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NUMERICAL SIMULATION AND EXPERIMENTAL INVESTIGATION OF PULSATING FLOW

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Summary: *The contribution presents numerical as well as experimental investigations of laminar and turbulent pulsating flow in an initial part of a rigid pipe. The simulations were performed with help of Fluent 6.3. Measurements of velocity profiles were done by an ultrasound velocity monitor UVP.*

1. Introduction

The primary task of the investigation was to compare different experimental techniques for unsteady flow measurements. Originally we supposed to use an experimental facility which was at disposal in a laboratory of IH where a stationary as well as a pulsatile flow could be studied. Unfortunately after preliminary tests on the original facility it was clear that the flow field is too disturbed even when a laminar flow regime was tested. It was necessary to rebuild the system and therefore the primary task was delayed. In this paper a set of first measurements is presented along with numerical simulations.

Pulsatile flow which is composed of a mean and oscillating component is investigated for a long time. Analytical solution of laminar flow mostly supposes that a time variation of a pressure gradient is known a priori as an initial input parameter. Recently, Unsal et al. (2005), presented a solution for a mass flow pulsation. Their solution is based on a complex variable, ψ, which is a function of non-dimensional frequency F.

$$F = \frac{R^2 f}{\nu} = \frac{\alpha^2}{2\pi} \tag{1}$$

$$\psi(F) = -\frac{4}{\pi F}\left\{1 + \frac{2i^{1/2} J_1\left[(2\pi F)^{1/2} i^{3/2}\right]}{(2\pi F)^{1/2} J_0\left[(2\pi F)^{1/2} i^{3/2}\right]}\right\} \tag{2}$$

where R is the pipe radius, f is the frequency, ν is viscosity, α is the Womersley parameter, J_0 and J_1 are the Bessel functions.

Separating the complex variable into real and imaginary parts the analytical solution for mass flow rate through pipe is obtained in the form

$$\frac{Q}{Q_s} = 1 + P^* |\psi| \sin\left[2\pi F - \tan^{-1}\left|\frac{\mathrm{Re}(\psi)}{Im(\psi)}\right|\right] = 1 + \frac{Q_{os}}{Q_s}\sin(2\pi F\tau - \Delta\Theta) \tag{3}$$

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where Q is the total discharge, Q_s is the stationary part, Q_{os} is the oscillating part, P^* is the non-dimensional pressure gradient, Re(ψ) and Im(ψ) are real and imaginary parts, ΔΘ is the phase shift and $|\psi|$ is ratio of amplitude of the mass flow rate and the pressure gradient.

$$|\psi| = \left[\mathrm{Re}(\psi)^2 + \mathrm{Im}(\psi)^2\right]^{1/2}$$
$$\Delta\Theta = \tan^{-1}\frac{\mathrm{Re}(\psi)}{\mathrm{Im}(\psi)} \tag{4}$$

In the following parts of the contribution a comparison between the analytical solution and experimental data will be presented.

Also we focus on the velocity measurements of the pulastile flow in an initial part of a pipe. In a review paper of Gundogdu & Carpinlioglu (1999) is one of the conclusion notes a call to determine a development length required to obtain a fully developed laminar pulsatile pipe flow. In the case of a steady laminar flow Durst et al. (2005) summarized available literature data and derived a relationship for the development length in the pipe flow

$$\frac{L}{D} = \left[(0.619)^{1.6} + (0.0567\,\mathrm{Re})^{1.6}\right]^{1/1.6} \tag{5}$$

where L is the pipe length, D is the pipe diameter and Re is Reynolds number.

2. Experimental set-up

The schematic view of the experimental facility is shown in Fig. 1. The system consists of

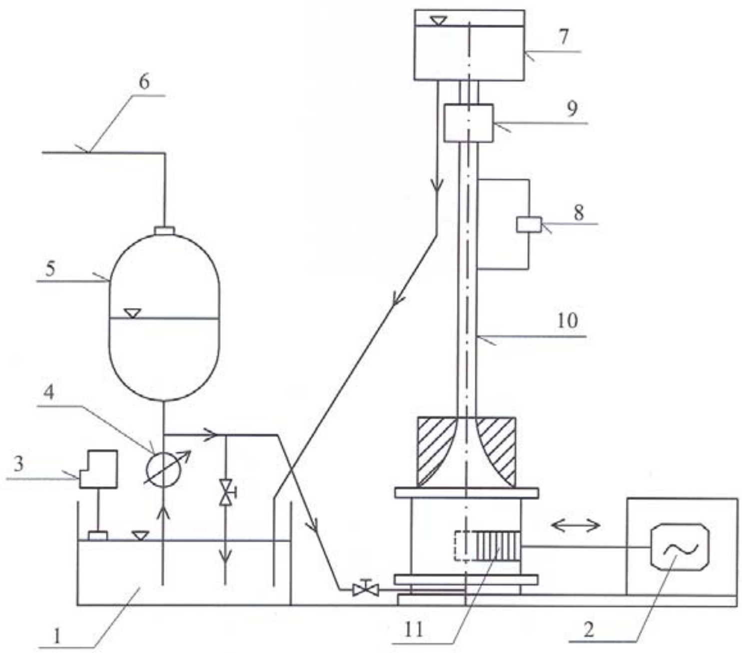

Figure 1. Schematic view of the experimental arrangement

a collecting box (1), a sinusoidal mechanism (2) with bellows (11), a float switch (3), a pump (4), a pressurized tank (5), an electromagnetic flow meter (9), a differential pressure transducer Validyne (8). Vertical pipe (10) of inner diameter 0.02 m is made from Plexiglas. Due to the space limit the total length of the pipe is only 1.25 m. The inlet part of the pipe is equipped with a smooth nozzle with area ratio equals 16. A pressure air (6) keeps constant conditions in the tank (5) to obtain the steady flow. The oscillating component of flow discharge is supplied from the sinusoidal mechanism, which alters a volume of bellows (11) in an input section. A stroke of the sinusoidal mechanism is adjustable and during the experiments two values were used: z = 5 mm and 13.4 mm, respectively.

The velocity profiles were measured by an ultrasound technique – UVP (ultrasonic velocity profile-meter). Principle of the UVP is based on the Doppler effect. After transmission of a short ultra sound pulse of given frequency, the UVP transducer receives echoes from particles suspended in the water stream. The transducer is able to measure simultaneously velocities in 128 points along the axis of the transducer. Three transducers of working frequency 4 MHz were used for the measurements of velocity field along the pipe. The transducers were placed outside the pipe and they were inclined from the flow direction by angles 70^o. Depending of the frequency of the pulsatile flow up to 2000 velocity profiles were picked up and stored on PC for subsequent processing. The transducers were placed at the following non-dimensional distances from the pipe inlet x/D = 6.5, 35.5 and 58, respectively.

The numerical simulations were performed with help of the program Fluent 6.3. The total length of the pipe for the numerical simulations was 3 m, diameter 0.02 m. The mesh size varied from 0.1 mm near the wall to 1 mm in the pipe centre. The computational domain was modelled as an axisymmetric (2D) case. Laminar solver and unsteady flow conditions were applied. A second-order upwind scheme was employed for discretization in the momentum equation and the pressure-velocity coupling was based on the Simplec method. In order to find a suitable time step several tests of a pulsatile flow were examined. It was proved that there exists a limit of the time step (t = 0.001 sec) below which the results were practically independent on the time step value. For higher values of the time step the results were modified mainly in the region near the wall. For the time derivates a second-order discretization was used. The boundary conditions were a given velocity waveform at the pipe inlet, pressure outlet and no slip conditions on the wall. Simulations were performed on an IBM working station with 4 processors Power 5+ and 32 GB RAM.

Hydraulic conditions of the present study are summarized in Table 1 where Re_S means Reynolds number based on values of the time averaged mean velocity.

Table 1.

steady flow
Re_S = 1000-5000
pulsatile flow
Re_S = 1000; 1500; 2000; 3000; 3900
$\alpha = R\sqrt{2\pi f / \nu}$ = 6.8-11.8
$\lambda = Q_{os} / Q_s$ = 0.36-3.1

3. Results and discussion

From the analytical solution given by the equations (1)-(4) it is evident that the phase shift $\Delta\Theta$ is a function of the parameter F. In Fig. 2 there is a plot of the measured data for the different parameter F and the hydraulic conditions. The phase shift was determined by a fitting of a sinusoidal function through the experimental data. For the laminar condition of the steady flow (Q_s<= 2 l/min ~ Re<=2000) the experimental data follow the analytical solution up to F~15 (α~ 9.7). In the case of Q_s= 3 l/min (~ Re=3000) the phase shift for lower values of the parameter F is considerably below the analytical curve but with increasing F approaches the analytical solution. For higher values of the parameter F all data significantly deviate from the analytical solution. This deviation roughly corresponds with a sharp increase of the mean pressure gradient. In Fig. 3 there is a plot of a dependence of the mean pressure gradient on the parameter F. There is evident a sudden change of the pressure gradient which indicates a possible transition from laminar to turbulent flow regime.

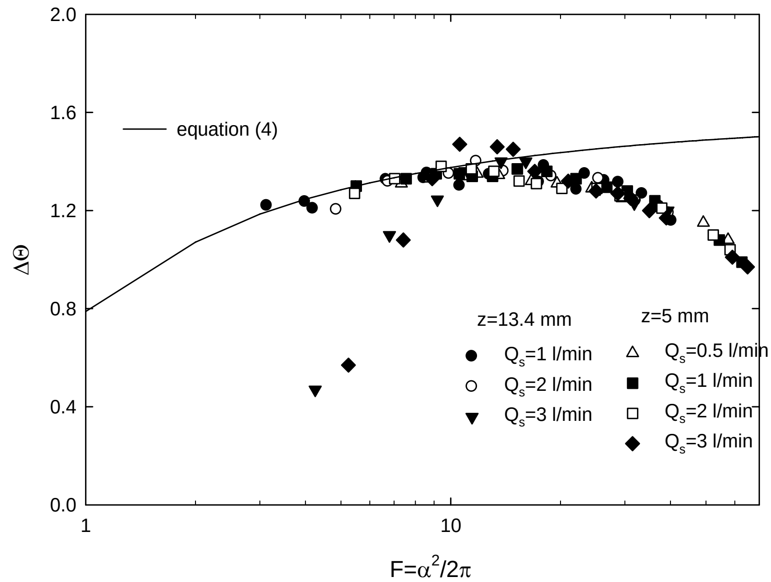

Figure 2. Dependence of phase shift on the parameter *F*

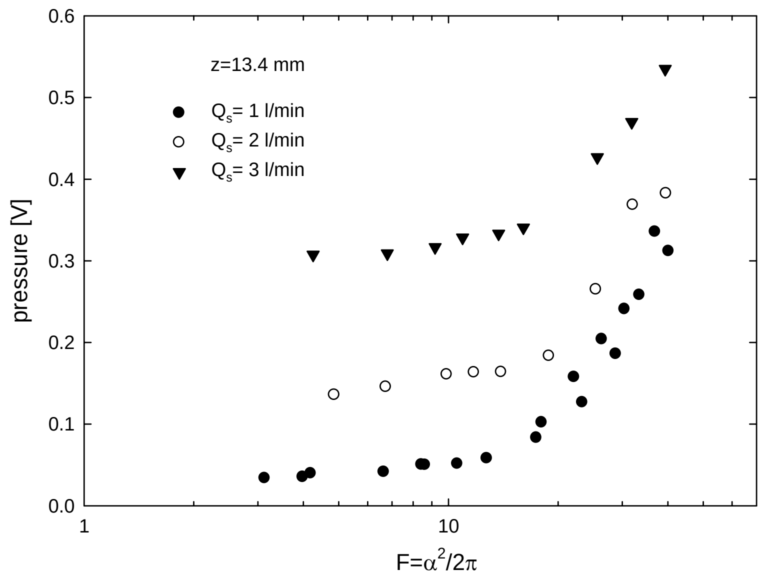

Figure 3. Dependence of time averaged pressure gradient on the parameter *F*

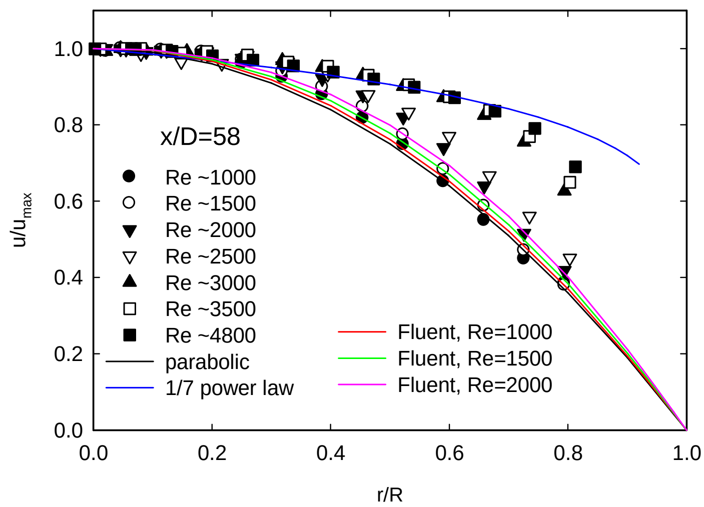

Figure 4. Velocity distribution of the steady pipe flow

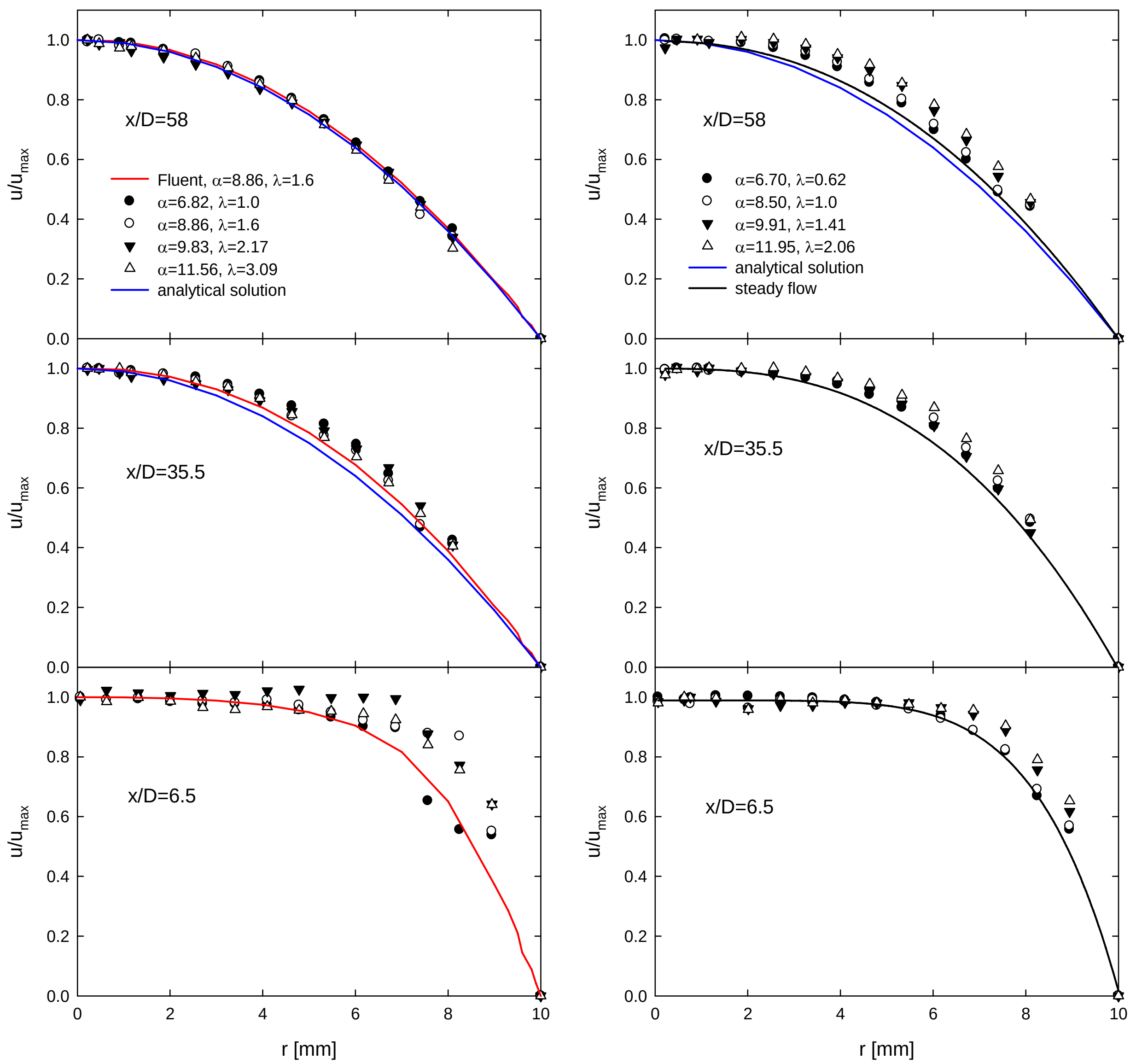

Figure 5. Velocity distribution of time averaged pulsatile flow -left Re_s=*1000*, right Re_s=*1500*

As was mentioned above the profiles of the axial velocity component were measured by the UVP method. Profiles of the steady flows for various Reynolds numbers measured at the distance x/D=58 from pipe inlet are shown in Fig. 4. As can be seen in the figure the velocity profiles follow the parabolic shape for lower values of Reynolds number. For Re about 2000-2500 there is a remarkable deviation from the parabolic shape but still the profiles can be considered as parabolic with an exponent higher than 2. The velocity profiles for Re > 3000 follow the 1/7 power law typical for turbulent flows. Numerical simulations of the steady flow with Reynolds numbers of 1000-2000 show similar tendency but the deviation from the parabolic shape is less.

Time averaged velocity distributions of the pulsatile flow are shown in Fig. 5. Left hand side of the figure shows the profiles for Re_s=1000, on the right there are the profiles for Re_s=1500 for various combinations of parameters α and λ. Numerical simulation was carried out for Re_s = 1000, α=8.86 and λ=1.6. While the time averaged velocity profile seems to be fully developed at the distance x/D=58 for Reynolds number Re_s=1000, for Re_s =1500 the velocity profiles are more flat compare with the velocity distribution of the steady flow. It indicates that the pulsatile flow needs a longer initial length to be fully developed. In Fig. 6 there is a plot of the amplitude of the oscillating part. Numerical simulation is in a good

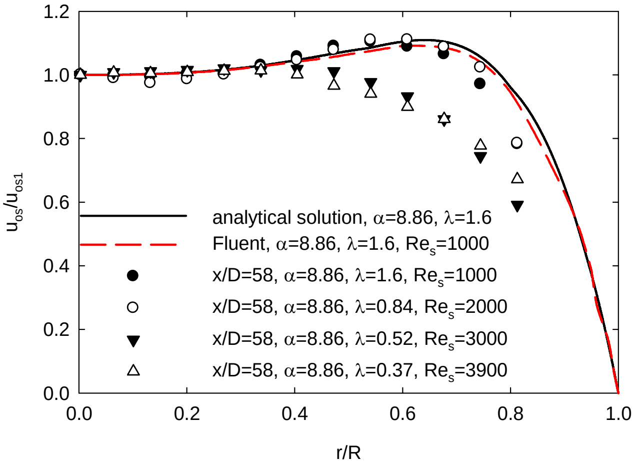

Figure 6. Amplitude of the oscillatory component

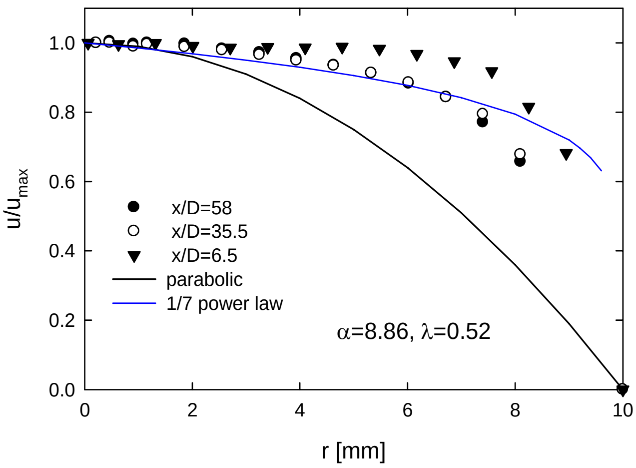

Figure 7. Velocity distribution of the turbulent pulsatile flow, Re_s=3900

agreement with the analytical solution. The experimental data for Re_s=1000 and 2000 follow the theoretical line for laminar regime but the data for Re_s=3000 and 3900 indicate an intermediate state between laminar and turbulent regimes. Also the velocity distribution for Re_s=3900 approaches rather the 1/7 power law than parabolic shape as can be seen in Fig. 7 and the profiles are practically the same as they were observed in the steady flow. Although Ohmi et al. (1982) reported that the laminar flow behavior was observed in a pulsatile flow for much higher values of Re_s than is the critical value (~2300) results of our experiments do not confirm this statement. This discrepancy may be explained by a geometry of the inlet part of the pipe.

Measured and simulated velocity data were used to estimate a development length of both the steady and the pulsatile flows. The results are shown in Fig. 8 together with the numerical simulations. The numerical results are independent of the flow type (steady or pulsatile). According to the equation (5) the steady laminar pipe flow should be fully developed in the distance x/(D.Re)=0.0567 and this is in an agreement with the numerical results. The experimental data of the steady flow roughly correspond with the numerical simulation. But in the case of the pulsatile flow it seems that the development length is about 20% longer than in the steady case.

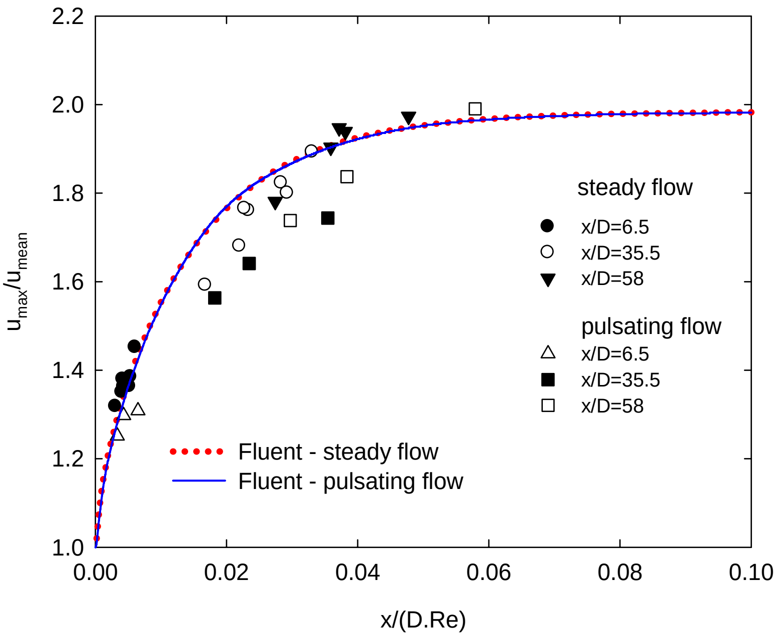

Figure 8. Development of the centre-line velocity along the pipe axis

4. Conclusion

The contribution presents the results of the steady and pulsatile flow in a beginning part of the pipe. The phase shift between the flow rate pulsation and pressure gradient remarkably deviates from the analytical solution for higher values of the parameter F. This deviation can be attributed to a laminar turbulent transition. A shape of the velocity profile typical for the turbulent flow was observed for Re_s >= 3000. The development length of the pulsatile flow is longer than in the steady flow.

5. Acknowledgement

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6. References

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