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FUZZY DISTRIBUTION OF INTERNAL FORCES IN SEISMICALLY LOADED FRAME STRUCTURE

P. Štemberk, J. Kruis, Z. Bittnar[1]

Summary: *In this paper, an approach to dynamic analysis based on the fuzzy set theory is presented. The dynamic analysis uses the response spectra method where the material parameters are considered fuzzy, which results in mode shapes and natural frequencies with fuzzy distribution. The subsequent design requires knowledge of internal forces, which in this case, are also in the form of fuzzy numbers. The resulting fuzzy distribution of internal forces in the structural elements reflects the degree of uncertainty contained in the input material parameters. This approach is explained in an illustrative example.*

1. Introduction

Concrete, as a convenient building material, inherently involves uncertainty about its composition, which is difficult to be eliminated completely, however, this uncertainty can be assessed by statistical, fuzzy, or other suitable tools. For design purposes, one may wish to conduct a statistical analysis, using the statistical characteristics of several measured events. In the case of earthquake, the measured data for each site of interest is not particularly dense, leaving the statistical characteristics with little relevance. On the other hand, the expected seismic load at a site can be alternatively expressed by the fuzzy sets, (Zadeh, 1965), which take into account the scarcity of seismic stations and the information about local sub-soil composition.

In this paper, an approach to dynamic analysis based on the fuzzy set theory is presented as a pre-step of the classical stochastic dynamic analysis. The material parameters of reinforced concrete are considered to be fuzzy quantities with a given distribution, i.e. fuzzy numbers with a desired shape of the membership function, (Valliappan and Pham, 1993). The dynamic analysis is performed with help of the fuzzy arithmetic on the α-cuts, (Kaufman and Gupta, 1985). The result of such an analysis is in the form of fuzzy numbers which compared with the stochastic approach is less expensive in terms of computation time and still it provides the designer with an idea of distribution of the sought quantity, (Kala, 2005). In order to further improve the computational efficiency, inspired by (Akpan et al., 2001), the concept of a surface response function, (Bucher et al., 1988; Rajashekhar and Ellingwood, 1993), is

[1] Ing. Petr Štemberk, Ph.D., doc.Ing. Jaroslav Kruis, Ph.D., Prof. Ing. Zdeněk Bittnar, DrSc.: Fakulta stavební, České vysoké učení technické v Praze; Thákurova 7; 166 29 Praha 6; tel.: +420.224 354 364, fax: +420.224 353 740; e-mail: stemberk@fsv.cvut.cz

utilized. This approach is demonstrated in an illustrative example of a 2D frame where the effect of uncertain material parameters transpires in corresponding distributions of natural modal shapes, natural frequencies of an analyzed two-dimensional frame. The results of the natural vibration analysis are then used in the investigation of structural vibration by the response spectrum. The methodology for application to seismic design is explained and the following procedure of reliability assessment is hinted. The question of how many α-cuts are necessary and how their number influences the result is also tackled.

2. Fuzzy numbers and fuzzy arithmetic

The uncertainty, which is present in input parameters, can be handled with help of the fuzzy set theory (Zadeh, 1965), where the uncertain quantities are defined in terms of fuzzy sets. Unlike in the classical set theory, the membership of an element to a fuzzy set includes the values between 0 and 1, where 0 means "does not belong" and 1 means "definitely belongs" to a fuzzy set. Usually, the fuzzy sets represent vague verbal evaluation. In cases when a fuzzy set represents a numeral, it is called a fuzzy number.

Fuzzy numbers

The notion of a fuzzy number arises from the experience of the everyday life when many phenomena which can be quantified are not characterized in the terms of absolutely precise numbers.

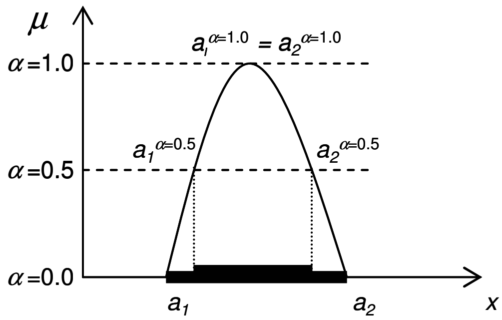

Fig.1. Normal fuzzy number and its α-cuts.

Fuzzy numbers are fuzzy sets which are defined on the set of real numbers. Their membership function assigns the degree of 1 to the central, also called nominal, modal or mean, value and lower degrees to other numbers which reflect their proximity to the central value according to the used membership function. The membership function should thus decrease from 1 to 0 on both sides of the central value. Such fuzzy sets are called fuzzy numbers. An example of a fuzzy number is shown in Fig.1, where μ represents the membership function and a1 and a2 stand for two real numbers on the real axis. The intervals defined for a specific value of the membership function, e.g., $\alpha = 0.7$, represent the so-called α-cuts. A fuzzy number can be equally expressed by either a nominal value and a membership function on each side of the nominal value or by a set of α-cuts.

Fuzzy arithmetic

A fuzzy arithmetic operation depends on the definition of a fuzzy number. In the cases when fuzzy numbers are defined by a set of α-cuts, the problem of fuzzy arithmetic is reduced to the well-known arithmetic operations on intervals, which are applied to each α-cut. In the case of real-life problems, such as dynamic analyses based on the finite element method, an extensive number of arithmetic operations are necessary and the formulation in the above terms is relatively expensive.

To eliminate the drawback of the α-cut formulation, new techniques for solving fuzzy linear equation systems have been developed, e.g. presented in (Buckley and Qu, 1991). However, these techniques are not easily applicable to robust problems, such as the fuzzy dynamic finite element analysis. Therefore, another technique for reducing the large number of combinations, originally developed for other problems, e.g. statistical analysis, was exploited – the concept of the response surface function.

3. Response surface function

Fuzzy analyses, as well as stochastic analyses, suffer from non-occurrence of analytical solutions in the case of non-deterministic input data. The non-occurence of analytical solution requires application of a suitable numerical method which discretizes the problem and solves it numerically. In order to reduce the necessary number of computation runs, the concept of the response surface function has been used many times, (Bucher et al., 1988; Rajashekhar and Ellingwood, 1993). The basic idea of the response surface function is to approximate a response of a structure by a number of selected parameters, which depend on the selected input parameters. The relation between the input and the output parameters should be as simple as possible. The polynomial function in the form

$$f^{(k)}(x) = a^{(k)} + \sum_{i=1}^{n} b_i^{(k)} x_i + \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij}^{(k)} x_i x_j, \tag{1}$$

is very popular. The superscript identifies an output parameter and n denotes the number of input parameters. The unknown coefficients *a*, *b* and *c*, can be obtained from, e.g. the least square method.

4. Response of structure to seismic load

In the case of known accelerograms, the response of a structure can be obtained by numerical integration of the equation of motion (Clough and Penzien, 1993; Bittnar and Šejnoha, 1996), which has the form

$$\boldsymbol{M} \frac{d^2 \mathbf{d}(t)}{dt^2} + \mathbf{C} \frac{d\mathbf{d}(t)}{dt} + \mathbf{K}\mathbf{d}(t) = \mathbf{f}(t), \tag{2}$$

where $\boldsymbol{M}$ denotes the mass matrix, $\boldsymbol{C}$ stands for the damping matrix, $\boldsymbol{K}$ denotes the stiffness matrix, $\boldsymbol{f}(t)$ expresses the load vector and $\boldsymbol{d}(t)$ is the vector of the nodal displacements which are computed. t stands for time. For design purposes, an artificial accelerogram is provided by the standards.

Prior to the introduction of powerful computers, the response of a structure to seismic load was computed with the help of response spectra. This method was also used for our purposes to preliminary estimate the structural behavior, and so it is briefly described.

A single degree of freedom subjected to seismic loading can be described by the equation

$$\ddot{v}(t) + 2\xi\,\omega\dot{v}(t) + \omega^2\, v(t) = -f\,\ddot{v}_g(t), \tag{3}$$

where $v(t)$ denotes the relative displacement, $\ddot{v}_g(t)$ denotes the ground acceleration, ω denotes the natural frequency, ξ damping ratio and f denotes the mode participation factor. The relative displacement can be expressed by the Duhamel integral

$$v(t) = \frac{f}{\omega}\int_0^t -\ddot{v}_g(\tau)\sin\omega(t-\tau)e^{-\xi\omega(t-\tau)}d\tau, \tag{4}$$

where τ denotes the time of load application and small values of ξ (less than 0.1) are assumed. Let the ground acceleration $\ddot{v}_g(\tau)$ be known and the damping ratio be estimated as $\xi = const$. Then, the relative displacement depends on the natural frequency and time. Let $v(\omega)_{\max}$ denote the maximum value of the relative displacement for a particular natural frequency. The displacement response spectrum is defined by

$$S_d(\omega) = v(\omega)_{\max}. \tag{5}$$

In other words, the maximum values of the relative displacements computed from Eq.(4) are computed for all possible natural frequencies and they create a curve which is denoted as the displacement response spectrum. The displacement response spectra are, however, usually expressed in terms of the period rather than in the terms of natural frequencies. The period of natural vibration is related to the natural frequency by the known relationship

$$T = \frac{2\pi}{\omega}. \tag{6}$$

The pseudo-velocity spectrum is defined by

$$S_v(\omega) = \omega S_d(\omega) \tag{7}$$

and the pseudo-acceleration spectrum is defined by

$$S_a(\omega) = \omega^2 S_d(\omega). \tag{8}$$

The response of a general structure to dynamic load can be formulated directly by the modal analysis. In that case, the system of the equations of motion is transformed to several single equations similar to Eq.(3), where the subscripts i have to be added. v_i denotes the coefficient of linear combination of the i-the eigenmode (it expresses the influence of the i-th eigenmode. The maximum value of the coefficient v_i is obtained from S_d or $S_a/\omega_i 2$, where ω_i is the i-th natural frequency.

The uncertainty in material parameters, described by fuzzy numbers, results in fuzzy natural frequencies and fuzzy eigenmodes. The response of a structure with uncertain parameters is therefore also uncertain. The total seismic response is then described by fuzzy numbers. The primal difference between the crisp and the fuzzy computation of seismic response with the help of the response spectrum is the fuzziness of the i-th period T_i. Therefore, the $v_i(T_i)_{\max}$ are also described by fuzzy numbers. The total displacements are the results of summation of contributions from particular modes.

5. Fuzzy reliability concept

The concept of reliability is traditionally connected with the probability evaluation, e.g. expressed by the reliability index, β, therefore, development of a novel reliability approach based purely on the fuzzy set theory is meaningful due to the necessity to revolutionize the notion of reliability which is already well-established, moreover, the reliability evaluation based on the fuzzy set theory cannot express the frequency of occurrence of an event in the sense known from the probability theory.

On the other hand, combination of the common reliability evaluation with the fuzzy set theory helps to take into account the vagueness contained in the stochastic information, known as the fuzzy randomness. The result of such reliability assessment is in the form of the reliability index, β, with fuzzy distribution. This method is explained in (Möller and Beer, 2004). However, this is not the objective of our work, since the primary objective in this paper is the preliminary analysis of a structure, which gives information of its possible behavior. We believe that it is reasonable to check first whether some event can occur at all, and once its occurrence is assured, it is desirable to evaluate the frequency of its occurrence, that means, to perform reliability assessment, which is due to heavy sampling much more computationally expensive.

6. Preliminary assessment of structure - example

As an example, the natural frequency analysis of a two-dimensional frame with four floors made of reinforced concrete is considered. Then, the possible distribution of internal forces in the frame is computed. Also, the possible distribution of a horizontal displacement of a joint is shown for an instance.

The overall height of the frame is 16 meters and the width is 5 + 5 meters. The dimensions of beams and columns are identical (0.5 x 0.5 m). It is assumed that the building was erected in four consecutive lifts. Each lift consists of placing concrete in three columns and in the beam which connects the upper ends of the columns. Therefore, it is further assumed that there are only four types of concrete whose composition can possibly differ. The influencing material parameters are the modulus of elasticity, E, and the density, ρ. E and ρ are fuzzy input parameters with nominal values of 30 GPa and 2500 kg/m^3, respectively, which can change by $\pm$ 10 % and are represented by fuzzy numbers with a linear membership function (triangular fuzzy numbers).

For our illustrative purposes, we need 925 response surface functions to describe the first five natural vibration modes, i.e. a response surface function to express each natural frequency and the horizontal and vertical displacements in each joint (23 joints on each of the four floors) for each natural mode shape. In order to obtain sufficient input and output data for calculation of the coefficients of the response surface functions, Eq. (1), it was decided to take three values (minimum, modal value, maximum) for each material parameter, E and ρ, that means $3^{2\times 4}$ (=6561) independent runs of the eigenvalue problem. The specific form of Eq.(1) in this example was

$$\begin{aligned} f^{(k)}(x) = {} & b_1^{(k)} E_1 + b_2^{(k)} E_2 + b_3^{(k)} E_3 + \\ & + b_4^{(k)} E_4 + b_5^{(k)} \rho_1 + b_6^{(k)} \rho_2 + \\ & + b_7^{(k)} \rho_3 + b_8^{(k)} \rho_4 + b_9^{(k)}, \end{aligned} \tag{9}$$

where E_1, E_2, E_3, E_4, ρ_1, ρ_2, ρ_3 and ρ_4 are input parameters and b_1, …, b_9 are output parameters. The computation is fully automated using the SIFEL package. The fourth mode shape is shown in Fig.2, where the dotted lines represent all possible envelopes of response, in other words, the minimum and maximum values, which correspond to the values obtained for α-cuts with $\alpha = 0$. The finite element model of this frame discretizes each frame section (beam and column) by five beam elements.

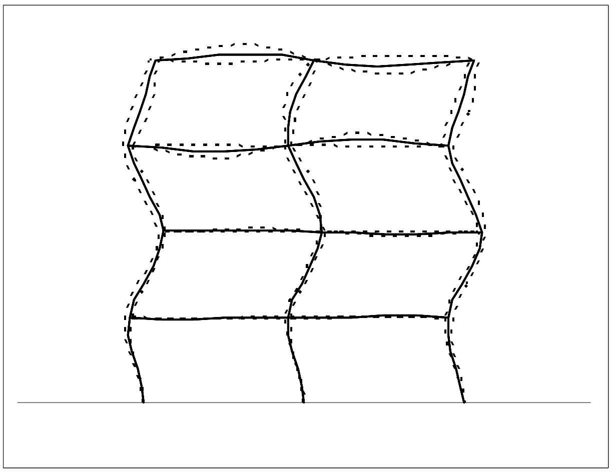

Fig.2. Fourth fuzzy mode shape of frame structure.

In order to verify the necessary number of input data (the number of α-cuts) for obtaining the response surface functions, the surface response functions were also calculated for five values, corresponding to the α-cut values with α equal to 0, 0.5 and 1, but that already meant $5^{2\times4}$ (=390625) independent runs of the eigenvalue problem. The improvement was negligible compared to the computational effort.

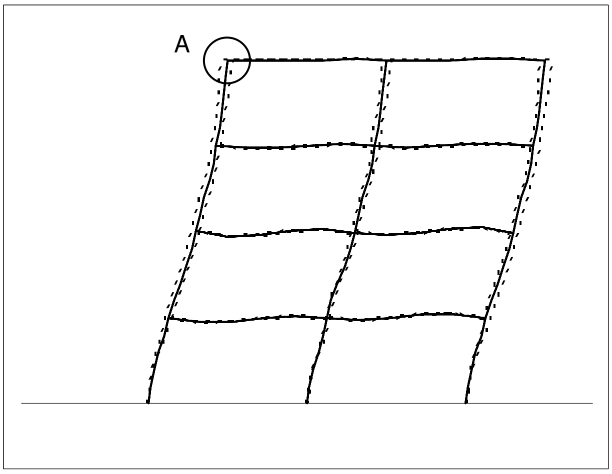

Fig.3. Distribution of displacements.

In the design of earthquake resistant structures, it is essential not to neglect any uncertainty as it may lead to an erroneous conclusion due to the dynamic simulation which may amplify such uncertainty beyond all limits. For those reasons it seems reasonable to express uncertain numerical data in terms of fuzzy numbers and use them as such in analyses to cover all possible solutions.

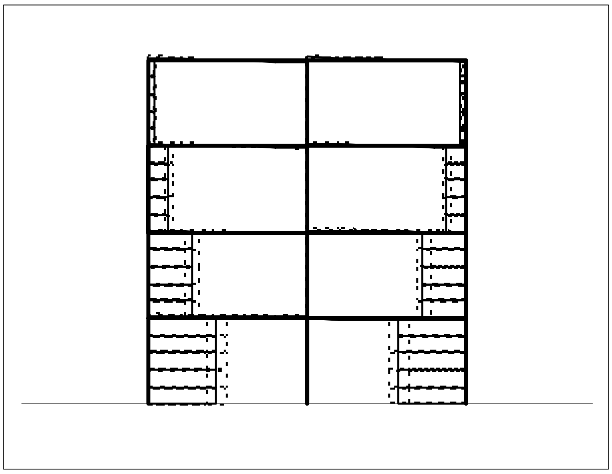

Fig.4. Distribution of normal forces.

Once the natural frequency analysis is finished, the acquired natural mode shapes are used for computation of the response of the RC frame to earthquake induced excitation, which is prescribed by a response spectrum given in a design standard. Following the procedure described above, the displacements, accelerations and internal forces are obtained. Fig. 3 shows the resulting distribution of displacements composed from the five natural mode shapes multiplied by the respective mode participation factors. Fig.4, Fig.5 and Fig.6 show the possible distributions of the internal forces. For an instance, the fuzzy distribution of the horizontal displacement of the joint marked by A in Fig.3 is shown in Fig.7. Provided with these results, the designer can already know whether the computed quantity exceeds the allowed limits, and if yes, then to which extent.

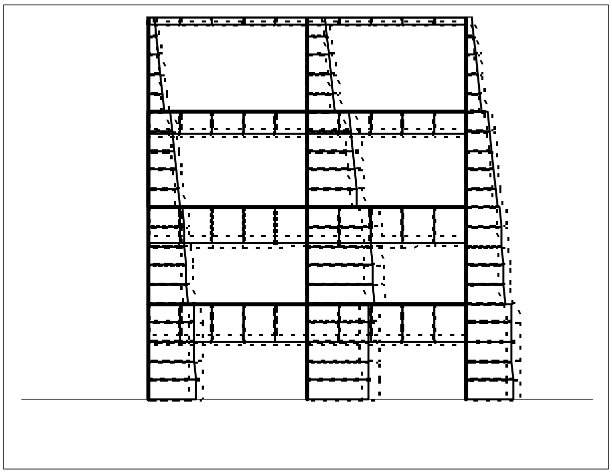

Fig.5. Distribution of shear forces.

At this moment the spread of the quantity reflects the uncertainty in the material parameters. In the case the results are too far from the allowed limits, the structure needs to be redesigned. It should be noted that the computation of this simple example lasted for about 20 minutes on an ordinary PC with the Pentium III processor. The real-life structures are much more complex and the 3-dimensional analysis is usually preferred. Bearing in mind that such computation can last for tens of hours, the advantage of this approach is obvious. The preliminary results obtained by the proposed method already provide the extra information on the effect of imprecision in the input data, compared with the deterministic analysis, and yet it is faster than the probability-based reliability-assessment, which should be better run at the stage of the design when it is sure that the structure would comply with the requirements of the design standards, or hygienic provisions.

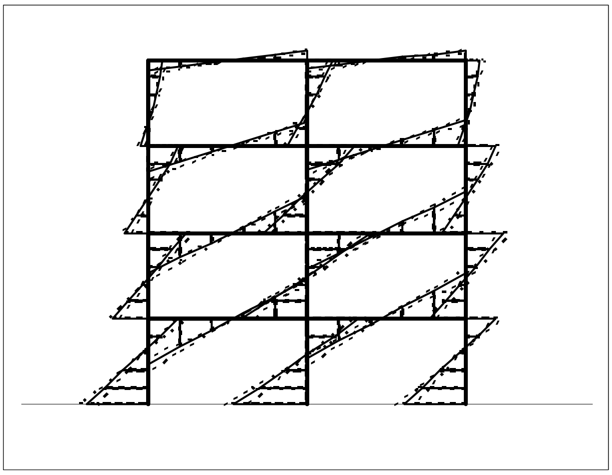

Fig.6. Distribution of bending moments.

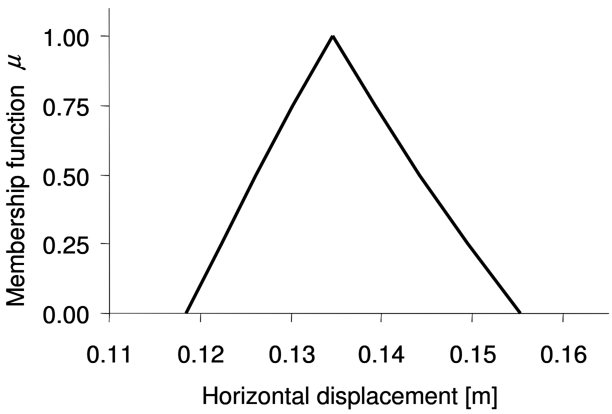

Fig.7. Horizontal displacement at joint A (design value).

7. Conclusions

This paper presents an approach to the preliminary assessment of structural behavior, which can serve as an efficient tool for verification that the structural response is within design limits even if the input data contain imprecision or vagueness. The efficiency is caused by the use of only the maximum and minimum values of the input parameter for each α-cut, which in our case are the input values corresponding to the values at α equal to 0 and 1. The mean output values are obtained by the ordinary single deterministic run for mean input values. Once the result of this verification is positive, the ordinary reliability analysis, which is more computationally expensive, can be run. The advantage of the proposed approach is that at the design stage time usually spent on the trial-and-error process is spared and allows to spend more time on the subsequent ordinary reliability assessment, which is quite computationally demanding, once it should comprise all the relevant sampling.

8. Acknowledgement

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9. References

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