National Conference with International Participation
ENGINEERING MECHANICS 2007
Svratka, Czech Republic, May 14 – 17, 2007

STATISTICALLY EQUIVALENT PERIODIC UNIT CELL FOR JOINTED ROCK MASSES

J. Gajdošík*, M. Šejnoha* and J. Zeman*

Summary: *This contribution deals with problem of solids weakened by cracks or joints. The rock containing crack is the model example of such a problem. Determination of properties is based upon the multi-scale homogenization. Owing to generally large dimensions of the analyzed problem (large underground caverns, deep seated tunnels, cracks in homogeneous mass, etc.) it is possible to treat joints, having sizes even of the order of meters, from the micromechanics point of view. The statistic description of spatial distribution of cracks is introduced. The common descriptors as e.g. crack density are compared with more sophisticated (containing more information) ones. The generation of the unit cells is based on the orientation and length of cracks.*

1 Introduction

Modeling of highly jointed rock masses presents a formidable challenge owing to the large complexity of the problem. It has been recognized long ago that discrete modeling of each joint is in such a case not only impractical, but computationally infeasible. Instead, elements of homogenization rooted in analyses of composites have been employed. Application of micromechanics based averaging techniques essentially demanding separation of scales is admissible, since rock joints, although of the order of meters, are still considerably smaller in size when compared with the dimensions of the analyzed problem (large underground caverns, deep seated tunnels, etc). Introduction of homogenization then transforms the original discontinuous body into a continuum with certain equivalent material properties as schematically depicted in Fig. 1.

In this context the jointed rocks can be viewed as a special class of solids weakened by cracks. Estimates of overall elastic moduli of such material systems can be obtained by several well known methods including the self-consistent (Laws and Dvorak, 1987) and Mori-Tanaka methods (Benveniste, 1987). A variant of the dilute approximation has been introduced in (Cai and Horii, 1993) to seek macroscopic nonlinear response of highly jointed rocks exploiting the dilation constitutive model presented in (Jing, 1990). A particular application of the Mori-Tanaka method to the analysis of cracked rocks can be found in (Deude *et al.*, 2002). A comprehensive survey of various averaging techniques available for a number crack geometries also

*Ing. Jan Gajdošík, Doc. Ing. Michal Šejnoha, Ph.D., Ing. Jan Zeman, Ph.D., Department of Mechanics, Faculty of Civil Engineering, Czech Technical University in Prague, e-mail: jan.gajdosik@fsv.cvut.cz, sejnom@fsv.cvut.cz, zemanj@cml.fsv.cvut.cz

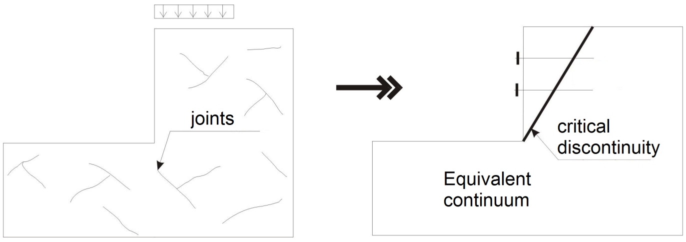


Figure 1: Jointed rock mass and equivalent continuum model

suggesting their potential drawbacks and essentially promoting the most simple ones, the dilute approximation and the Mori-Tanaka method, is given in (Kachanov, 1992). Although computationally very attractive, the observed deficiencies of simple averaging techniques (Gajdošík *et al.*, 2007) may open the way to a more accurate approach presented in the framework of statistically equivalent periodic unit cells (Zeman and Šejnoha, 2001).

2 Concept of statistically equivalent periodic unit cell

Images of real rock masses generated by Global positioning system receivers combined with geologic mapping, digital photography, remote sensing and tomography that show rock formation including distribution of fractures (joints) will soon become commonly available. It is therefore imperative to develop a general modeling concept that would take detailed information about real "microstructure" into account and if possible in an efficient way. A concept of statistically equivalent periodic unit cell (SEPUC) developed in our previous works for various classes of two-phase composites, see e.g. (Zeman and Šejnoha, 2006), presents one particular option.

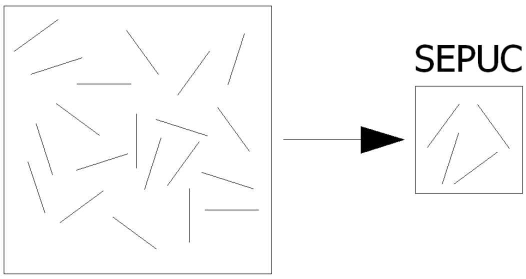


Figure 2: Concept of statistically equivalent periodic unit cell

The leading idea of this approach, evident from Fig. 2, is to replace a complex non-periodic microstructure by a certain periodic unit cell (PUC), which still optimally resembles the original microstructure in a proper sense. To reduce the problem complexity the periodic unit cell is described by a substantially smaller number of parameters. It has been found by Zeman and Šejnoha (2006) that the predictive capabilities of the resulting SEPUCs strongly depend on the microstructure quantification by suitable geometrical descriptors.

Owing to its simplicity the first descriptor we tested was the second order intensity function

$K(r)$ (Pyrz, 1994) given by

$$K(r) = \frac{A}{N^2} \sum_{k=1}^{N} I_k(r), \tag{1}$$

where $I_k(r)$ is the number of points (centers of joints) within a circle with radius r centered at the k-th joint, N is the total number of joints in the sample and A is the sample area. Periodicity of the analyzed sample is often assumed to account for points outside the sampling area. Examples of this function for various crack patterns are shown in Fig. 3.

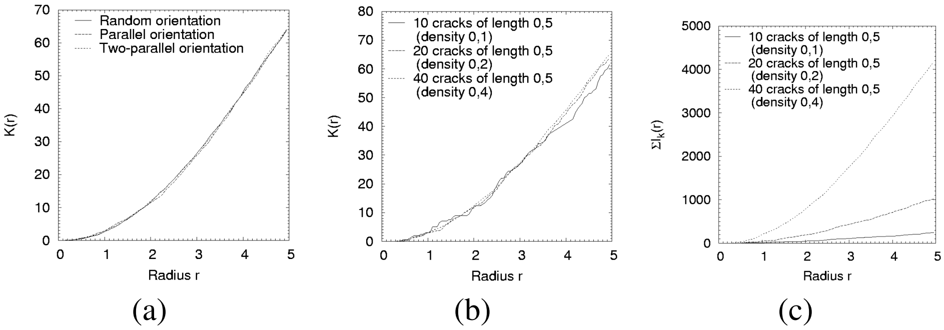


Figure 3: $K(r) - r$ plot: (a) Various orientations of cracks - the same density and the same crack length, (b) variable density - different number of cracks of the same length, $\sum I_k - r$ plot: (c) same as (b)

It becomes evident from these plots that this function, as a descriptor of the points distribution, is not capable of capturing the essential differences between various crack patterns. It cannot discriminate between different crack orientations, Fig. 3(a), different crack lengths, and even different densities, Figs. 3(b). Certain differences arise, by no surprise, when removing the scaling factor from Eq. 1, Figs. 3(c). But for the case of different crack orientations the functions $\sum I_k - r$ also coincide. It appears that applicability of this function is thus limited to artificial or well defined crack patterns such as parallel orientation providing the same crack pattern is assumed for both the real microstructure and statistically equivalent periodic unit cell.

More sophisticated geometrical descriptors are therefore needed if we wish to address in situ observed crack patterns. Intuitively, a variant of the above technique that incorporates certain elements of the Lineal path function (Torquato, 2002) can be adopted. Such a descriptor, hereafter referred to as the Lineal cross function can be evaluated as follows: In view of the original formulation of the Lineal path function consider again a sampling template in Figure 4 placed into a medium. Next, for a given segment length and orientation trace the cracks being crossed by this segment. The function is then defined as the sum of lengths of cracks being crossed by this segment raised to power of two and suitably normalized such that

$$X(r, \alpha) = \frac{1}{N_{x,y}\overline{L}^2} \sum_{x,y} L_{x,y}^2(r, \alpha), \tag{2}$$

where $N_{x,y}$ represents the number of throws of a given segment into the medium, $\overline{L}$ is the average crack length and $L_{x,y}(r, \alpha)$ is the length of a crack crossed by a segment of length r in the direction of α at the position x, y. For the calculation purposes the template, instead of being randomly thrown into the medium, is successively placed into the prescribed grid much in the spirit of the deterministic sampling.

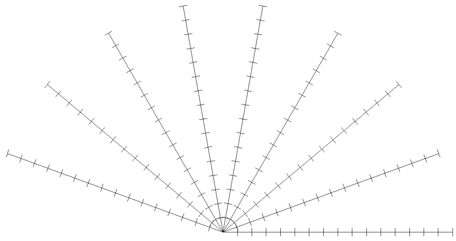


Figure 4: **Example of sampling template**

Obviously, other descriptors such as a certain variant of the direct measure of elastic homogenized response might also prove useful. These, however, primarily due to space limitation will be examined in the forthcoming paper.

3 Definition of SEPUC

Suppose that an appropriate idealized geometrical model of a periodic unit cell is given, see previous section for concrete examples. Now it suffices to assume that a geometrical model is fully characterized by an N-dimensional vector of parameters $\boldsymbol{v}$. Once the geometry of the unit cell is specified, a corresponding digital image can be generated and used to determine the statistical descriptor X. Then, the following measure of similarity between the original microstructure and a periodic unit cell can be introduced:

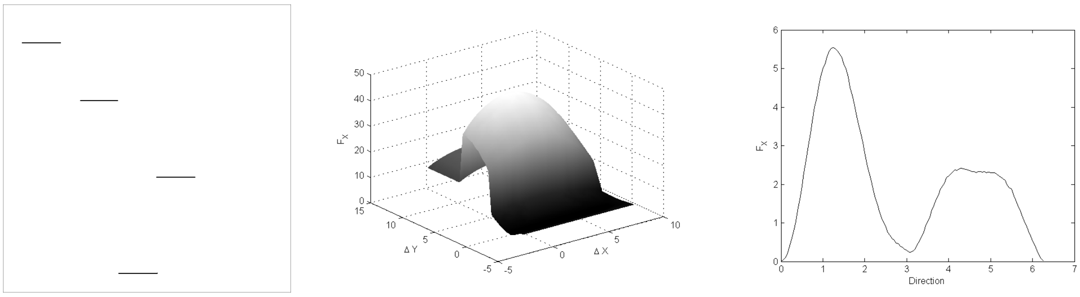
$$F_X(\boldsymbol{v}) = \sum_i \sum_j \left(\overline{X}(i,j) - X(i,j)\right)^2, \tag{3}$$

where $\overline{X}$ is the lineal cross function corresponding to the target medium. The parameters of the SEPUC are then simply found by minimizing the objective function provided by Equation (3). Note that the solution of the introduced optimization problem, as evident from the next section, requires minimization of a multi-dimensional and multi-modal objective function.

4 Objective function

Before proceeding with the actual search for the SEPUC we first performed a simple test of applicability of the proposed descriptor Equation (2) as well as the objective function Equation (3). This test falls into a category of reconstruction media problems. To that end, a rectangular area of dimensions 10 x 15 containing 4 parallel cracks was assumed (Figure 5(a)). To observe the character of the objective function, one of the cracks was first repeatedly placed into a number of different locations. The resulting variation of the objective function is plotted in Figure 5(b). In the next step, the positions and directions of all cracks in tested specimen were fixed except for the orientation of the same “free” crack. The corresponding result appears in Figure 5(c).

As evident from Figure 5 the objective is characterized by a great number of local extremes (highland landscape) and by large areas with relatively small change in the function value (plateau resembling shape). Although relatively small in size, 12 unknowns are to be determined - 4 cracks with 3 independent parameters (x-position, y-position and direction), the



(a) (b) (c)

Figure 5: **Test of the objective function: (a) - assumed crack morphology, (b) - objective function as a result of change of one crack position, (c) - objective function as a result of change of one crack orientation.**

problem of reconstructing the original medium, Figure 5(a), already presents a formidable challenge for the optimization algorithm to succeed. Two particular algorithms, both exploiting the elements of evolution strategies, were tested.

First the program SADE (Hrstka *et al.* (2003)) developed at the Department of mechanics, Faculty of Civil Engineering, Czech technical university in Prague, was tested. To run the optimization problem a set of random microstructures with the same number of cracks as assumed for the original microstructure but with different locations and orientations were generated with the goal to eventually match the original microstructure. This would correspond to a value of the objective function (Equation (3)) equal to zero since in this case the two functions $\overline{X}$ and X coincide.

This algorithm, however, failed to deliver the expected results. A relative success, at least for the present problem, we celebrated with the method SOMA (Zelinka, 2004). With the help of this algorithm we therefore proved the suitability of both the proposed geometrical descriptor (the Lineal cross function) and objective function as tools for finding the desired SEPUC, which is the topic of our current research.

The use of the proposed descriptor is further promoted by the results displayed in Figure 6. This figure shows plots of the Lineal cross function for various crack patterns which differ both in terms of crack orientation and size, while the crack density is kept constant. Unlike the second order intensity function the Lineal cross function much better reflects the microstructural details. Note that color images provide even insight into the quality of this function.

5 Conclusion

A new geometrical descriptor (the Lineal cross function) to be used for the determination of SEPUC was introduced in this paper. This descriptor together with the objective function formulated on the bases of the least square method, Equation (3) seem to be sufficiently robust to achieve this goal for both random and well defined crack configurations.

Since the ultimate goal is to define a road-map for the homogenization of rocks with a realistic formation joints weakened by crack, a step that incorporates nonlinear multiscale computing, the possibility of using stiffness matrix obtained from linear homogenization as the fitness function for the nonlinear problem also appears worthwhile to examine.

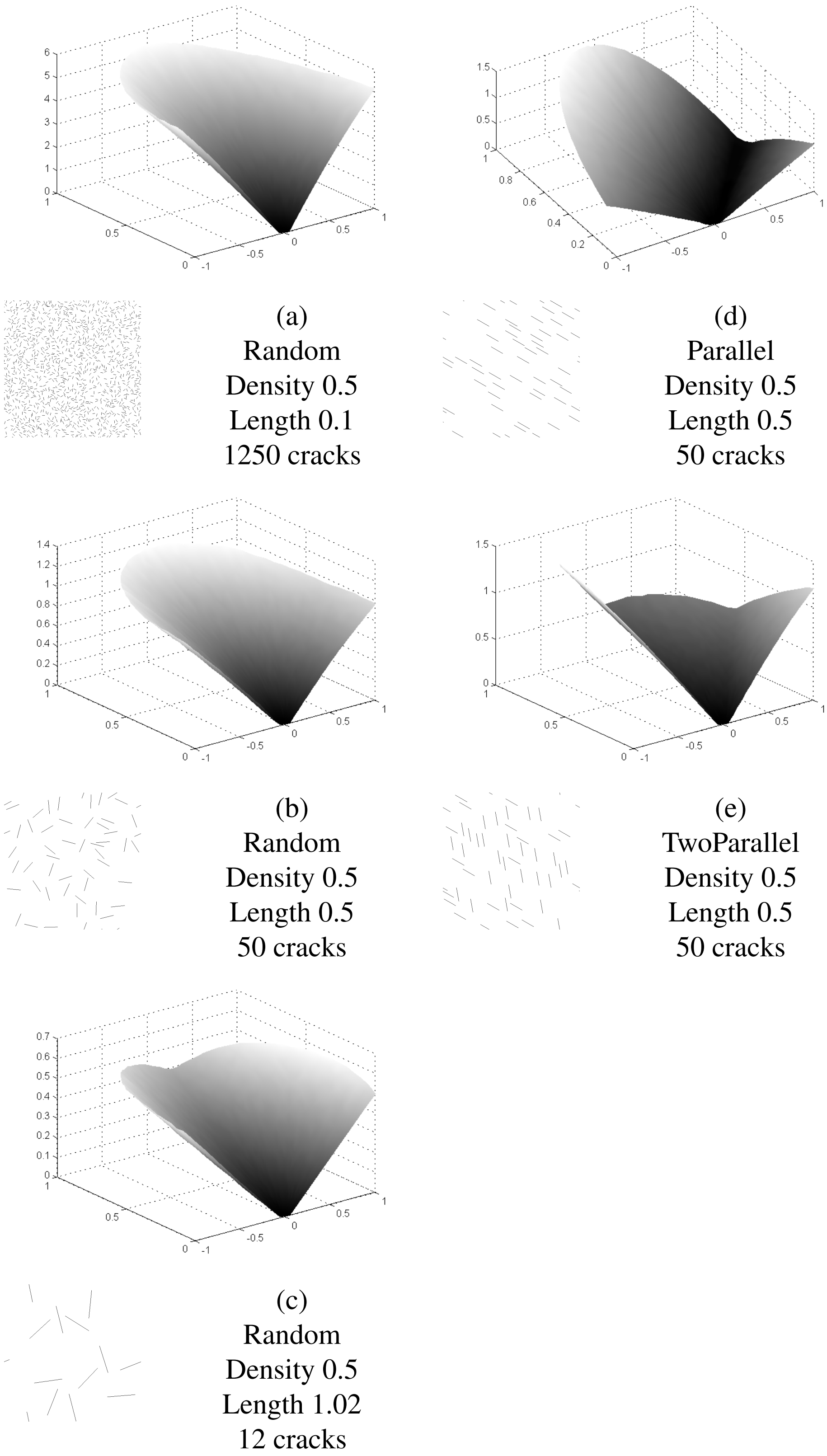


Figure 6: **Examples of lineal cross function: (a),(b),(c) - comparison of function calculated over crack sets with crack equal to 0.5 and various crack length. In case (c) is obvious, that for small scale the statistical isotropy is corrupted. In the sample (d) are cracks oriented in one particular direction (30 degrees). In the sample (e) are cracks oriented in one particular direction (30 and 80 degrees).**

Acknowledgments

The financial support provided by by the research project CEZ MSM 6840770003 and partially also by the GAČR Grant No. 106/03/H150 is gratefully acknowledged.

References

Benveniste, Y. (1986), "On the Mori-Tanaka's method in cracked bodies" *Mechanics Research Communications*, Vol. 13(4), pp. 193-201.

Cai, M. and Horii, H. (1993), "A constitutive model and FEM analysis of jointed rock masses", *Int. J. Rock. Mech. Min. Sci. & Geomech*, Vol. 30(4), pp. 351-359.

Deudé, V., Dormieux, L., Kondo, D. and Maghous, S. (2002), "Micromechanical Approach to Nonlinear Poroelasticity: Application to Cracked Rocks", *Journal of Engineering Mechanics*, Vol. 128(8), pp. 848–855.

Gajdošík, J., Šejnoha, M., Zeman, J. (2007) "Homogenized response of jointed rock masses with periodic fields", *Proceedings of International Conference on Computational Mechanics and Experimental Engineering*, 3-8 January 2007, Miami, USA, on CD rom.

Hrstka, O., Kučerová, A., Lepš, M. and Zeman, J. (2003), "A competitive comparison of different types of evolutionary algorithms", *Computers and Structures*, 81(18-19):1979-1990.

Jing J. (1990), "A two-dimensional constitutive model of rock joints with pre- and post-peak behaviour" *Proc. Int. Conf. on Rock Joints*, Barton and Stephansson (Ed.), Balkema, Rotterdam, pp. 633-638.

Kachanov, M. (1992), "Effective elastic properties of cracked solids: critical review of some basic concepts", *Applied Mechanics Review*, Vol. 45(8), pp. 304-335.

Laws, N. and Dvorak, G.J. (1987), "The effect of fiber breaks and aligned penny-shaped cracks on the stiffness and energy release rates in unidirectional composites", *International Journal of Solids and Structures*, Vol. 23(9), pp. 1269-1283.

Pyrz R. (1994), "Correlation of microstructure variability and local stress field in two-phase materials", *Materials Science and Engineering*, Vol. A177, pp. 253-259.

Torquato, S. (2002), "Random heterogeneous materials: Microstructure and macroscopic properties", *(Springer: New York)*

Zelinka I., In, "G. Onwubolu, B.V.Babu, New Optimization Techniques in Engineering", Springer-Verlag, ISBN 3-540-20167X,2004, Chap. 7 "SOMA - Self Organizing Migrating Algorithm", Chap. 25 "Mechanical Engineering Problem Optimization by SOMA"

Zeman, J. and Šejnoha, M. (2001), "Numerical evaluation of effective properties of graphite fiber tow impregnated by polymer matrix", *Journal of the Mechanics and Physics of Solids*, Vol. 49(1), pp. 69-90.