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AN ESTIMATION OF FATIGUE LIFE UNDER GENERAL STRESS

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Summary: *The paper deals with an old problem of praxis. Structures, in general, are exposed to random excitations, which generate normal and torsional stresses. While an estimation of fatigue life is rather well known for uniaxial stress, quite different situation holds for combined random stress. A new approach to the title problem is described lower. It is based on cumulation of stress energy density in peaks of a complex damaging stress.*

1. Introduction

Fatigue of metals is a very serious problem of mechanical structures exposed to dynamic loading. This is a reason why the phenomenon has been pursuited over 150 years. In spite of it, results of investigations are not perfect namely in cases of random and combined loading. There are many hypotheses for estimating of fatigue life, however, all of them should be used with caution.

The general way of displaying fatigue properties of specimens tested under harmonic loading are diagrams of S-N curves. They plot numbers of cycles N_a of harmonic loading to failure as functions of applied stress amplitudes σ_a or τ_a. The points of experimental results are plotted in swapped axes in such a way that the dependent N_a is on the horizontal and independent stress amplitude on vertical axis. To create such diagrams is rather expensive because of long-term tests of specimens on special testing machines.

Harmonic loading occurs rather rarely in operational conditions. Usually, measured stresses are random processes generated by a random environment of an observed object. Typical representatives of such objects are vehicules. Random character of stresses obstructs a direct application of S-N curves for estimation of fatigue life. However, many hypotheses of damage cumulation have been developed, which may serve for the purpose. The first, who invented how to transform damage caused by a random loading into harmonic one, was Pålmgren (1924). His idea was extended by Langer (1937), who split the total time of damage in two parts – the time of latent damage, and the period of crack propagation. More than 20 years after Pålmgren, Miner (1945) published the same formula $\left(\sum n/N = 1\right)$ for damage caused by random stresses in America. Since that time, this procedure is denoted as the Palmgren-Miner rule in Europe, and as the Miner rule in United States of America. Modifications of this rule of linear cumulation of damage appeared later in many hypotheses of other authors. One of the most used hypothesis belongs to Corten and Dolan (1956). It enables the user to control slope of the S-N curve for calculating the damage.

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All above mentioned hypotheses are applicable for lifetime estimation in conditions of uniaxial stresses. The accuracy of estimates rapidly increased when the 'rain-flow' algorithm developed by Matsuishi and Endo (1968) appeared. Unfortunately, majority of machine parts works in conditions of multiaxial loading, which generates combined stress. This fact disables to use rain-flow method, because no closed hysteresis loops are generated. In consequence there is no reliable method for estimating fatigue life of parts exposed to multiaxial stress.

2. Stress energy density

Many years ago, Balda (1981) published his heuristic uniaxial method, which transformed standard S-N curves $[s_a,\, N_a]$ of steel, where s_a was an amplitude of stress σ_a or τ_a and N_a a number of harmonic cycles to fracture, into similarly looking diagram $[a,\, E_d]$ with a an aggressivity of stress process and E_d a sum of squares of stress peaks. It supplied quite good results when applied to data from the article of Pfeiffer (1975). The method has been reinvented recently, when searching the suitable method for fatigue life prediction of combine stressed parts. However, better theoretical background had to support it.

It is clear that any fatigue damage is caused by an energy cumulated in *plastic* deformations of a tested part. Unfortunately, to find this energy is not a simple problem. However, an *elastic* energy density may be obtained from the formulae of linear elasticity:

Let $\boldsymbol{\varepsilon} = [\,\varepsilon_x,\, \varepsilon_y,\, \varepsilon_z,\, \gamma_{yz},\, \gamma_{zx},\, \gamma_{xy}\,]^T$ and $\boldsymbol{\sigma} = [\,\sigma_x,\, \sigma_y,\, \sigma_z,\, \tau_{yz},\, \tau_{zx},\, \tau_{xy}\,]^T$

be vectors of relative deformations and stresses, respectively. Linear transformations hold between both vectors:

$$\boldsymbol{\varepsilon} = \boldsymbol{\Phi}\,\boldsymbol{\sigma} \qquad \text{and} \qquad \boldsymbol{\sigma} = \boldsymbol{\kappa}\,\boldsymbol{\varepsilon}\,, \tag{1}$$

where

$$\boldsymbol{\Phi} = \frac{1}{E}\left[\begin{array}{ccc|ccc} 1 & -\nu & -\nu & & & \\ -\nu & 1 & -\nu & & 0 & \\ -\nu & -\nu & 1 & & & \\ \hline & & & 2\,(1+\nu) & 0 & 0 \\ & 0 & & 0 & 2\,(1+\nu) & 0 \\ & & & 0 & 0 & 2\,(1+\nu) \end{array}\right] \tag{2}$$

and

$$\boldsymbol{\kappa} = \frac{E}{(1+\nu)(1-2\,\nu)}\left[\begin{array}{ccc|ccc} 1-\nu & \nu & \nu & & & \\ \nu & 1-\nu & \nu & & 0 & \\ \nu & \nu & 1-\nu & & & \\ \hline & & & \frac{1-2\nu}{2} & 0 & 0 \\ & 0 & & 0 & \frac{1-2\nu}{2} & 0 \\ & & & 0 & 0 & \frac{1-2\nu}{2} \end{array}\right], \tag{3}$$

with Young modulus E, Poisson's ratio ν and stiffness matrix $\boldsymbol{\kappa} = \boldsymbol{\Phi}^{-1}$. Hence, the elastic energy density is described by the equation

$$\Lambda \;=\; \frac{1}{2}\,\boldsymbol{\varepsilon}^T\boldsymbol{\kappa}\,\boldsymbol{\varepsilon} \;=\; \frac{1}{2}\,\boldsymbol{\sigma}^T\boldsymbol{\Phi}\,\boldsymbol{\sigma}\,. \tag{4}$$

Uniaxial stress

It is convenient to use second formula from the equation (4) because stresses can be calculated directly from acting generalized forces. Hence, the stress energy density for uniaxial loading becomes

$$\Lambda_{1a} = \frac{\sigma_a^2}{2E} \qquad \text{or} \qquad \Lambda_{1a} = \frac{2(1+\nu)\,\tau_a^2}{2E} \tag{5}$$

in every peak and valley of the loading cycle, respectively. It is clear, that quantity E_d used in the transformed fatigue life diagram instead of N_a was simply scaled cumulative stress energy density $E_d = 2E\sum_{2N_a}\Lambda_{1a}$. Let the aggressivity a used in the new diagram instead of σ_a is a function $a = g(\boldsymbol{p})$, where $\boldsymbol{p}$ is a vector of test parameters. Processing of Pfeiffer (1975) data has shown, that the statistical moments of extremes are the significant parameters of the aggressivity function. If the distribution is more or less symmetric, it is fair to expect that a is mainly a function of a magnitude of extremes and their distribution. Hence, it has been accepted to express the aggressivity a as function of the standard deviation s_σ of extremes of the stress process $\sigma(t)$ as the square root of the second moment, and the normalized fourth moment μ_4 of extremes, kurtosis. The aggressivity has been tested in a trial form $a = s_\sigma\,\mu_4$, which manifested a good fit for Pfeiffers' tests in tension-pressure.

Biaxial stress

A biaxial stress arises, when vector $\boldsymbol{\sigma}$ has two nonzero elements. It is also a case of one normal stress σ and one shear τ. The instantaneous stress energy density then becomes

$$\Lambda_2 = \frac{1}{2}\,\boldsymbol{\sigma}^T\boldsymbol{\Phi}\,\boldsymbol{\sigma} = \frac{\sigma^2 + 2(1+\nu)\,\tau^2}{2E}\,. \tag{6}$$

Comparing this result with the first formula from equation (5), one observes, that the nominator is essentially square of some, say damaging, stress

$$\sigma_d^2 = \sigma^2 + (k_c\,\tau)^2\,. \tag{7}$$

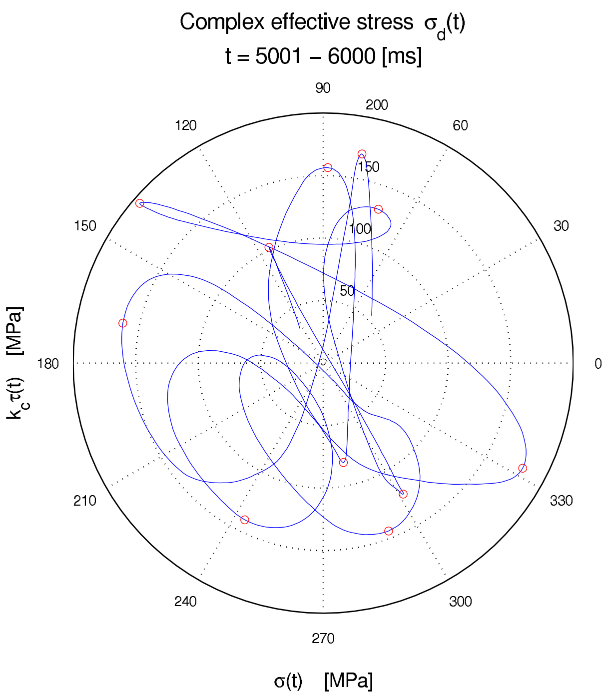

Fig. 1. Trajectory of complex stress

Since $\nu = 0.3$ for steel in elastic state, $k_c^2 = 2.6$. It is already for the third time, when we have obtained the identical formula for fatigue strength of material under combined stress condition. For the first time it has been presented in Balda and Svoboda (2002), and after in Balda et all (2003). In the later reference, it has been declared that $k_c = \sigma_c/\tau_c$ in order that uniaxial ultimate fatigue stresses σ_c and τ_c as limits of combined stresses are not biased.

Unfortunately, there is no straight way how to obtain real $\sigma_d(t)$ from the equation (7). Nevertheless, the equation can be understood as a modulus of complex stress function

$$\sigma_d(t) = \sigma(t) + i\,k_c\,\tau(t)\,. \tag{8}$$

A sample trajectory of damaging stress under equation (8) is rather complicated in the complex plane of σ_d (see Fig. 1).

Maximal deviations from the origin are denoted by small circles. It is clear, that it is difficult to say how hysteresis curves are getting close, and what are the amplitudes of cycles. Hence, we may conclude that the rainflow algorithm for decomposition of the combined stress process is inapplicable. Fortunately, we have the formula (6) for the stress energy density. A total stress energy density in extremes of module squares $|\sigma_{da}|^2$ is a measure of accumulated energy during the whole test in a tested part. The aggressivity of the damaging process can be obtained in a similar way as for the uniaxial loading.

It is always advantageous to work with non-dimensional quantities. This is the reason why relative total stress energy density has been introduced in the form

$$E_d = \frac{\sum_{n=1}^{N} \sigma_{da,n}^2}{R_m^2} \leq 1 , \tag{9}$$

and also relative aggressivity a of a combined stress process

$$a = \frac{s_{\sigma_{da}}\, g(\mu_{4a})}{R_m} , \qquad 0 < a \leq 1 . \tag{10}$$

Both quantities have been used for plotting fatigue lifetime curves in non-dimensional coordinates.

3. Experimental verification

Thanks to the long-term research of multiaxial fatigue in the Institute of Thermomechanics, there is a collection of many experimental results obtained from tests on tube specimens loaded by a general combination of tension-pressure and torque processes.

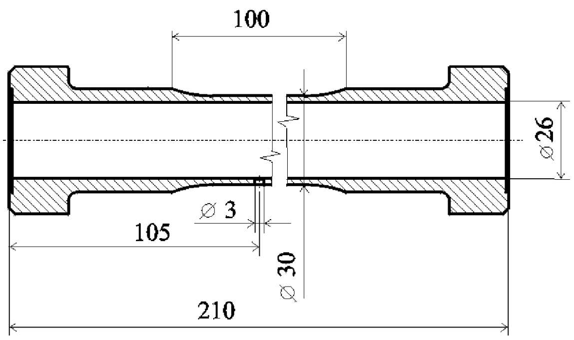

Fig. 2. Notched tube specimen

Fig. 2 depicts the tube specimen that has been used for all tests during several years of investigations. The specimens have been tested on a computer controlled electro-hydraulic testing machine INOVA ZUZ 200-1 enabling combined loading. Testing programs covered tests of both notched and plain specimens. Measurements were carried out for both uniaxial stresses $\sigma(t)$ or $\tau(t)$ and their random non-proportional combination.

Tab. 1. Parameters of specimens

Chemical contents [%]					Mechanical properties [Mpa]					
C	Mn	Si	P	S	R_m	R_e	σ_c	τ_c	σ_c^*	τ_c^*
0,18	1,29	0,50	0,28	0,14	550	450	220	135	120	80

Mechanical properties denoted by symbols without asterisks belong to plain tube, while those with asterisks are valid for specimen with a drilled lateral hole 3 mm in diameter. It is remarkable that the value of the coefficient k_c^2 evaluated from the ratio of fatigue ultimate stresses is $k_c^2 = (\sigma_c/\tau_c)^2 = 2.6557$ is very close to the theoretical value $k_c^2 = 2.6$, which is valid for plain tubes. The coefficient $k_c^{*2} = (\sigma_c^*/\tau_c^*)^2 = (120/80)^2 = 2.25$ for notched tubes.

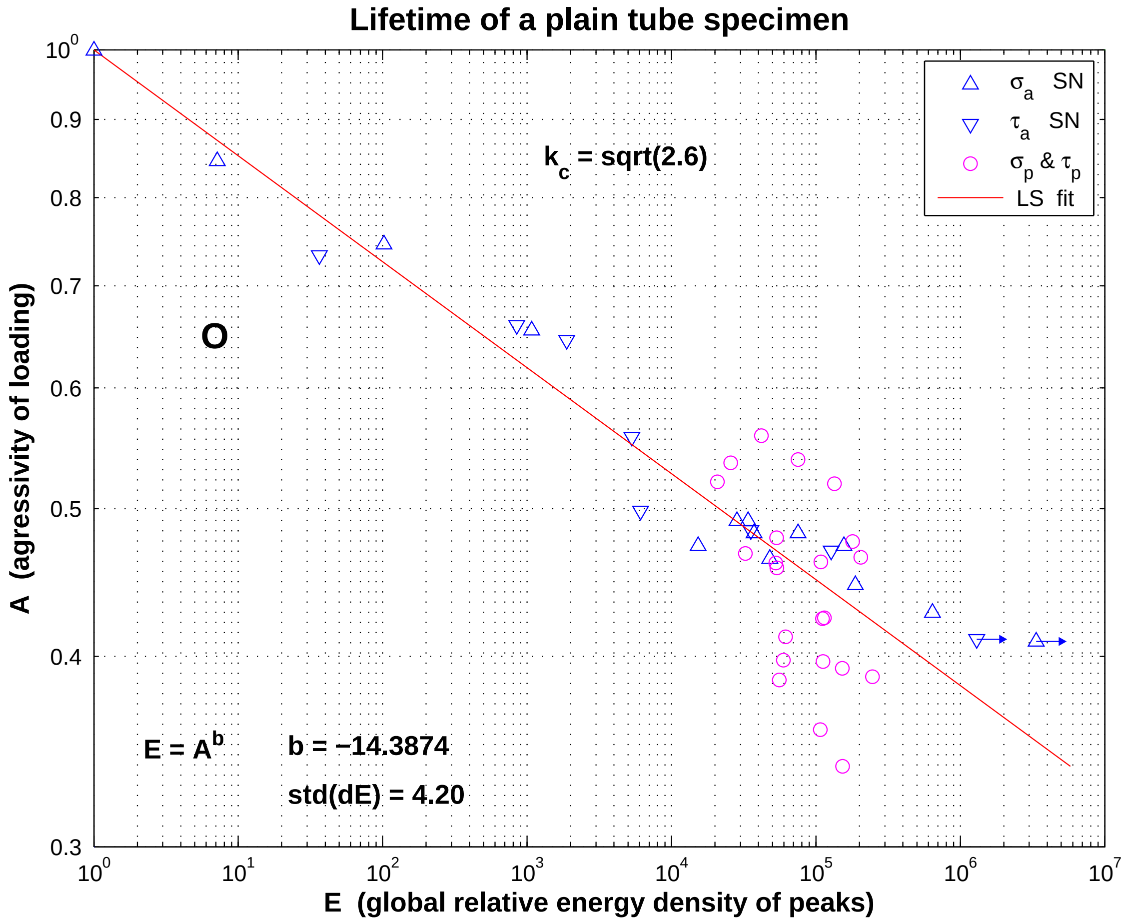

Fig. 3. Lifetime curve of plain tubes

Fig. 3 shows results of processing 21 uniaxial harmonic tests of plain tubes fatigue lives, which are marked by triangles in the figure. There are also circle markers in the diagram, of 22 fatigue lives of plain tubes non-proportionally loaded by a various combinations of random force and torque processes characterized by a coefficient $\kappa_{4a} = s_{\tau a}/s_{\sigma a}$. Symbols s_a are standard deviations of $\sigma(t)$ and $\tau(t)$ extremes. In both cases, the aggressivity of the loading has been evaluated via the equation (10) with $g(\mu_{4a}) = \mu_{4a} = m_{4a}/\sigma_{da}^4$. It is obvious that the scattering of test results of combined loading is large. It means that the function $g(\cdot)$ depends not only on μ_{4a} but also on the ratio of standard deviations of shear an normal components κ_{4a}.

The figure contains some basic parameters of the processing. Firstly, there is a value of the coefficient k_c on the top center of the diagram. The theoretical value $k_c^2 = 2.6$ has been used in Fig. 3. A least square fit of the data in logarithmic axes has been prepared for the global relative stress energy density E_d as an exponential function of the aggressivity. The form is printed in the left lower corner of the diagram with fitted coefficient b included. In log-log axes, the curve fitted from all points of fatigue lives is a straight line going through the point $(1; 1)$, which corresponds the strength R_m of material in tension. Just below the value of b-coefficient, there is standard deviation of point of stress energy density points.

The bold symbol O, on the left from the fitted line represents an order of a program version. Versions of the program `fatgED` vary by slight differences in calculating global stress energy density and the aggressivity. The program assembled in `MATLAB` evaluates fatigue lives for both harmonic and combined loading, and forms tests records and diagrams.

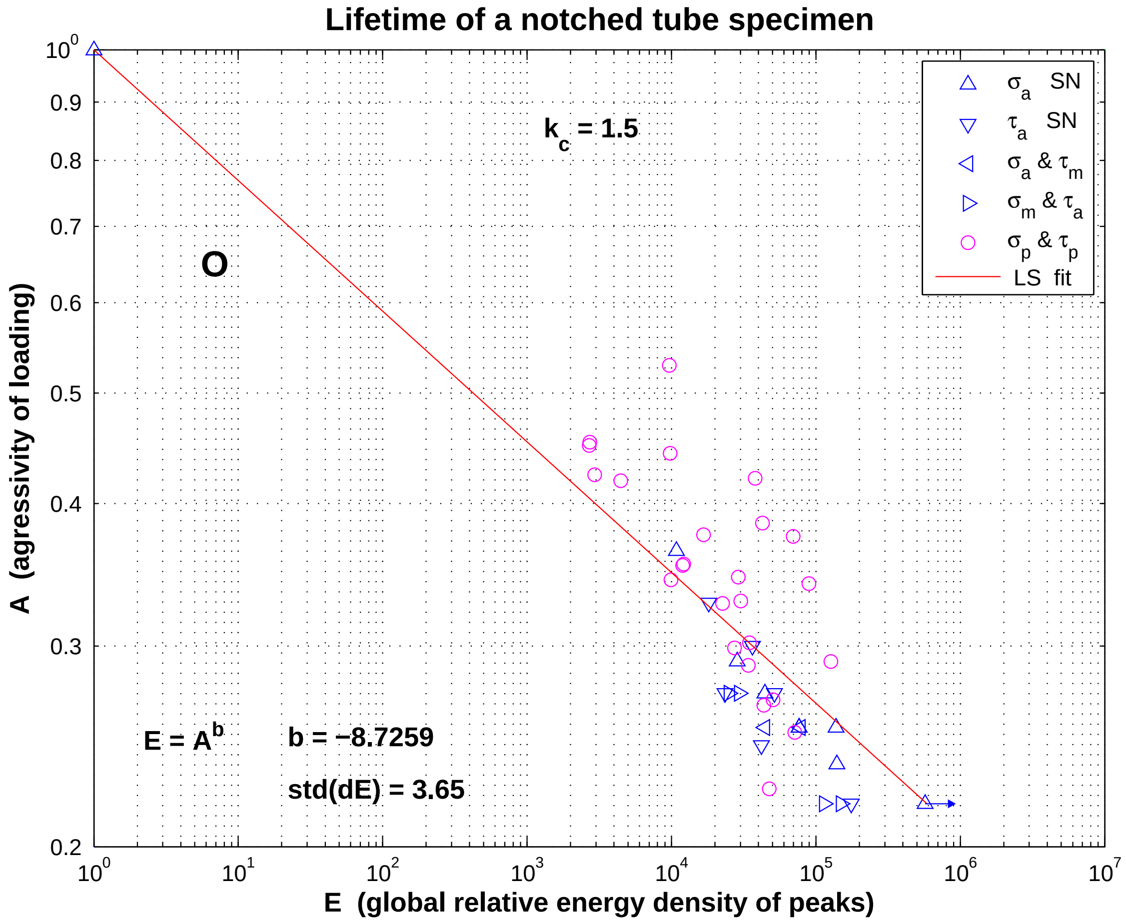

Fig. 4. Lifetime curve of notched tubes

Similar situation raises up for notched tube specimens as seen from Fig. 4. Results of 19 harmonic tests are marked by triangles in the figure. Note that left and right oriented triangles belong already to special combined loading, when one component is dynamic while the other static. It is obvious that not only uniaxial tests with $\sigma(t)$ or $\tau(t)$ but also those with static components are creating one well grouped set. If the same held for uniaxial loading with static components, the new fatigue life diagram were as important as the Haigh's diagram for ultimate fatigue stress limit.

The collection of harmonic tests points is complemented by 25 random tests results, combined stress with various κ inclusive, denoted by circles in the diagram. They have larger scattering just like in case of plain tubes. It is expected that the more complex formula for function $g(\cdot)$ might change the slope of the least-square fit line, and lower the standard deviation of the final cumulative stress energy density denoted as `std(dE)=3.65`$= s_E$ in the diagram. It means that 68 % of results comes into the band $\pm s_E$ round the least squares line, provided the result distribution be normal. Note that the coefficient k_c^* used for evaluation both energy density and aggressivity has been chosen 1.5, which corresponds to the ratio σ_c^*/τ_c^*.

Comparing figures 3 and 4, one concludes that there is certain similarity between new fatigue life curves and standard S-N curves. Both curves are approximately linear in log-log axes, and their descent is steeper for notched specimens. Liner course of the curve has been validated by the plain tube specimen tests under high levels of stresses (see Fig. 3). However, this behavior should also be checked by tests performed on notched specimens by harmonic uniaxial loading.

4. Conclusions

The contribution describes an attempt to unify many approaches for evaluation of fatigue lives of arbitrarily loaded parts into one. It is based on the fact that the driving substance of the fatigue damage is the stress energy density cumulated on a crack tip such causing its propagation.

The idea is not quite new. It appeared already a quarter of century ago in much simpler case of uniaxial fatigue. This contribution presents the new method, which transforms all kinds of stressing into one effective damaging stress and calculates two artificial quantities i.e. a cumulative stress energy density in stress extremes and an aggressivity of loading. The new method is supported by almost 90 experimental results of fatigue tests performed on plain and notched tube specimens under both uniaxial and combined loading.

The final form of the method needs to flesh out the formula for the evaluation of the loading aggressivity with the aim to lower an error of a fatigue life estimation. Experiments with the method should be complemented by special periodic and proportional process loading tests.

Acknowledgment

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