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ARTICULAR CARTILAGE INSTANTANEOUS LOADING RESPONSE

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Summary: *Articular cartilage is often modeled as a biphasic mixture (a solid porous matrix swollen by a fluid that may move slowly through the matrix pores). The anisotropic matrix is often assumed to be linearly elastic and the fluid to be inviscid, both being intrinsically incompressible. The viscous dissipation of the model is due to the frictional drag of the fluid through the matrix pores. The creep and stress relaxations experiments yield the equilibrium compliance of the tissue. At equilibrium the fluid pressure tends to zero and all the load is carried only by the matrix. However, for suddenly applied or quickly changing loads, the mixture behaves as a single-phase incompressible material. The fluid is pressurized, carries most of the load and makes the mixture stiffer. For small strains in the matrix, the paper presents this instantaneous compliance once the equilibrium compliance is known.*

1. Introduction

Articular cartilage forms a thin layer on the surfaces of synovial joints and serves as a bearing and shock absorbing material. Articular cartilage is composed of a relatively small amount of cells and a large amount of an intercellular composite solid matrix (mostly proteoglycans and collagen fibrils) swollen by water. A part of this water is free and can be expelled upon compaction of the solid porous phase.

A natural mathematical approach to articular cartilage is through a biphasic mixture (a porous, permeable, solid matrix with a fluid flowing through it). For example, a successful theoretical biphasic model of articular cartilage has been extensively studied by Mow and co-workers: Mow et al. (1980), Armstrong et al. (1984), Mak et al. (1987, 1989). Their model can describe both the equilibrium and kinetic response of the tissue. Both the elastic solid matrix and the interstitial fluid are immiscible and intrinsically incompressible. The fluid (assumed to be inviscid for simplicity) may be moved through the tissue by a pressure

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gradient. The viscous dissipation of the cartilage tissue is dominated by the frictional drag of the fluid through the porous permeable collagen-proteoglycan solid matrix. This model and its extensions have been validated experimentally for small strains of the matrix. Some commonly used experiments (steady direct permeability experiment (Mow et al., 1980), transient confined and unconfined creep and stress-relaxation experiments (Armstrong et al., 1984), indentation creep (Mak et al., 1987, 1989)) have been made and compared with the mathematical solutions to obtain material parameters, i. e. the equilibrium elastic compliance of the matrix and permeability. In creep or stress relaxation experiments the fluid is partly expelled of the tissue, the fluid pressure in the pores tends to zero and at equilibrium all the load is carried by the matrix. The equilibrium compliances in tension and compression differ considerably (Akizuki et al. (1986), Cohen et al. (1993)) as different parts of the matrix respond to tension and to compression. While collagen fibers are stiff in tension, hydrophilic proteoglycan aggregates contribute to the compressive stiffness. However, non-linear bi-modulus models would greatly complicate the analysis. As the matrix compression is predominantly perpendicular and the extension parallel to the articular surface, linear transversely isotropic or orthotropic (and even homogeneous) models agree more closely with the observed sites of cartilage failure (Donzelli et al., 1999) than isotropic models do. These anisotropic models, approximating the tension-compression nonlinearity, predict not only high shear stress at the subchondral-bone interface and separation of the cartilage layer, but are also consistent with lesions observed at the articular surface and provide better curve fitting data of cartilage early response indentation data (Cohen et al., 1993).

In a short time span, due to a low matrix permeability, the interstitial fluid is pressurized and makes the mixture stiffer for suddenly or quickly changing loads. The purpose of this paper is to find theoretically the instantaneous response of the above biphasic model to a suddenly applied load if the equilibrium compliances are known. In fact, at the time of the load application, the model behaves as a single-phase incompressible elastic material. Its compliance at that time will be expressed with the help of the equilibrium compliance.

2. Orthotropy of the matrix

The equilibrium and continuity equations of the biphasic model in question (with the incompressible phases) are of the form (Mow et al., 1980)

$$\sigma_{ij,j} = 0, \quad (k_{ij} p_{,j})_{,i} = D_t u_{k,k}, \text{ with } \sigma_{ij} = -p\delta_{ij} + \sigma'_{ij}. \tag{1}$$

Eqs. $(1)_1$, $(1)_2$, $(1)_3$ express, respectively, the stress equilibrium (for zero body and inertial forces), the flow rate balance for a volume element and the sum of the partial stresses. The Cartesian coordinates x_i are used and $i, j, k = 1, 2, 3$. Summation over 1, 2, 3 is assumed for the repeated pairs of subscripts i, j, k. A comma followed by a subscript indicates the partial derivative with respect to the corresponding coordinate. D_t denotes the material time derivative. Symbols $\sigma_{ij}, \sigma'_{ij}, p, u_i, k_{ij}$ and δ_{ij} denote the total stress tensor, effective (or elastic) stress tensor (that is due to matrix deformation), interstitial fluid pressure, matrix displacement vector, matrix permeability tensor and unit tensor, respectively. The matrix small deformation tensor is $\varepsilon_{ij} = (u_{i,j} + u_{j,i})/2$. Tensors $\sigma_{ij}, \sigma'_{ij}, \delta_{ij}, \varepsilon_{ij}, k_{ij}$ are symmetric.

The matrix is assumed orthotropic and Cartesian coordinates x_1, x_2, x_3 are chosen along the orthotropy axes. Tensors ε_{ij} and σ'_{ij} are joined in the orthotropic Hooke law as

$$\begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix} \cdot \begin{bmatrix} \sigma'_{11} \\ \sigma'_{22} \\ \sigma'_{33} \end{bmatrix}, \tag{2}$$

$$\varepsilon_{12} = (1/2\mu_{12})\sigma'_{12}, \quad \varepsilon_{13} = (1/2\mu_{13})\sigma'_{13}, \quad \varepsilon_{23} = (1/2\mu_{23})\sigma'_{23} \ ,$$

where

$$c_{\alpha\alpha} = 1/E_\alpha, \quad c_{\alpha\beta} = -\nu_{\beta\alpha}/E_\beta, \quad \alpha \neq \beta \,. \tag{3}$$

Here and in what follows, $\alpha, \beta, \gamma = 1, 2, 3$ and, in contrast to i, j, k, no summation is made over pairs of repeated subscripts α, β, γ. Here, $\nu_{\alpha\beta} = -\varepsilon_{\beta\beta}/\varepsilon_{\alpha\alpha}$ $(\alpha \neq \beta)$, $E_\alpha = \sigma'_{\alpha\alpha}/\varepsilon_{\alpha\alpha}$ and $\mu_{12}, \mu_{13}, \mu_{23}$ are the equilibrium Poisson ratios, equilibrium Young moduli with the strains due to the uni-axial stress $\sigma'_{\alpha\alpha}$, and shear moduli, respectively. If $p \to 0$ for time $t \to \infty$ for a stationary loading, then Eq. $(1)_3$ yields $\sigma_{ij} \to \sigma'_{ij}$. Thus, tensor c_{ij} in Eq. $(2)_1$ defines the equilibrium compliance. The symmetry condition $c_{ij} = c_{ji}$ (due to the existence of the elastic potential) yields

$$\nu_{\beta\alpha}/E_\beta = \nu_{\alpha\beta}/E_\alpha \,, \ \ \alpha \neq \beta \,. \tag{4}$$

It follows from Eq. (4) that only three of the six Poisson ratios are independent. The determinant of c_{ij} in Eq. $(2)_1$ is assumed to be different from zero, and thus, system $(2)_1$ can be inverted. The form of this inversion is omitted here. There are nine independent material equilibrium constants of the mixture, here called equilibrium compliances, i. e. E_k, three of the six equilibrium Poisson ratios ν_{ij} and μ_{12}, μ_{13}, μ_{23}.

Assume that a biphasic body consolidates under a stationary load suddenly applied at the boundary at time $t = 0$. Let $p = 0$ for $t > 0$ at the boundary, i. e. the interstitial fluid can freely pass the outer boundary. However, it holds that $p \neq 0$ at $t = 0$ there. After a sufficiently long time, pressure p (present in both phases) should tend to zero inside the body (if the pores are interconnected), and components ε_{ij} tend in time to fixed values, assumed here to be small compared with unit. At that time, the matrix already bears the whole load. Consider now a unit cube cut off the material. Apply various stationary loads σ_{ij} at its walls and measure all respective consolidated values of ε_{ij}. Assume that for any number of measurements, Eqs. (2) (with $\sigma_{ij} = \sigma'_{ij}$) yield a unique set of nine equilibrium compliances.

Now, consider a general load, applied suddenly on a biphasic body and then varying with time. For two intrinsically incompressible phases (the fluid in the pores and the solid phase of the pore walls), which is the present case, the mixture behaves as an incompressible single-phase elastic solid at the time of the load application (Armstrong et al., 1984). In fact, at that moment, though the pressure gradient and the fluid flow rate are non-zero in the body, the fluid flow through the pores and the boundary is still zero. Both the pores and the matrix deform, but their volume becomes still unchanged. It is the purpose of this note to calculate the compliances (here called instantaneous) of the above single-phase incompressible material (at the time of the load application) using the equilibrium compliances.

See Pipkin (1976) as to the theory of internal constraints in linear elasticity. In what follows, the quantities with a bar refer to the single-phase incompressible material (with $\bar{\varepsilon}_{ii}=0$). For this material, total stress $\bar{\sigma}_{ij}$ is the sum of the elastic stress, $\bar{\sigma}'_{ij}$, and the reaction to the constraint, $-\bar{p}\delta_{ij}$, where $\bar{p}$ is called the hydrostatic pressure:

$$\bar{\sigma}_{ij}=-\bar{p}\delta_{ij}+\bar{\sigma}'_{ij}. \tag{5}$$

The part $-\bar{p}\delta_{ij}$ does not contribute to the strain. Note that Eq. (5) is of the same form as Eq. $(1)_3$. The counterparts of Eqs. $(2)_1$, (3)-(4) for the incompressible material are of the form

$$\begin{bmatrix}\bar{\varepsilon}_{11}\\ \bar{\varepsilon}_{22}\\ \bar{\varepsilon}_{33}\end{bmatrix}=\begin{bmatrix}\bar{c}_{11} & \bar{c}_{12} & \bar{c}_{13}\\ \bar{c}_{21} & \bar{c}_{22} & \bar{c}_{23}\\ \bar{c}_{31} & \bar{c}_{32} & \bar{c}_{33}\end{bmatrix}\cdot\begin{bmatrix}\bar{\sigma}_{11}\\ \bar{\sigma}_{22}\\ \bar{\sigma}_{33}\end{bmatrix}, \tag{2a}$$

with

$$\bar{c}_{\alpha\alpha}=1/\bar{E}_{\alpha},\quad \bar{c}_{\alpha\beta}=-\bar{\nu}_{\beta\alpha}/\bar{E}_{\beta},\quad \alpha\neq\beta, \tag{3a}$$

and

$$\bar{\nu}_{\beta\alpha}/\bar{E}_{\beta}=\bar{\nu}_{\alpha\beta}/\bar{E}_{\alpha},\ \ \alpha\neq\beta. \tag{4a}$$

$\bar{E}_k,\bar{\nu}_{ij}$ refer to the instantaneous parameters of the single-phase incompressible material and are here called the instantaneous compliances. As the fluid transport is connected with a change in the volume of the matrix pores and not with matrix shear, the shear moduli of both media are the same ($\mu_{12}=\bar{\mu}_{12}$, etc.).

Summing up three Eqs. (2a) and using the incompressibility condition $\bar{\varepsilon}_{ii}=0$ gives for a single $\bar{\sigma}_{\alpha\alpha}\neq 0$ with the use of Eqs. (3a), (4a) three conditions

$$\bar{\nu}_{12}+\bar{\nu}_{13}=1,\quad \bar{\nu}_{21}+\bar{\nu}_{23}=1,\quad \bar{\nu}_{31}+\bar{\nu}_{32}=1. \tag{6}$$

Eqs. (4a) and (6) yield $\bar{\nu}_{ij}$ as

$$\bar{\nu}_{\alpha\beta}=\frac{\bar{E}_{\gamma}(\bar{E}_{\alpha}+\bar{E}_{\beta})-\bar{E}_{\alpha}\bar{E}_{\beta}}{2\bar{E}_{\beta}\bar{E}_{\gamma}}. \tag{7}$$

Here α,β,γ take, respectively, the values 1, 2, 3 and their all (not only cyclic) permutations.

Both models (biphasic and single-phase) become equivalent at the time of the load application if the stresses $\sigma_{\alpha\alpha}=\bar{\sigma}_{\alpha\alpha}$ cause the same pressures $p=\bar{p}$ and strains $\varepsilon_{\alpha\alpha}=\bar{\varepsilon}_{\alpha\alpha}$ at that time. Thus, let us set $\varepsilon_{\alpha\alpha}=\bar{\varepsilon}_{\alpha\alpha}$, $\sigma_{\alpha\alpha}=\bar{\sigma}_{\alpha\alpha}$ and $p=\bar{p}$ for any α and use constitutive equations (2), (2a) to obtain with the aid of $(1)_3$, (3), (3a)

$$\left(\frac{1}{\bar{E}_1}-\frac{1}{E_1}\right)\bar{\sigma}_{11}-\left(\frac{\bar{\nu}_{21}}{\bar{E}_2}-\frac{\nu_{21}}{E_2}\right)\bar{\sigma}_{22}-\left(\frac{\bar{\nu}_{31}}{\bar{E}_3}-\frac{\nu_{31}}{E_3}\right)\bar{\sigma}_{33}=a_1\bar{p}, \tag{8}$$

$$-\left(\frac{\bar{\nu}_{12}}{\bar{E}_1}-\frac{\nu_{12}}{E_1}\right)\bar{\sigma}_{11}+\left(\frac{1}{\bar{E}_2}-\frac{1}{E_2}\right)\bar{\sigma}_{22}-\left(\frac{\bar{\nu}_{32}}{\bar{E}_3}-\frac{\nu_{32}}{E_3}\right)\bar{\sigma}_{33}=a_2\bar{p}\,,$$

$$-\left(\frac{\bar{\nu}_{13}}{\bar{E}_1}-\frac{\nu_{13}}{E_1}\right)\bar{\sigma}_{11}-\left(\frac{\bar{\nu}_{23}}{\bar{E}_2}-\frac{\nu_{23}}{E_2}\right)\bar{\sigma}_{22}+\left(\frac{1}{\bar{E}_3}-\frac{1}{E_3}\right)\bar{\sigma}_{33}=a_3\bar{p}\,,$$

where with the use of Eq. (4)

$$a_1=\frac{1}{E_1}\left(1-\nu_{12}-\nu_{13}\right),\quad a_2=\frac{1}{E_2}\left(1-\nu_{21}-\nu_{23}\right),\quad a_3=\frac{1}{E_3}\left(1-\nu_{31}-\nu_{32}\right). \tag{9}$$

Summing Eqs. (8) gives with the use of Eqs. (6)

$$-a_1\bar{\sigma}_{11}-a_2\bar{\sigma}_{22}-a_3\bar{\sigma}_{33}=\left(a_1+a_2+a_3\right)\bar{p}\,, \tag{10}$$

which yields $p=\bar{p}$ as a function of $\bar{\sigma}_{\alpha\alpha}$ and E_k,ν_{ij}. $\bar{\sigma}_{11},\bar{\sigma}_{22},\bar{\sigma}_{33}$ in Eqs. (8) and (10) are arbitrary and independent. Choosing $\bar{\sigma}_{11}\neq 0,\bar{\sigma}_{22}=0,\bar{\sigma}_{33}=0$ (and similarly for $\bar{\sigma}_{22}$ and $\bar{\sigma}_{33}$) in Eqs. (8) and (10), we get $\bar{E}_\alpha$ by excluding $\bar{p}$ from Eq. (10) and one of Eqs. (8) for each α in the form

$$\frac{1}{\bar{E}_\alpha}=\frac{1}{E_\alpha}-\frac{a_\alpha^2}{\sum_{\beta=1}^{3}a_\beta}. \tag{11}$$

By taking, for example, ν_{12}, ν_{23} and ν_{31} as independent (ν_{21},ν_{32} and ν_{13} are then given through Eq. (4)) and using Eqs. (9) and (4), after some algebra, Eq. (11) gives $\bar{E}_k$ in the form

$$\frac{\bar{E}_\alpha}{E_\alpha}=\frac{\left(1-2\nu_{\alpha\beta}\right)E_\gamma/E_\alpha+\left(1-2\nu_{\beta\gamma}\right)E_\gamma/E_\beta+\left(1-2\nu_{\gamma\alpha}\right)}{1+\left(1-2\nu_{\beta\gamma}\right)E_\gamma/E_\beta-\left(E_\gamma\nu_{\alpha\beta}+E_\alpha\nu_{\gamma\alpha}\right)^2/E_\alpha E_\gamma}. \tag{12}$$

Here α,β,γ are, respectively, the cyclic permutations of 1, 2, 3. Eq. (12) gives $\bar{E}_k$ as functions of the equilibrium material parameters of the biphasic material, E_k,ν_{ij}. Remind that Eq. (7) gives $\bar{\nu}_{ij}$ as functions of $\bar{E}_k$ and that the shear moduli of both media are the same. Thus, all the material parameters of the equivalent single-phase incompressible material, i. e. immediate compliances $\bar{E}_k$, $\bar{\nu}_{ij}$, have been obtained with the help of the equilibrium parameters of the biphasic material, i. e. equilibrium compliances E_k,ν_{ij}.

The inversion of the matrix of system (2a) is not trivial as its determinant is zero. Look for the inversion in the form

$$\begin{bmatrix}\bar{\sigma}_{11}\\ \bar{\sigma}_{22}\\ \bar{\sigma}_{33}\end{bmatrix}=-\begin{bmatrix}\bar{p}\\ \bar{p}\\ \bar{p}\end{bmatrix}+\begin{bmatrix}\bar{C}_{11}&0&0\\ 0&\bar{C}_{22}&0\\ 0&0&\bar{C}_{33}\end{bmatrix}.\begin{bmatrix}\bar{\varepsilon}_{11}\\ \bar{\varepsilon}_{22}\\ \bar{\varepsilon}_{33}\end{bmatrix}, \tag{13}$$

i. e. assume that the stiffness matrix is diagonal. Insert into Eq. (2a) for $\bar{\sigma}_{\alpha\alpha}$ from Eq. (13), use the incompressibility condition $\bar{\varepsilon}_{ii}=0$ to exclude one of $\bar{\varepsilon}_{\alpha\alpha}$ and obtain a set of equations containing always some couple of $\bar{\varepsilon}_{\beta\beta}, \bar{\varepsilon}_{\gamma\gamma}$ ($\alpha \neq \beta \neq \gamma$) that can be chosen independently. Setting always only one in these couples different from zero, a set of linear equations for three $\bar{C}_{\alpha\alpha}$ is obtained. Though the number of these equations is higher than 3, all these equations have one solution for the main stiffnesses $\bar{C}_{\alpha\alpha}$ ($\alpha = 1, 2, 3$) that can be written (after using Eqs. (3a), (6), (7)) in the form

$$\bar{C}_{\alpha\alpha} = \frac{\bar{\nu}_{\beta\gamma}}{1 - \bar{\nu}_{\alpha\beta}\bar{\nu}_{\beta\alpha}} \bar{E}_{\alpha} = \frac{2\bar{E}_{\alpha}\bar{E}_{\beta}\bar{E}_{\gamma}\left[\bar{E}_{\alpha}\left(\bar{E}_{\beta} + \bar{E}_{\gamma}\right) - \bar{E}_{\beta}\bar{E}_{\gamma}\right]}{4\bar{E}_{\alpha}\bar{E}_{\beta}\bar{E}_{\gamma}^{2} - \left[\bar{E}_{\gamma}\left(\bar{E}_{\alpha} + \bar{E}_{\beta}\right) - \bar{E}_{\alpha}\bar{E}_{\beta}\right]^{2}} . \tag{14}$$

Again, here α, β, γ take, respectively, the values 1, 2, 3 and their cyclic permutations. This substantiates the assumed form of Eq. (13). Values of $\bar{C}_{\alpha\alpha}$ must be available if the initial problem is solved in displacements rather then in stresses, which is often more convenient.

3. Conclusion

The instantaneous compliances for a linear model of biphasic mixtures (a porous elastic matrix and an inviscid fluid slowly flowing through the matrix pores, both intrinsically incompressible) have been obtained theoretically using the equilibrium compliances and assuming small strains in the matrix. For the orthotropic matrix, the instantaneous and equilibrium Young moduli and Poisson ratios have been dealt with. Eq. (14) gives the instantaneous main stiffnesses as functions of the instantaneous Young moduli.

To illustrate, let us compare the instantaneous and equilibrium Young moduli for the human humeral head cartilage. Cohen et al. (1993) performed creep indentation tests at small strains and obtained the following equilibrium moduli for the cartilage matrix considered transversely isotopic and homogeneous across the whole thickness. When assuming transverse isotropy with the isotropy axis in the x_3-axis (perpendicular to the cartilage surface), it holds for this special case of orthotropy

$$E_1 = E_2, \quad \nu_{13} = \nu_{23}, \quad \nu_{31} = \nu_{32}, \quad \nu_{12} = \nu_{21}, \quad \mu_{13} = \mu_{23}, \quad \mu_{12} = E_1 / 2\left(1 + \nu_{12}\right)$$

and Eq. (7) yields

$$\bar{\nu}_{31} = \bar{\nu}_{32} = 1/2, \quad \bar{\nu}_{13} = \bar{\nu}_{23} = \bar{E}_1 / 2\bar{E}_3, \quad \bar{\nu}_{12} = \bar{\nu}_{21} = 1 - \bar{E}_1 / 2\bar{E}_3 .$$

By comparing the above experiments with the theoretical solutions, they found $E_3 = 0.46\text{MPa}$ and $E_1 = 5.8\text{MPa}$. For zero equilibrium Poisson ratios (see Cohen et al. (1993)), Eq. (12) gives the instantaneous Young moduli (equal to the main stiffnesses in this case!) $\bar{E}_3 = 3.36\text{MPa}$ and $\bar{E}_1 = 6.23\text{MPa}$. It is apparent that the instantaneous Young moduli are higher than the equilibrium ones, especially for the perpendicular direction and that the instantaneous state is more isotropic. For zero equilibrium values of $\nu_{\alpha\beta}$ the instantaneous Poisson ratios become $\bar{\nu}_{13} = \bar{\nu}_{23} = 6.3$, $\bar{\nu}_{12} = \bar{\nu}_{21} = -5.3$, $\bar{\nu}_{31} = \bar{\nu}_{32} = 1/2$, i. e. values markedly different from zero (their equilibrium values). It is interesting to note that the above ratio $\bar{E}_3 / E_3$ corresponds to that measured using the velocity of ultrasound propagation in some other cartilage specimens (Knecht et al., 2006).

Let a biphasic solid of the above material be suddenly loaded at time $t = 0$. The load is assumed to vary with time and the time-dependent response of the solid is looked for. The step-load could be replaced with a load that increases smoothly and quickly from zero to the value of the step. For a good approximation, the time steps during the steep increase of the load should be very small. However, there is another possibility how to solve this time-dependent problem. In fact, first, to solve the time-independent problem for the initial value of the load and a single-phase incompressible elastic material. Second, to find the time-dependent solution for the biphasic material and the former solution should be used as the initial conditions for the latter. Moreover, for low matrix permeability (such as $k_{ij} \approx 10^{-16}\,\mathrm{m}^4 / \mathrm{N} \cdot \mathrm{s}$ for articular cartilage) and a short time span, or for a periodic loading with a short period (of order of several seconds), the single-phase solution could serve as a good approximation.

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