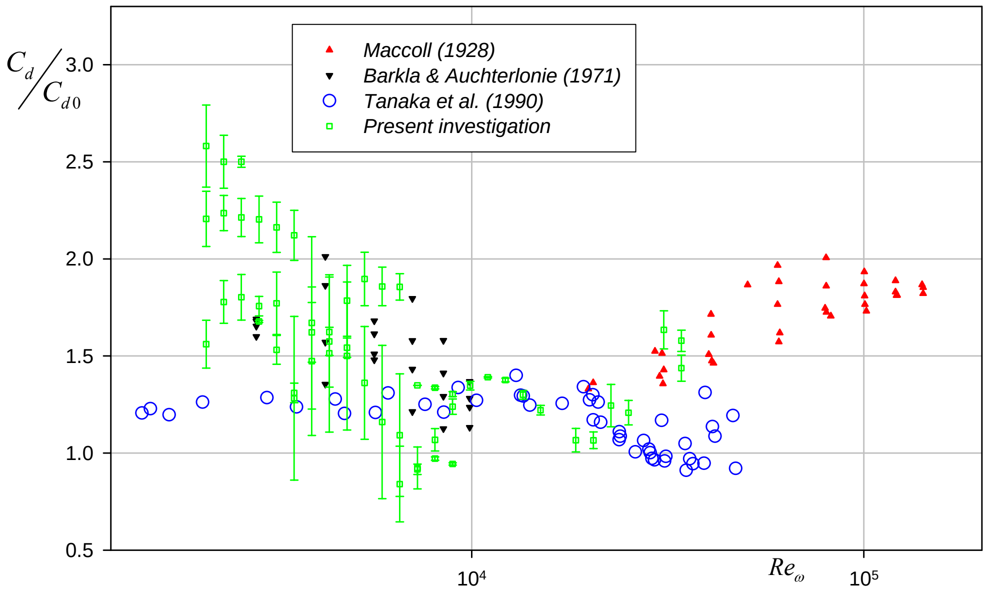

Figure 6 Influence of rotational motion on the relative drag coefficient C_d / C_{d0}.

4. 2. Drag moment coefficient.

Relationship between the drag moment coefficient C_s and Re_ω is presented in Figures 7 and 8. The ratio C_S / C_{S0} versus Re is presented in Figure 9. The general tendency can be formulated - with increasing value of Re the drag moment coefficient C_S also increases if the rotational Reynolds number $Re_\omega > 5 \times 10^3$. For the lower Reynolds number the opposite is valid. However, the available experimental data are insufficient to make general conclusions.

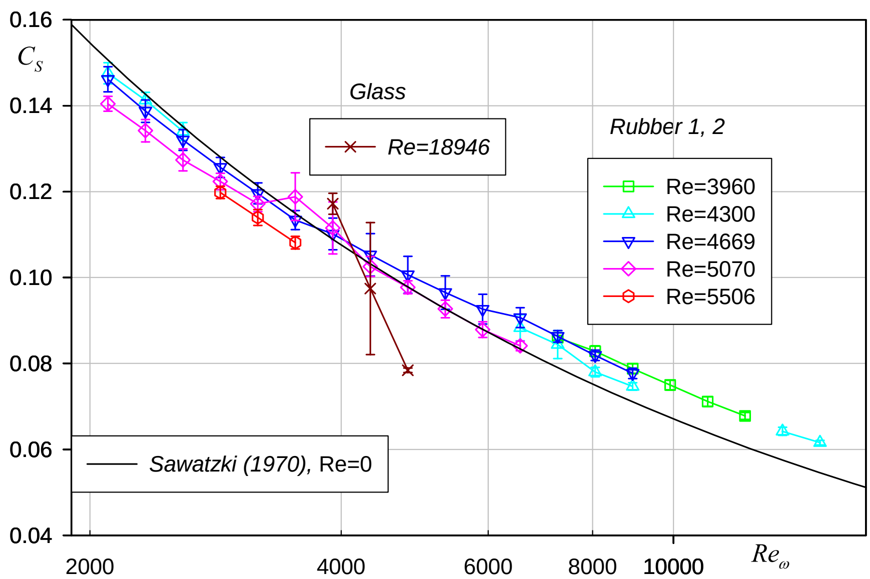

Figure 7 Drag moment coefficients C_S versus rotational Reynolds number Re_ω. Comparison with *Sawatzki (1970)* data (for only rotation) - Rubber 1, 2, Glass.

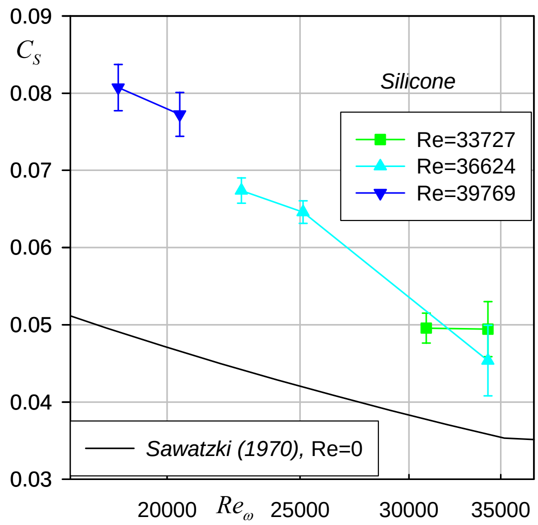

Figure 8. Drag moment coefficients C_S versus rotational Reynolds number Re_ω, comparison with *Sawatzki (1970)* data. Silicone ball.

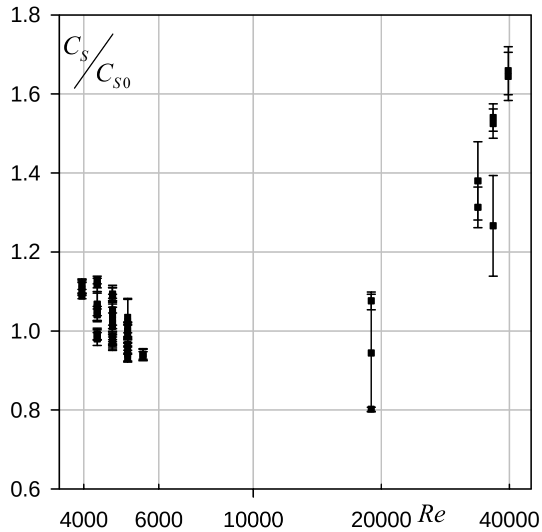

Figure 9. The relative drag moment coefficient C_S / C_{S0} versus Re_ω. Influence of translational motion on the drag moment coefficient.

5. Conclusions

Experimental investigation of the drag force, drag moment and its dimensionless coefficients of the spherical particle moving translationally and simultaneously rotating in calm water were conducted.

The drag coefficient was found to be in satisfactory agreement with the drag coefficient measured by other authors in the relevant ranges of Reynolds numbers. The drag moment coefficient was compared only with available data for the stationary rotating sphere.

Effects of translational motion on the drag moment coefficient and of the rotational motion on the drag coefficient were observed. It was found that translational motion generally increases the drag moment coefficient and the rotation of the ball generally increases the drag coefficient.

With increasing value of particle Reynolds number Re the drag moment coefficient C_S also increases if rotational Reynolds number $Re_\omega > 5 \times 10^3$. For the lower rotational Reynolds number the opposite effect was observed.

However, the available experimental data are sufficient to make only preliminary conclusions.

6. Acknowledgements

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7. Notation

C_d - drag force coefficient;
C_{d0} - drag force coefficient, $Re_\omega = 0$;
C_m - added mass coefficient;
C_M - Magnus force coefficient;
C_S - drag moment coefficient;
C_{S0} - drag moment coefficient, $Re = 0$;
d_p - spherical particle diameter;
$\vec{F}_B$ - Basset force;
$\vec{F}_d$ - drag force;
$\vec{F}_g$ - submerged gravitational force;
$\vec{F}_m$ - added mass force;
$\vec{F}_M$ - Magnus force;
$\vec{g}$ - gravity acceleration;
J - particle moment of inertia;
$\vec{M}$ - drag moment
$\vec{n}$ - unit vector normal to the trajectory;
Re - particle Reynolds number;
Re_ω - rotational particle Reynolds number;
r_p - spherical particle radius;
u_r - particle-liquid relative velocity;
$x(t)$, $y(t)$ - coordinates of particle trajectory;
$\varphi(t)$ - particle angle of rotation;
Γ - ratio of the spherical particle surface velocity to the particle-liquid relative velocity;
μ - fluid dynamic viscosity;
ν - fluid kinematic viscosity;
ρ - particle density;
ρ_f - fluid density;
Ω - particle volume;
$\vec{\tau}$ - unit vector tangential to the trajectory;
ω - angular velocity of sphere;

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