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MODIFICATION OF DYNAMIC PROPERTIES OF CIRCULAR PLATE USING CONSTRAINING LAYERS

M. Naď*

Summary: *A method for modification of dynamic properties of circular plate is presented. The modal characteristics (natural frequencies, mode shapes) of circular plate are changed by using of constraining layers. The part of circular plate is embedded into layers for which the different geometrical parameters and material properties are supposed. The effect of mass and stiffness structural modifications created using this approach on modal properties of circular plate is analysed.*

1. Introduction

Vibration and acoustic requirements are becoming increasingly important in the design of mechanical structures. The need to vary the structural behaviour to solve noise and vibration problems often occurs at the design or prototype stages (Bodnicki 2001, Nánási 2003), giving rise to the so-called structural modification problem. Structural modification is a procedure aimed at identifying the changes required in a structural system to modify its dynamic behaviour (natural frequencies, structural modes, frequency response).

Structural modification with connection of modal analysis technology refers to a technique to modify local physical properties of structure in order to change or optimize its dynamic characteristics. The dynamic characteristics of a structure, usually referred to as its natural frequencies and mode shapes, are determined by its mass, stiffness and damping properties. The properties outlined by these distributions are called the spatial properties of the structure. The spatial properties are often quantified by a mathematical model of the structure, such as a finite element model. This model translates the physical properties of the structure, such as its dimensional, geometrical and material properties, into distributed mass, damping and stiffness properties. For structural modification using a finite element model, it is possible to determine the modification in terms of mass, damping and stiffness changes. However for a real-life structure, it is more important to determine the structural modification in terms of physical parameter (such as thickness, length, Young modulus, density, etc.) changes.

The main objectives of structural dynamic modification techniques are generally to reduce vibration levels, shift resonance frequencies, improve dynamic stability, place optimally the

* Ing. Milan Naď, CSc.: Department of Applied Mechanics, Faculty of Materials Science and Technology in Trnava, Slovak University of Technology; Paulínska 16; 917 24 Trnava; phone.: +421 33 55 11 733, fax: +421 33 55 11 668; e-mail: milan.nad@stuba.sk

modal points, perform modal synthesis and optimise the weight and cost. There are two main reasons for structural modification. First, an existing structure may exhibit unsatisfactory dynamic characteristics. Second, the design of a structure which is known to experience a dynamic working environment needs to satisfy some well defined criteria such as averting vibration resonance. The dynamic characteristics of a structure usually referred to as its natural frequencies and mode shapes, are determined by its mass, stiffness and damping distributions. For a real-life structure, it is more important to determine the structural modification in terms of physical parameter changes.

In the present paper the change of dynamical properties of circular plate which is clamped on inner radius and embedded into both-sided constraining layers covering part of the plate on inner radius is investigated. A basic structural models and parametric study of dynamic properties of this structure will be discussed in following sections. It provides not only the information of the dynamics of circular plate on inner radius clamped and embedded into constraining layers but the results can be used also for its suitable dynamic modification. The finite element method (program ANSYS) is used to solve dynamic properties of this plate

2. Formulation of the structural modification problem

Generally, for a damped mechanical system, the equation of motion can be represented as follows

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{B}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{F}(t), \tag{1}$$

where $\mathbf{M}$ is the mass matrix, $\mathbf{B}$ is the damping matrix and $\mathbf{K}$ is the stiffness matrix of the system. The vectors $\mathbf{x}, \dot{\mathbf{x}}, \ddot{\mathbf{x}}$ and $\mathbf{F}(t)$ are vectors of the displacements, velocities, accelerations and forces, respectively.

The undamped system without force excitation is defined by

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{0}. \tag{2}$$

Equations (1) and (2) can be transformed using the transformation equations

$$\mathbf{x}(t) = \mathbf{\Phi}\mathbf{q}(t), \qquad \mathbf{\Phi}^T\mathbf{M}\mathbf{\Phi} = \mathbf{I}, \qquad \mathbf{\Phi}^T\mathbf{K}\mathbf{\Phi} = \mathbf{\Lambda}, \tag{3}$$

where $\mathbf{\Phi} = [\boldsymbol{\phi}_1, \boldsymbol{\phi}_2, ..., \boldsymbol{\phi}_N]$ is matrix of modal vectors.

Eigenvalue problem of system (2) is written by

$$(\mathbf{\Phi}^\mathbf{T}\mathbf{K}\mathbf{\Phi} - \omega^2\mathbf{\Phi}^\mathbf{T}\mathbf{M}\mathbf{\Phi})\mathbf{q} = (\mathbf{\Lambda} - \omega^2\mathbf{I})\mathbf{q} = \mathbf{0}. \tag{4}$$

where $\lambda = \omega^2$ is eigenvalue and ω is natural angular frequency.

If modification of the system has to be incorporated through changes in layer parameters, the mass and stiffness properties of the undamped mechanical system are also modified and equation (1) becomes

$$(\mathbf{M} + \Delta\mathbf{M})\ddot{\mathbf{x}} + (\mathbf{K} + \Delta\mathbf{K})\mathbf{x} = \mathbf{F}(t) \tag{5}$$

and for modified system without external excitation can be expressed as

$$(\mathbf{M} + \Delta\mathbf{M})\ddot{\mathbf{x}} + (\mathbf{K} + \Delta\mathbf{K})\mathbf{x} = \mathbf{0}. \tag{6}$$

The matrices $\Delta\mathbf{M}$ and $\Delta\mathbf{K}$ characterise mass and stiffness modifications in the spatial model. The practical modification is not carried out on matrices but on physical components or parameters of the structure.

Using equations (3), eigenvalue problem of modified system (6) is

$$[\mathbf{\Phi}_m^T(\mathbf{K}+\Delta\mathbf{K})\mathbf{\Phi}_m - \omega_m^2\mathbf{\Phi}_m^T(\mathbf{M}+\Delta\mathbf{M})\mathbf{\Phi}_m]\mathbf{q}_m = \mathbf{0}\,, \qquad (7)$$

where ω_m is the natural angular frequency of modified system.

Equation (7) provides the new natural angular frequencies (ω_m) and new mode shapes (ϕ_m) of the system after structural modification (Srivastava & Kundra, 1994). The natural angular frequency after modification of the circular plate for i-th mode shape can be generally expressed by relation

$$\omega_{m,i} = \omega_{0,i} \pm \Delta\omega_i\,, \qquad (8)$$

where $\omega_{0,i}$ is natural angular frequency of structure before modifications for i-th mode shape, $\Delta\omega_i$ is change of natural angular frequency caused by structural modifications.

3. Numerical example

We consider a circular plate of a inner radius r_1, outer radius r_2 and thickness h. The plate is clamped on inner radius and embedded into both-sided elastic layers of thickness h_v covered part of plate on inner radius. The model of this plate is shown on Fig. 1.

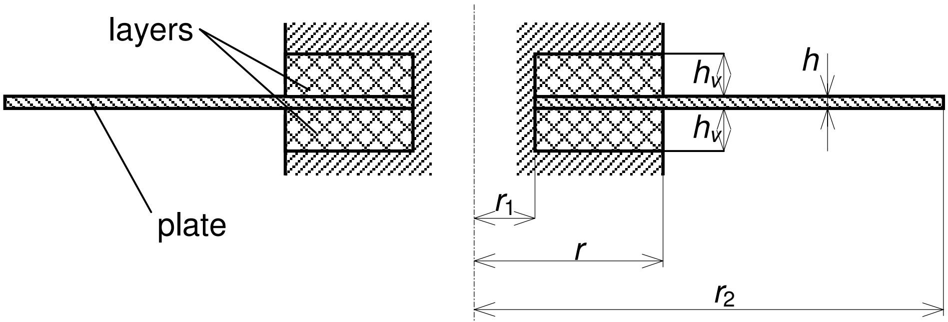


Fig. 1 Model of circular plate in constraining layers

The analysis of the layer parameters on dynamic properties of circular plate (Fig.1) are studied. The material properties and geometrical parameters are shown in Table 1.

Table 1 Material properties and geometrical parameters

Outer radius of plate	r_2	[m]	0,15
Inner radius of plate	r_1	[m]	0,015
Thickness of plate	h	[m]	0,0015
Radius of layers	r	[m]	0,015 ÷ 0,15
Thickness of layers	h_v	[m]	0,001 ÷ 0,030

	plate	layers
Young modulus [GPa]	210	0,001 ÷ 100
Poisson number [-]	0,3	0,3
Density [kg.m^{-3}]	7800	1000

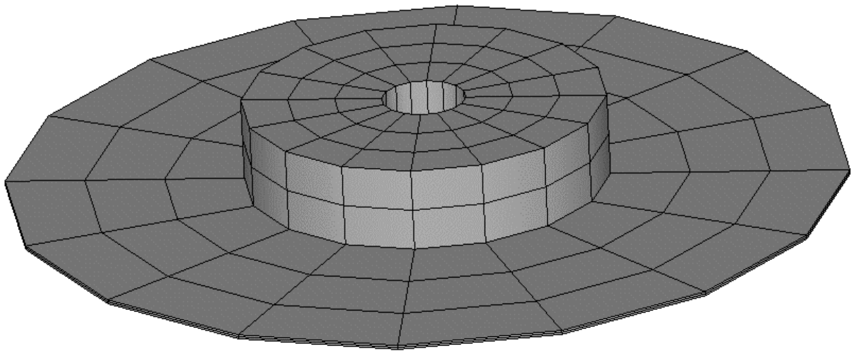
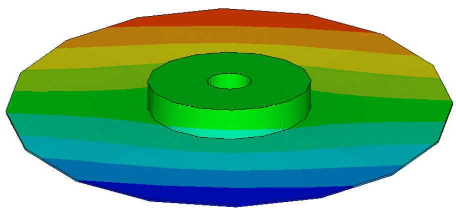
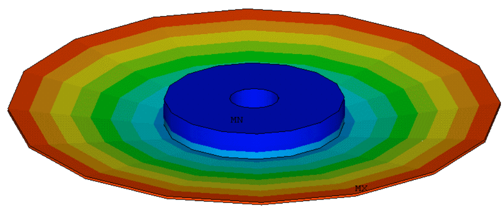
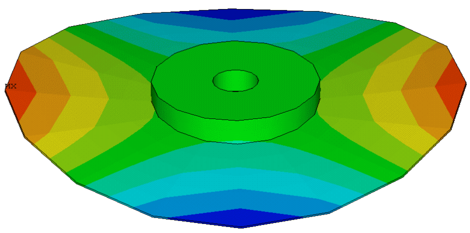
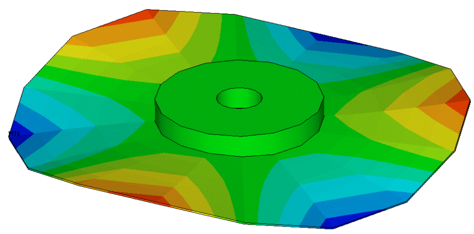


Fig. 2 Finite element model of the plate

The effect of change of Young modulus on first four natural frequencies is shown in Fig. 3 (r = 0,055 m, h_v = 0,02 m). The individual mode shapes are represented by nodal lines and nodal circles and each mode shape is described by number of nodal circles/number of nodal lines (Table 2). On this figure we can see that if Young modulus is increasing then natural frequencies are increasing too. The dependence of the first natural frequency on Young modulus for different thickness of layers is shown on Fig. 4. The influence of the radius of layers on first natural frequency for different Young modulus is shown on Fig. 5 and similarly for different thickness of layers is shown on Fig. 6.

Table 2 Mode shapes of circular plate embedded into constraining layers

 0/1	 0/0
 0/2	 0/3

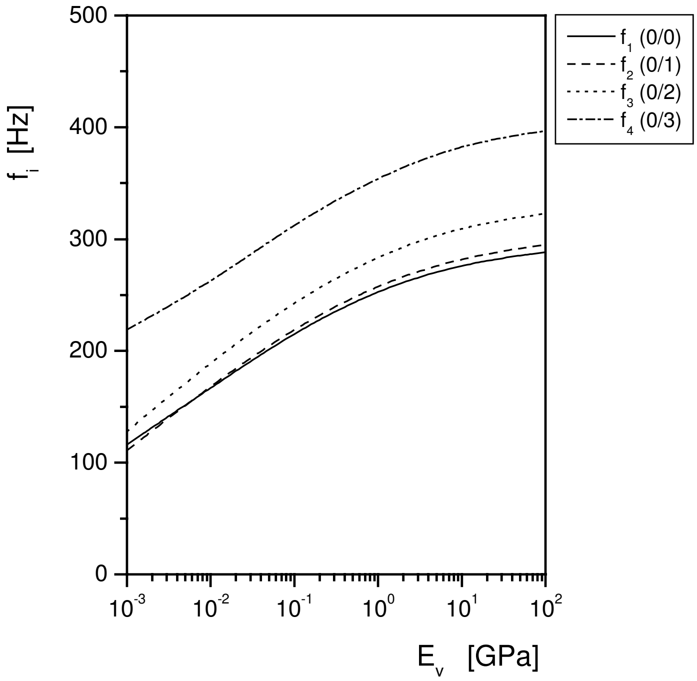


Fig. 3 Dependence of the first four natural frequencies on Young modulus of layers (r = 0,055 m, h_v = 0,02 m).

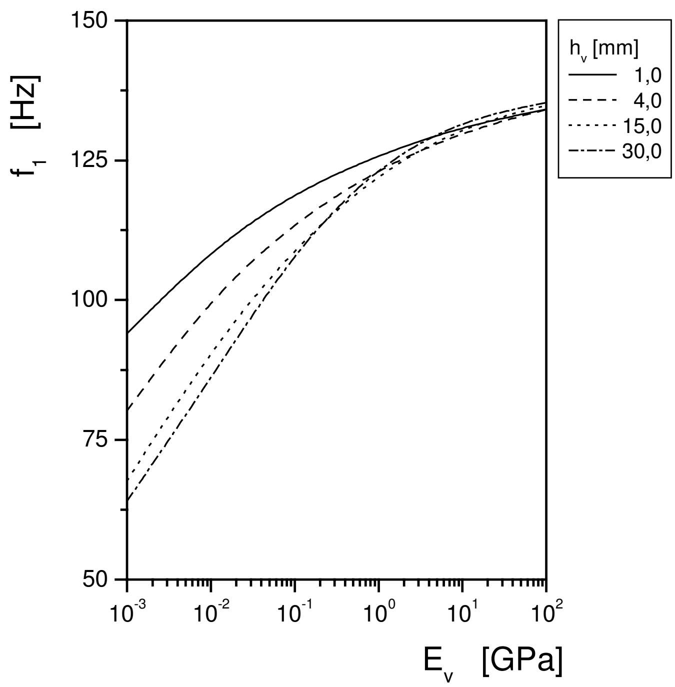


Fig. 4 Dependence of the first natural frequency on Young modulus of layers for different layer thickness (r = 0,055 m).

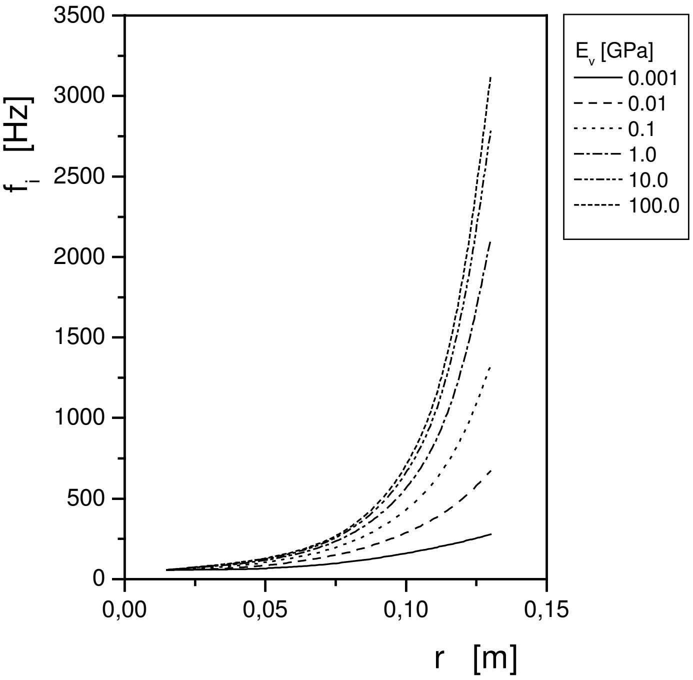


Fig. 5 Dependence of the first natural frequency on radius of covering layers for different Young modulus of layers (h_v = 20 mm)

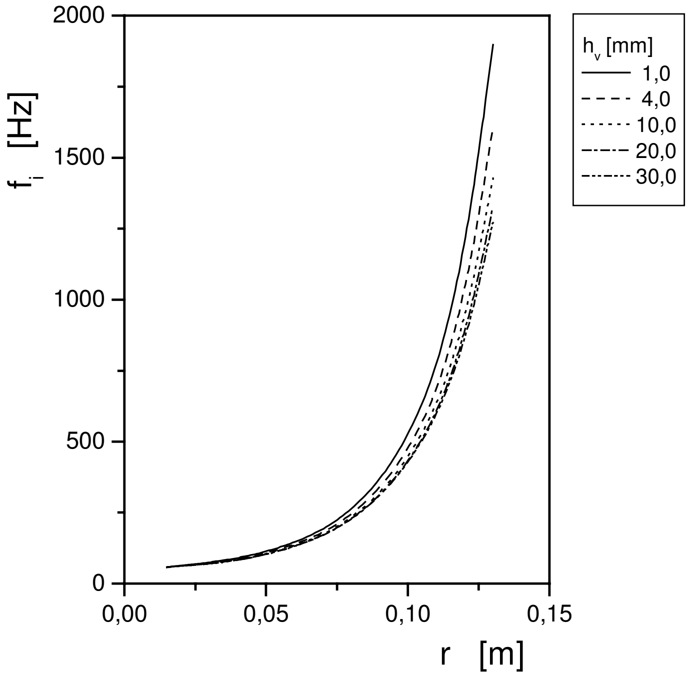


Fig. 6 Dependence of the first natural frequency on radius of covering layers for different layer thickness (E_v = 0,1 GPa)

4. Conclusion

A method of structural modification of mechanical system and shifting its natural frequency to a desired values has been presented. The modification is done in physical parameters of mechanical system.

The required properties of circular plate can be obtained by structural parameters (thickness and radius of layers, Young modulus of layers) of constraining layers in which the plate is embedded. These structural modifications of the layers cause that the mass and stiffness properties of structure are changed. Consequently the modal properties of circular plate are changed, too. Influence of the parameters of layers on natural frequencies are presented. From presented results we can say that modal properties of this structure are mainly affected by changing of the Young modulus of the material of layers. Finally it can be noted that this manner of structural modification is very effective mainly for frequency shifting.

5. Acknowledgement

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6. References

Bodnicki M.: *Application of commutation phenomena in DC micromotor for identification of its rotor angular displacement*. Proceedings of 5. Franco-Japanese Congress & 3. European-Asian "Congress of Mechatronics". 9-11.10.2001, Besançon,

Nánási T.: *Vibration of rotating annular plate*. In Proceeding of the 8^{th} International Acoustic Conference, Kočovce, 2003, pp. 59-62,

Srivastava R. K., Kundra T.K.: *Dynamic modifications of vibrating structures*. Journal of Structural Engineering, Vol. 20, No. 4, 1994, pp. 185-194.