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SANDWICH COMPOSITE BAR ELEMENT FOR GEOMETRIC NONLINEAR ANALYSIS

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Summary: *The contribution deals with the straight sandwich bar element derived by new nonincremental full geometric nonlinear approach. We assume a two-node straight sandwich composite bar finite element with three layers of double symmetric rectangular cross-section area. The homogenisation of the material properties is made for polynomial variation of elasticity modulus and polynomial variation of constituent´s volume fraction at the top and bottom layers. Stiffness matrix of the composite bar contains transfer constants, which accurately describe the polynomial uni-axial variation of effective Young´s modulus. In the numerical experiments different mixture rules have been considered for calculation of the effective longitudinal elasticity modulus of the composite (FGM´s) bar. The results obtained will be compared with solid element analysis in the ANSYS simulation programme.*

1. Introduction

The composite structures (e.g. laminate, sandwich structures, or FGM´s) are often used in engineering applications. Their FE analyses require creating very fine mesh of elements even for relatively small sized bodies, what increases computational time, particularly in nonlinear analyses. Macro-mechanical modelling of the composites is based on material properties homogenisation.

The simplest mixture rules, which determine average effective material properties, are based on the assumption that the composite material property is the sum of the material properties of each constituent multiplied by its volume fraction. To increase the accuracy of the composite material properties calculation the new homogenisation techniques and the improved mixture rules have been applied [2,4,6]. Recently application of the multiscale computation is prevails [1,5].

In this contribution we deal with the straight sandwich bar element intended to perform nonincremental full geometric nonlinear analysis.

We assume a two-node straight sandwich composite bar finite element with double symmetric rectangular cross-section area (Figure 1). Debonding of the layers is not considered.

The homogenisation of the material properties is made for three layered sandwich bar with constant material properties of the middle layer and polynomial variation of the effective elasticity modulus and volume fraction of fibre and matrix at the top/bottom layer (Figure 1).

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To derive of the bar element matrices, the effective longitudinal elasticity modulus have been considered [9,10]. The uni-axially polynomial variation of fibre elasticity modulus E_f and the matrix elasticity modulus E_m is given as polynomials

$$E_f(x) = E_{fi}\,\eta_{E_f}(x) = E_{fi}\left(1+\sum_{k=1}^{q}\eta_{Efk}x^k\right)$$

$$E_m(x) = E_{mi}\,\eta_{E_m}(x) = E_{mi}\left(1+\sum_{k=1}^{q}\eta_{Emk}x^k\right) .$$

The fibre v_f and matrix v_m volume fractions of the constituents are chosen by expressions

$$v_f = 1 - v_m = v_{fi}\,\eta_{v_f}(x) = v_{fi}\left(1+\sum_{k}\eta_{vfk}x^k\right)$$

$$v_m = 1 - v_f = v_{mi}\,\eta_{v_m}(x) = v_{mi}\left(1+\sum_{k}\eta_{vmk}x^k\right) .$$

The effective longitudinal elasticity modulus is then given by

$$E_L(x) = v_f(x)\,E_f(x) + v_m(x)\,E_m(x) . \tag{1}$$

The bar element with varying stiffness is loaded in linear elastic load state. The effective longitudinal elasticity modulus changes as the polynomial

$$E_L(x) = E_{Li}\,\eta_{E_L}(x) \tag{2}$$

where $E_{Li} = v_{fi}\,E_{fi} + (1 - v_{fi})E_{mi}$ is the effective longitudinal elasticity modulus at node i and

$$\eta_{E_L}(x) = 1 + \frac{\eta_{v_f}(x)\eta_{E_f}(x) + \eta_{v_m}(x)\eta_{E_m}(x)}{E_{Li}} \tag{3}$$

is the relation for longitudinal effective elasticity modulus.

In this contribution a new approach to evaluation of equlibrium equations suggested by Murín [7] is presented. In this solution no linearisation of the variation of Green-Lagrange strain tensor is used. Thus we can obtain the exact nonlinear nonincremental formulation of the element stiffness matrices. When total Lagrangian formulation is used, nonlinearised equations can be derived from the equlibrium of internal and external work

$$\int_{{}^0\mathrm{V}} {}^tC_{ijrs}\,e_{rs}\,\delta e_{ij}\,d\mathrm{V} + \int_{{}^0\mathrm{V}} {}^tC_{ijrs}\left(e_{rs}\,\delta\eta_{ij} + \eta_{rs}\,\delta e_{ij} + \eta_{rs}\,\delta\eta_{ij}\right)d\mathrm{V} = \int_{{}^0\mathrm{A}} {}^tf_i\,\delta u_i\,d\mathrm{A} + {}^tF_i^k\,\delta u_k \tag{4}$$

written in conventional notation. After implementation of correspondent approximation of the displacement functions $u_i = \phi_{ik}\,u_k$ we can modify the equation (4) for FEM requirements to the form

$$\begin{aligned}
&\frac{1}{4}\int_{{}^0\mathrm{V}} {}^tC_{ijkl}(\phi_{km,l}+\phi_{lm,k})(\phi_{in,j}+\phi_{jn,i})u_i^m\,d\mathrm{V} + \\
&+\frac{1}{4}\int_{{}^0\mathrm{V}} {}^tC_{ijkl}\,\phi_{pm,k}\,\phi_{pr,l}(\phi_{in,j}+\phi_{jn,i})u_i^m\,u_i^r\,d\mathrm{V} + \\
&+\frac{1}{2}\int_{{}^0\mathrm{V}} {}^tC_{ijkl}\,\phi_{pr,i}\,\phi_{pn,j}(\phi_{km,l}+\phi_{lm,k})u_i^m\,u_i^r\,d\mathrm{V} + \\
&+\frac{1}{2}\int_{{}^0\mathrm{V}} {}^tC_{ijkl}\,\phi_{pm,k}\,\phi_{pv,l}\,\phi_{rq,i}\,\phi_{rn,j}\,u_i^m\,u_i^v\,u_i^q\,d\mathrm{V} = \int_{{}^0\mathrm{A}} {}^tf_i\,\phi_{in}\,d\mathrm{A} + {}^tF_i^n
\end{aligned} \quad . \tag{5}$$

Now we have in (5) a basic relation, which can be used for an arbitrary finite element derivation.

2. The bar element with varying stiffness

If the concept of transfer functions and constants published by Rubin [8] is used in the derivation of the stiffness relation, we obtain local nonlinear stiffness matrix of the element for linear elastic material.

Stiffness matrix of the composite bar contains transfer constants, which accurately describe the polynomial uni-axial variation of the effective Young´s modulus. After substitution of the new straight bar shape functions

$$u(x) = \phi_{i1}\, u_i + \phi_{k1}\, u_k = \left(1 - \frac{d'_{2E_L}(x)}{d'_{2E_L}}\right) u_i + \frac{d'_{2E_L}(x)}{d'_{2E_L}} u_k \tag{6}$$

into (5), the nonlinear stiffness matrix of the element has the following form

$$\mathbf{K} = \frac{AE_{Li}}{d'_{2E_L}} \left[1 + \frac{3}{2}(\lambda - 1) \frac{\overline{d'_{2E_L}}}{(d'_{2E_L})^2} + \frac{1}{2}(\lambda - 1)^2 \frac{\overline{\overline{d'_{2E_L}}}}{(d'_{2E_L})^3} \right] \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}. \tag{7}$$

where d'_{2E_L}, $\overline{d'_{2E_L}} = \int_0^{L^0} (d''_{2E_L}(x))^2 dx$, $\overline{\overline{d'_{2E_L}}} = \int_0^{L^0} (d''_{2E_L}(x))^3 dx$ are the transfer constants for elastic loading case and $\lambda = \frac{u_k - u_i}{L^0} + 1$ is the stretching. These transfer constants can be computed by using simple numerical algorithm published by Kutiš and Murín [3].

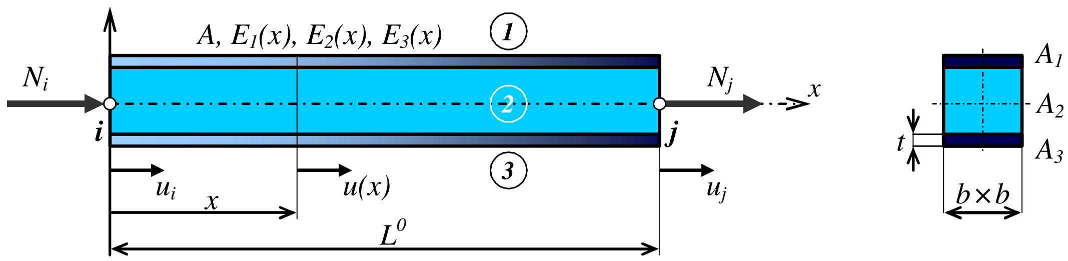

Figure 1: Symmetric sandwich bar element with variation of stiffness in initial state

Then the internal force in the bar element can be calculated using the formulae

$$N_i = -\frac{AE_{Li}}{d'_{2E_L}} \left[1 + \frac{3}{2}(\lambda - 1) \frac{\overline{d'_{2E_L}}}{(d'_{2E_L})^2} + \frac{1}{2}(\lambda - 1)^2 \frac{\overline{\overline{d'_{2E_L}}}}{(d'_{2E_L})^3} \right] (\lambda - 1) L^0 \,. \tag{8}$$

Final, the resulting system of nonlinear equations of the type $\mathbf{K\,u = F}$ is usually solved using Newton-Raphson method. In this solution process, the full tangent stiffness matrix was expressed by

$$\mathbf{K}_T = \frac{\partial \mathbf{F}}{\partial \mathbf{u}} = \frac{AE_{Li}}{d'_{2E_L}} \left[1 + 3(\lambda - 1) \frac{\overline{d'_{2E_L}}}{(d'_{2E_L})^2} + \frac{3}{2}(\lambda - 1)^2 \frac{\overline{\overline{d'_{2E_L}}}}{(d'_{2E_L})^3} \right] \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}. \tag{9}$$

3. Numerical experiments

In the numerical experiments the accuracy and efficiency of the new nonincremental geometric nonlinear bar element equations with varying of effective material properties was examined. We assume a three layered two-node sandwich bar with double symmetric rectangular cross-section (Figure 1). As a typical example of geometrically nonlinear behaviour the three-hinge mechanism was analysed (Figure 2).

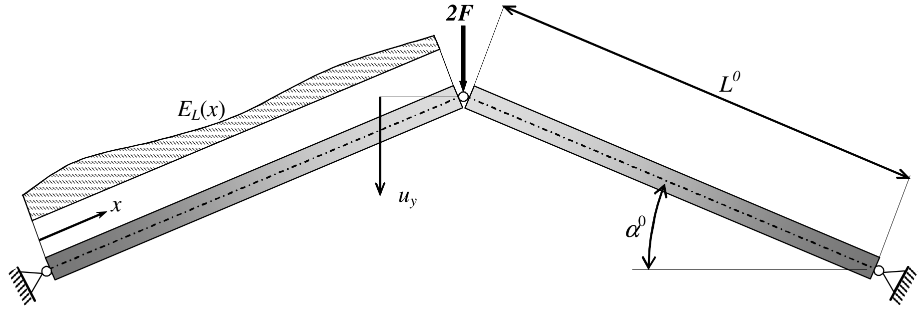

Figure 2: Von Mises bar element structure

Two different approaches published in [9,10] and [4] have been considered for calculation of the effective longitudinal elasticity modulus of the composite (FGM´s) bar with both polynomial variation of constituent´s volume fraction and polynomial longitudinal variation of the elasticity modulus.

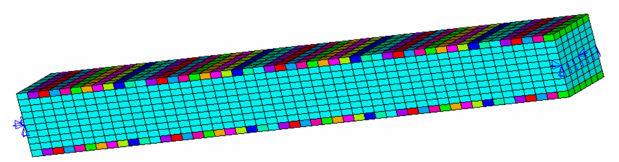

Figure 3: ANSYS model with 3000 solid elements to compare against our approach

The elasticity modulus of the faces (layers 1 and 3 in Figure 1) is described by polynomials $E_1(x)$ and $E_3(x)$. Elasticity modulus of the core $E_2(x)$ (layer 2) is constant. These material properties were used to compare our results with solution in ANSYS.
To obtain variation of the effective longitudinal elasticity modulus we are using extended mixture rules (labeled as MR) [9,10] and improved homogenisation techniques described by Love and Batra (LB) [4]. In addition we assume polynomial variations of the component's volume fraction given by expressions $v_f(x) = 0{,}5(1 + 6x)$ for the fibre volume fraction and $v_m(x) = 1 - v_f(x) = 0{,}5(1 - 6x)$ for the matrix volume fraction.
After implementation of above mentioned homogenisation procedures we have obtained the effective longitudinal elasticity modulus for the new bar element, summarized in Table 1.
To examine the accuracy of the new bar element the software Mathematica was used.

Table 1: Variation of elasticity moduli used in numerical examples

	extended mixture rules (Murín [9,10] - MR) [GPa]	**improved mixture rules** (Love and Batra [4] - LB) [GPa]
solid element model in ANSYS	$E_1(x) = E_3(x) = 327{,}5 + 435x$ $E_2(x) = 255$	$E_1(x) = E_3(x) = 320{,}4075 + 478{,}68x + 40{,}77x^2$ $E_2(x) = 255$
new bar element	$E_L(x) = 269{,}5 + 87x$	$E_L(x) = 268{,}0695 + 95{,}736x + 8{,}154x^2$

In all numerical examples the following geometric parameters have been used (Figures 1 and 2) used: $L^0 = 0{,}1$ m, $\alpha^0 = 10°$
$A^0 = 0{,}01 \text{x} 0{,}01$ m^2, $t = 0{,}001$ m.

Results of both, ANSYS and new bar element solutions are presented in two graphs. First graph shows relation between common hinge displacement vs. axial force(Figures 5). Second graph shows relation between common hinge displacement vs. global reaction (Figures 6).

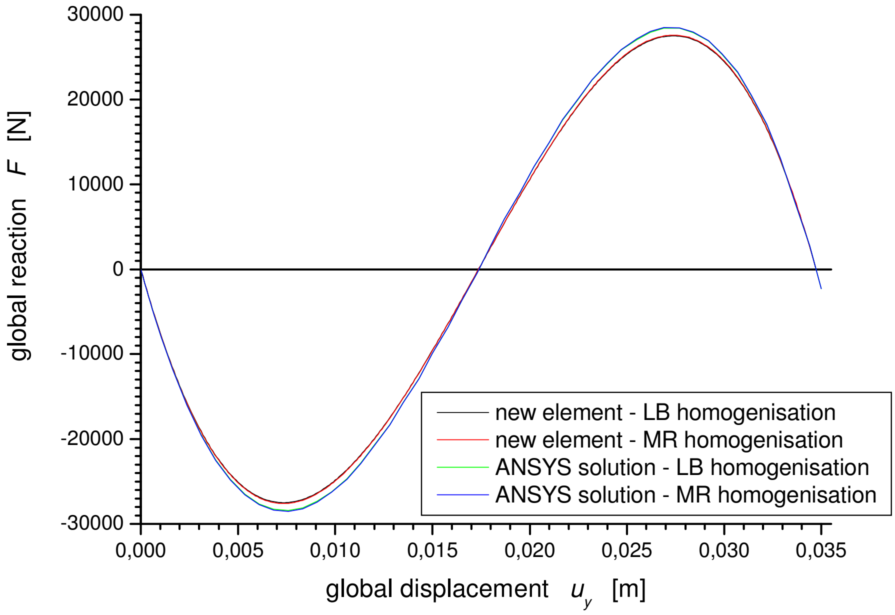

Figure 5: Common hinge displacement vs. global reaction

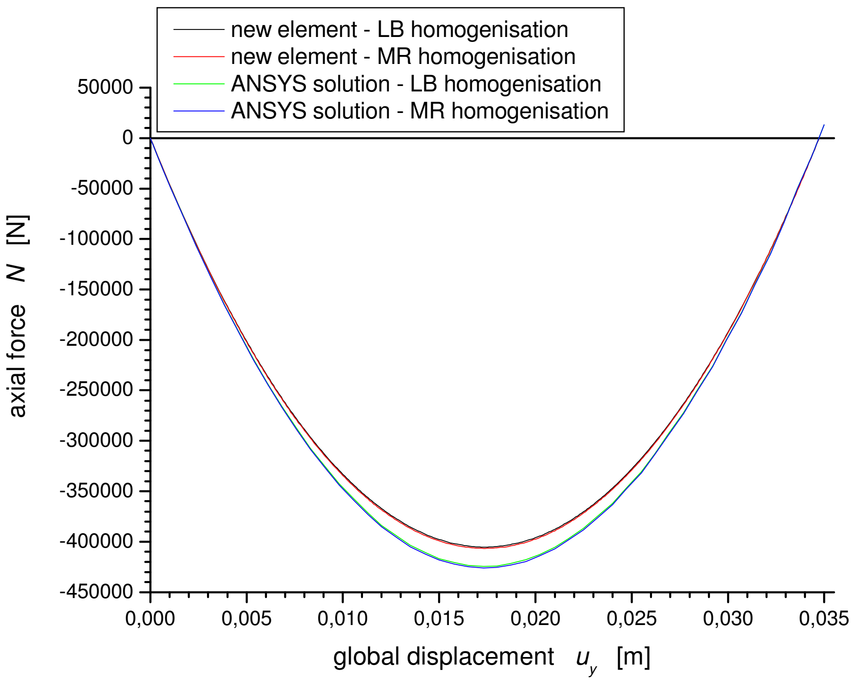

Figure 6: Common hinge displacement vs. axial force

4. Conclusion

The results of numerical experiments are presented in this contribution using the above mentioned mixture rules. The obtained results are compared with solid analysis in the ANSYS simulation programme. Findings show good accuracy and effectiveness of this new finite element. Difference between ANSYS and new element results are less then 3,2% for the global reaction and 4,5% for axial force. The results obtained with this element do not depend on the mesh density.

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6. References

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