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EFFECT OF DIFFERENT ALTERNATIVES OF SELF-EXCITATION AND DAMPING ON THE VIBRATION QUENCHING

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Dedicated to my dear friend Prof. Dr. H. Ecker on the occasion of his 50th birthday

Summary: A *two-mass system consisting of a basic self-excited subsystem mounted on a foundation subsystem is investigated. Several alternatives of self-excitation expressed by a function of velocity and deflection of the basic mass and several alternatives of foundation subsystem damping are considered. The efficiency of different damping components is analyzed and the conditions for the full vibration suppressing is formulated.*

1. Introduction

Self-excited vibration represents an important phenomenon in physical and mechanical systems. There exist different sources of self-excitation, which results in different mathematical models describing important properties of the self-excitation. In most cases the self-excited vibration represents a danger for the save run of different systems and devices. Therefore, it is necessary to use means for vibration suppressing or, at least, for reducing the vibration intensity.

There exists a lot of literature dealing with the analysis of self-excited systems and the basic theory can be found in any book on oscillatory systems. In most these books the attention is given, first of all, to the explanation of the excitation mechanism and to mathematical models (see e.g. [1], [2]). Less attention is given to different means for vibration suppressing. This is dealt especially in book [3]. The active means using parametric excitation represents quite a new approach (see [4] to [15]).

One important group of these suppressing means is represented by additional subsystems (e.g. a tuned absorber or foundation mass) where the suppressing effect is due to the action of damping (e.g. absorber mass or foundation mass motion). It is evident that for the different types of self-excitation the efficiency of different types of damping can be even substantially different.

This will be illustrated on a two-mass system where the upper mass m_1 mounted on a spring having stiffness k_1 is self-excited and this basic subsystem is attached to a foundation

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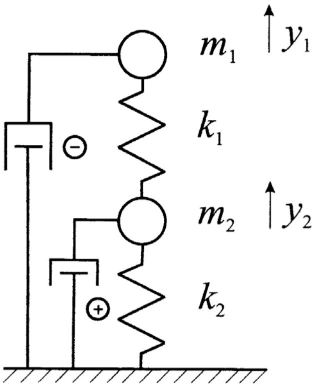

Fig. 1 Schematic representation of the system.

subsystem characterized by mass m_2 and spring having stiffness k_2 (see Fig. 1). The deflections are y_1, y_2. The foundation mass motion is damped. Both self-excitation and the foundation damping will be described by different terms to analyze the effect of different terms. The analysis is limited to self-excitation described by the terms being functions of the velocity and deflection of mass m_1 i.e. by functions of $\dot{y}_1, y_1$. In a similar way the foundation damping forces are described by the terms being functions of $\dot{y}_2, y_2$.

2. Differential equations of motion

Expressing formally the action of self-excitation as $f_1(\dot{y}_1, y_1)$ and of the foundation damping $f_2(\dot{y}_2, y_2)$ the system in question is governed by the following equations:

$$m_1\ddot{y}_1 + k_1(y_1 - y_2) + \varepsilon f_1(\dot{y}_1, y_1) = 0, \tag{2.1}$$
$$m_2\ddot{y}_2 - k_1(y_1 - y_2) + k_2 y_2 + \varepsilon f_2(\dot{y}_2, y_2) = 0.$$

We can suppose that all terms in the functions f_1 and f_2 comprise the corresponding velocities. Using the time transformation $\omega_1 t = \tau\,(\omega_1 = \sqrt{k_1/m_1})$ equations (2.1) get the form:

$$y_1'' + y_1 - y_2 + \varepsilon F_1(y_1', y_1) = 0, \tag{2.2}$$
$$y_2'' - M\,(y_1 - y_2) + q^2 y_2 + \varepsilon F_2(y_2', y_2) = 0,$$

where

$$F_1(y_1', y_1) = \frac{1}{k_1} f_1(y_1', y_1), \quad F_2(y_2', y_2) = \frac{M}{k_1} f_2(y_2', y_2), \quad M = \frac{m_1}{m_2}, \quad q^2 = \frac{k_2/m_2}{k_1/m_1}.$$

Equations (2.2) can be transformed into the quasi-normal form using transformation

$$y_1 = x_1 + x_2, \qquad y_2 = a_1 x_1 + a_2 x_2 \tag{2.3}$$

where

$$a_1 = M/(q^2 + M - \Omega_1^2),\; a_2 = M/(q^2 + M - \Omega_2^2),$$
$$(\Omega^2)_{1,2} = \frac{1}{2}(1 + q^2 + M \pm \sqrt{(1 + q^2 + M)^2 - 4q^2}).$$

Note: It can be proved that (see [5]) the following relations are valid:

$$a_1 > 0, \quad a_2 < 0, \quad a_1 a_2 = -M. \tag{2.4}$$

In this way questions (2.2) get the form:

$$x_1'' + \Omega_1^2 x_1 + \frac{\varepsilon}{a_1 - a_2}\left[-a_2 F_1(x_1' + x_2',\, x_1 + x_2) + F_2(a_1 x_1' + a_2 x_2',\, a_1 x_1 + a_2 x_2)\right] = 0, \qquad (2.5)$$

$$x_2'' + \Omega_2^2 x_2 + \frac{\varepsilon}{a_1 - a_2}\left[a_1 F_1(x_1' + x_2',\, x_1 + x_2) - F_2(a_1 x_1' + a_2 x_2',\, a_1 x_1 + a_2 x_2)\right] = 0.$$

Seeking the single-frequency vibration the harmonic balance method can be used to determine the approximate solution. The solution with the first mode can be sought in the form:

$$x_1 = X_1 \cos \Omega\tau, \quad x_2 = 0, \quad (X_1 > 0), \qquad (2.6)$$

the solution with the second mode in the form:

$$x_1 = 0, \quad x_2 = X_2 \cos \Omega t, \quad (X_2 > 0). \qquad (2.7)$$

Inserting these into equations (2.5) and comparing the coefficients at $\cos \Omega\tau$ (considering above mentioned assumption on the form of functions F_1, F_2) the following results are obtained for the first and second mode vibrations:

$$\Omega = \Omega_1, \qquad \Omega = \Omega_2. \qquad (2.8)$$

When comparing the coefficients at $\sin\Omega\tau$ the following relations are obtained for the first mode:

$$\frac{\Omega}{\pi}\int_0^{2\pi/\Omega_1} [-a_2 F_1(-\Omega_1 X_1 \sin \Omega_1\tau,\, X_1 \cos \Omega_1\tau,) + F_2(-a_1\, X_1 \sin \Omega_1\tau, a_1 X_1 \cos \Omega_1\tau)] \sin \Omega_1 \tau d\tau = 0.$$

(2.9)

Similarly for the second mode we obtain:

$$\frac{\Omega}{\pi}\int_0^{2\pi/\Omega_2} [a_1 F_1(-\Omega_2 X_2 \sin \Omega_2\tau,\, X_2 \cos \Omega_2\tau,) - F_2(-a_2\, X_2 \Omega_2 \sin \Omega_2\tau, a_2 X_2 \cos \Omega_2\tau)] \sin \Omega_2 \tau d\tau = ($$

(2.10)

In the further Section several alternatives of functions F_1 F_2 are analyzed.

3. Analytical results

Using the approach described in the previous section and different types of self-excitation and foundation damping the following result are obtained. We shall suppose that all coefficients of the terms describing the self-excitation as well as of the foundation damping are positive. Let us start with the most used mathematical model: van der Pol self-excitation.

System I $\quad F_1(y_1', y_1) = (-\beta + \delta\, y_1^2) y_1'. \qquad (3.1)$

The following alternatives of foundation damping are considered:

Ia $\quad F_2(y_2', y_2) = \kappa\, y_2', \qquad (3.2)$

Ib $\quad F_2(y_2', y_2) = \gamma\, y_2^2\, y_2', \qquad (3.3)$

Ic $\quad F_2(y_2', y_2) = \kappa\, y_2' + \vartheta \operatorname{sgn}(y_2'), \qquad (3.4)$

Alternative Ia

Using the method of harmonic balance equation (2.9) get the form

$$-a_2[-\Omega_1 X_1(-\beta+\frac{1}{4}\delta X_1^2)]-a_1\kappa\Omega_1 X_1=0. \tag{3.5}$$

From which follows:

$$X_1^2=\frac{4}{\delta}(\beta+\frac{a_1}{a_2}\kappa) \tag{3.6}$$

Considering that $a_1/a_2<0$ for the real value of X_1 reads:

$$\beta>-\frac{a_1}{a_2}\kappa. \tag{3.7}$$

Then relation

$$\kappa>-\frac{a_2}{a_1}\beta. \tag{3.8}$$

This is the condition for the full suppressing of the vibration with the first mode. Similarly the relation for X_2 reads:

$$X_2^2=\frac{4}{\delta}(\beta+\frac{a_2}{a_1}\kappa). \tag{3.9}$$

The condition for the full suppression of the second mode vibration reads:

$$\kappa>-\frac{a_1}{a_2}\beta. \tag{3.10}$$

Alternative Ib

Using the same approach the following results are obtained:

$$X_1^2=4\beta/(\delta-\frac{a_1}{a_2}\gamma), \qquad X_2^2=4\beta/(\delta-\frac{a_2}{a_1}\gamma). \tag{3.11}$$

We can see, considering that $\delta-(a_1/a_2)\gamma$ and $\delta-(a_2/a_1)\gamma$ are positive, that both X_1 and X_2 always exist although the progressive foundation damping reduces the vibration.

Alternative Ic

For this alternative equation (2.9) get the form

$$-a_2[\Omega_1 X_1(\beta-\frac{1}{4}\delta X_1^2)-a_1(\Omega_1 X_1\kappa+\frac{4}{\pi}\vartheta)=0. \tag{3.12}$$

This can be written in the form

$$\Phi_1(X_1)=\Omega_1 X_1(\beta+\frac{a_1}{a_2}\kappa)=-\frac{4}{\pi}\frac{a_1}{a_2}\vartheta+\frac{1}{4}\delta X_1^3\Omega_1=\Phi_2(X_1). \tag{3.12a}$$

For the second vibration mode the following relation is valid:

$$\Phi_1(X_2) = \Omega_2 X_2(\beta + \frac{a_2}{a_1}\kappa) = -\frac{4}{\pi}\frac{a_2}{a_1}\vartheta + \frac{1}{4}\delta X_2^3 \Omega_2 = \Phi_2(X_2). \tag{3.13}$$

The solution for X_1 (or X_2) can be achieved as section points of the curves $\Phi_1(X_1)$ and $\Phi_2(X_1)$ (respectively of $\Phi_1(X_2)$ and $\Phi_2(X_2)$). The first curve is a straight line going through the origin and the second curve is an increasing function with increasing X_1 (or X_2). This for X_1 (or X_2) has a positive value. There exist three alternatives (see schematically in Fig. 2): No section points, two section points exist, the curves touch one another.

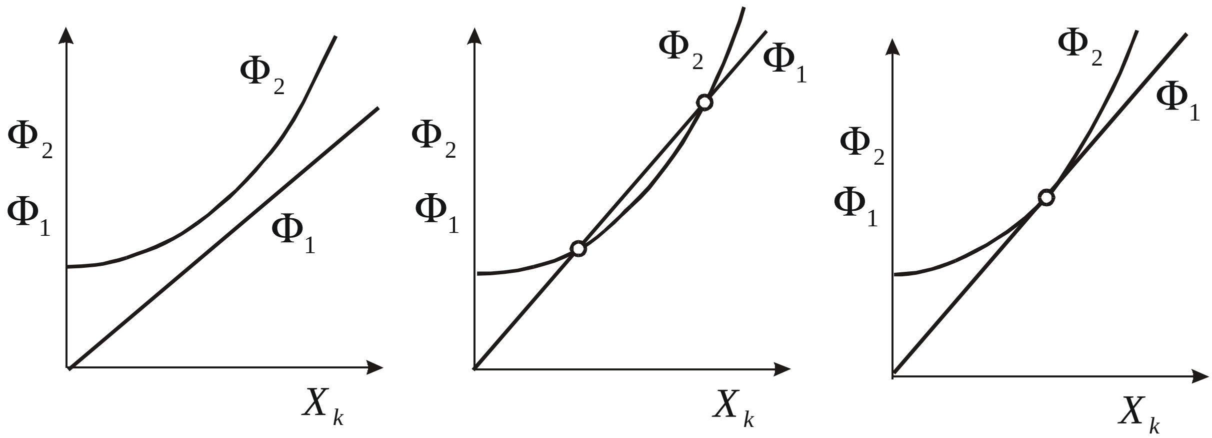

Fig. 2 Schematic representation of curves $\Phi_1(X_k), \Phi_2(X_k)\ (k=1,2)$,

courses of the curves.

In the first case no vibration with the corresponding vibration mode can occur. In the second case the section point with lesser X_1 corresponds to unstable solution, the point with greater X_1 to the stable solution. There exist two domains of attraction for initial conditions: one belongs to non-oscillatory solution (of course corresponding to certain mode) the other to the stable vibration. The third case represents such a boundary case between the previous ones.

System II

This does not belong to the class of self-excited systems characterized by the negative linear viscous damping. Here the self-excitation is described by

$$F_1(y_1', y_1) = -\Theta \operatorname{sgn}(y_1') + \kappa_1 y_1'. \tag{3.14}$$

Foundation damping is given by the following alternatives:

(a) $$F_2(y_2', y_2) = \kappa_2 y_2', \tag{3.15}$$

(b) $$F_2(y_2', y_2) = \vartheta_2 \operatorname{sgn}(y_2'). \tag{3.16}$$

Using again the same approach the following results are obtained:

Alternative IIa $$X_1 = \frac{4}{\pi}\Theta \Big/ \left[\Omega_1(\kappa_1 - \frac{a_1}{a_2}\kappa_2)\right], \tag{3.17}$$

$$X_2 = \frac{4}{\pi}\Theta / \left[\Omega_2 (\kappa_1 - \frac{a_2}{a_1}\kappa_2) \right]. \tag{3.18}$$

We can see that in no case X_1 or X_2 can reach zero value; the vibration amplitude is reduced only due to the action of the positive foundation damping.

Alternative IIb
$$X_1 = \frac{4}{\pi}(\Theta + \frac{a_1}{a_2}\vartheta_2), \tag{3.19}$$

$$X_2 = \frac{4}{\pi}(\Theta + \frac{a_2}{a_1}\vartheta_2). \tag{3.20}$$

In opposite to the previous case a full suppressing of the vibration can be achieved.

System III

Here the term is also nonlinear but of the second order, i.e. the self-excitation is characterized as follows:

$$F_1(y_1', y_1) = -\beta_1 |y_1| y_1' + \delta_1 y_1^2 y_1'. \tag{3.21}$$

The foundation damping is characterized as follows:

(a) $$F_2(y_2', y_2) = \vartheta_2 \operatorname{sgn}(y_2'), \tag{3.22}$$

(b) $$F_2(y_2', y_2) = \kappa_2 y_2' + \delta_2 |y_2| y_2'. \tag{3.23}$$

Alternative IIIa

For amplitudes X_1 X_2 the following relations are valid:

$$\frac{1}{4}\delta_1 \Omega_1 X_1^3 - \frac{4}{\pi}\frac{a_1}{a_2}\vartheta_2 = \frac{4}{3\pi}\beta_1 \Omega_1 X_1, \tag{3.24}$$

$$\frac{1}{4}\delta_1 \Omega_2 X_2^3 - \frac{4}{\pi}\frac{a_2}{a_1}\vartheta_2 = \frac{4}{3\pi}\beta_1 \Omega_2 X_2 \tag{3.25}$$

The situation is similar to that for Alternative Ic.

Alternative IIIb

Amplitudes X_1, X_2 can be obtained from relations:

$$(X_1)_{1,2} = \frac{4}{\delta_1}\left[\frac{2}{3\pi}\left(\beta_1 + \frac{a_1}{a_2}\delta_2\right) \pm \sqrt{\left[\frac{2}{3\pi}\left(\beta_1 + \frac{a_1}{a_2}\delta_2\right)\right]^2 + \frac{1}{4}\delta_1 \frac{a_1}{a_2}\kappa_2} \right], \tag{3.26}$$

$$(X_2)_{1,2} = \frac{4}{\delta_1}\left[\frac{2}{3\pi}\left(\beta_1 + \frac{a_2}{a_1}\delta_2\right) \pm \sqrt{\left[\frac{2}{3\pi}\left(\beta_1 + \frac{a_2}{a_1}\delta_2\right)\right]^2 + \frac{1}{4}\delta_1 \frac{a_2}{a_1}\kappa_2} \right]. \tag{3.27}$$

For this alternative the conditions

$$\left[\frac{2}{3\pi}\left(\beta_1+\frac{a_1}{a_2}\delta_2\right)\right]^2>\frac{1}{4}\delta_1\left|\frac{a_1}{a_2}\right|\kappa_2\,, \tag{3.28}$$

$$\left[\frac{2}{3\pi}\left(\beta_1+\frac{a_2}{a_1}\delta_2\right)\right]^2>\frac{1}{4}\delta_1\left|\frac{a_2}{a_1}\right|\kappa_2\,. \tag{3.29}$$

must be met in order to get real amplitudes X_1, X_2. From the above results it follows that a full suppression can be achieved.

4. Conclusion

A general rule can be formulated: A full suppression of self-excited vibration can be achieved when the foundation damping component has the same form as the negative component of the self-excitation of the basic subsystem and, furthermore, certain conditions are met.

A full suppression can also be achieved in the case of the nonlinear higher order form of the negative part of the self-excitation when certain conditions are met even if the terms of the foundation damping have not exactly the same character as the mentioned negative part of the self-excitation.

One aim of this contribution is to initiate further and deeper analysis especially as for the nonlinearly self-excited systems.

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