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DESCRIPTION OF MATERIAL PROPERTIES OF HARDENING CONCRETE INSIDE DECK OF COMPOSITE BRIDGE

I. Monzón Hualde[1], P. Štemberk[1], O. Lojkásek[2]

Summary: *During execution, concrete bridges are represented by several varying structural systems. In order to reduce the effect of the system variation, the execution of concrete bridges requires special procedures, when the construction equipment needs to be placed, or ride on the concrete, which did not reach its design properties. This paper shows an analysis of a concrete bridge deck. For the analysis, the material parameters of concrete are approximated by functions, which take into account the effect of composition and local temperature. A simple thermal analysis is used to express the thermal field distribution in the concrete deck which affects the progressing hardening. Finally, a mechanical analysis is conducted which gives the possible deformation of the bridge deck caused by a mobile concrete pump and other equipment.*

1. Introduction

The execution of a concrete structure in general, and especially a composite bridge, is affected the compressive strength of concrete. The criterion for form stripping is that structure must have reached a loading capacity such that it can support, at least, itself with a certain degree of safety and without excessive deformations or crack formation. This means that a certain strength and certain bonding between the reinforcement and the concrete must be attained. Several properties of concrete, such as the tensile and the shear strength, E-modulus, ultimate strain, etc, are of interest here. All these properties can be related with good approximations to the compressive strength at the early age.

In this paper, approximations of compressive strength and other concrete parameters are proposed. These functions are dependent upon local temperature and water–cement ratio of concrete in question. The temperature at each location of concrete deck is obtained by means of simple thermal analysis based on the finite elements method.

This paper also shows a analysis of deformation of the deck of bridge based on Chen model of plasticity, where the material parameters are approximated by scaling functions proposed previously. This model was originally derived for solid materials, but some studies verify the possibility of using it not only for hardened concrete but also for hardening concrete with yet evolving structure.

[1] Isabel Monzón Hualde, Ing. Petr Štemberk, Ph.D.: Fakulta stavební, České vysoké učení technické v Praze; Thákurova 7; 166 29 Praha 6; tel.: +420.224 354 364, fax: +420.224 353 740; e-mail: stemberk@fsv.cvut.cz
[2] Ing. Otakar Lojkásek: JHP Mosty spol. s r.o.; Ústřední 60; 102 00 Praha 10; tel.: +420.272 700 452, fax: +420.272 703 703; e-mail: lojkasek@jhp-mosty.cz

2. Approximation of evolution of mechanical parameters of concrete

2.1 Definition of evolutionary function

An evolutionary function was introduced by Petr Štemberk (Ph.D., 2003). This function describes the evolutionary changes in the microstructure of solidifying and hardening concrete, and is given by

$$h(t_n) = a_5 \cdot \left(\frac{a_3 \cdot t_n^{a_2}}{a_1 + a_3 \cdot t_n^{a_2}} \right)^{a_4} , \tag{1}$$

where t_n is a normalized time with respect to the normalized final setting time ($t/6$, t is age in hours) and a_1, a_2, a_3, a_4 and a_5 are empirical parameters.

In this paper, several functions for these parameters are proposed regarding the compressive strength evolution expressed by the Evolutionary Function. The compressive strength increases exponentially in the early stage. The exponential period cease after a few days. The way the strength increases depends on a large number of factors, the most important of these are:

- Cement
- Water – Cement ratio (w/c)
- Admixtures
- Curing conditions: temperature, access of water

As has been mentioned earlier, in this report only variations in temperature and water–cement ratio are taken into consideration in the approximations of the parameters.

The compressive strength has been adjusted to the evolutionary function, which describes the evolutionary changes in the microstructure of solidifying and hardening concrete, of the compressive strength. For each temperature and water-cement ratio, the values of parameters a_i (i=1, 2, 3, 4, 5) have been proposed.

2.2 Effect of temperature

One of the most important influencing factors on the rate of strength increase is the temperature. A high initial temperature during the early stage causes a lower strength in the hardened concrete. As a rule, high temperatures accelerate the hardening rate and low temperatures reduce it.

On the basis of the graph *"Compressive strength gain f_{cc} in concrete with curing temperatures"* (Byfors 1980), the curves of the compressive strength have been adjusted to the evolutionary function of the compressive strength. For each temperature values of parameters a_i (i=1, 2, 3, 4, 5) have been obtained. From these values, the following expressions depending on temperature exclusively are proposed:

$$a_1 = 0{,}0067 \cdot T^3 - 0{,}4997 \cdot T^2 + 11{,}016 \cdot T - 51{,}76$$
$$a_2 = 0{,}005 \cdot T - 1{,}1643$$
$$a_3 = -0{,}0011 \cdot T^3 + 0{,}0661 \cdot T^2 - 1{,}0523 \cdot T - 5{,}8483$$
$$a_4 = -0{,}0121 \cdot T + 1{,}7693$$

$$a_5 = -0{,}4193 \cdot T + 43{,}148$$

2.3. Effect of water–cement ratio

The relation between the compressive strength and the water-cement ratio is used in concrete mix proportions. The proportions are adapted to give certain strength at a certain age and the relation with water-cement ratio must be supplemented with the age.

The water–cement ratio has no notable effect on the time when the strength development begins. On the other hand, a lower water-cement ratio entails a higher rate of growth, once the gain in strength has started, a low water-cement ratio does mean a higher strength. The compressive strength does not increase in a similar manner for different water-cement ratios. The deviations are the greatest for low water-cement ratio and the greatest after 24-48 hours (at 20ºC), nevertheless the deviations occur throughout the early age.

On the basis of the graphic *"Compressive strength, f_{cc} gain in concrete with various w_0/c"* (Byfors 1980), the approximations of the parameters a_i (i=1, 2, 3, 4, 5) (depending exclusively on the water-cement ratio) have been obtained similarly:

$$a_1 = 77{,}483 \cdot (w/c)^3 - 116{,}56 \cdot (w/c)^2 + 28{,}669 \cdot (w/c) + 21{,}221$$

$$a_2 = 1$$

$$a_3 = -437{,}46 \cdot (w/c)^3 + 1116{,}9 \cdot (w/c)^2 - 940{,}72 \cdot (w/c) + 271{,}58$$

$$a_4 = -99{,}984 \cdot (w/c)^3 + 223{,}71 \cdot (w/c)^2 - 160{,}17 \cdot (w/c) + 43{,}67$$

$$a_5 = -67{,}018 \cdot (w/c)^3 + 166{,}01 \cdot (w/c)^2 - 184{,}63 \cdot (w/c) + 98{,}758$$

2.4. Combined effect of temperature and water-cement ratio

On the basis of the previous results, the following expressions for the parameters (dependent on local temperature and water-cement ratio) have been proposed.

$$a_1 = \left(103{,}17 \cdot (w/c)^2 - 159{,}44 \cdot (w/c) + 67{,}27\right) \cdot \left(0{,}000705 \cdot T^3 - 0{,}0526 \cdot T^2 + 1{,}158 \cdot T - 5{,}47\right)$$

$$a_2 = 0{,}008 \cdot T + 1{,}08$$

$$a_3 = \left(372{,}35 \cdot (w/c)^2 - 587{,}13 \cdot (w/c) + 225{,}28\right) \cdot \left(-0{,}00011 \cdot T^3 + 0{,}0066 \cdot T^2 - 0{,}105 \cdot T + 0{,}6\right)$$

$$a_4 = \left(31{,}746 \cdot (w/c)^2 - 47{,}778 \cdot (w/c) + 23{,}032\right) \cdot \left(-0.002 + 0{,}3\right)$$

$$a_5 = \left(37{,}037 \cdot (w/c)^2 - 108{,}52 \cdot (w/c) + 84{,}481\right) \cdot \left(-0{,}0118 \cdot T + 1{,}26\right).$$

3. Thermal analysis

The temperature history can be obtained from real measurements or from a thermal analysis. In this paper, we focused on thermal stress analysis of the concrete structure during construction using a FEM-based analysis, it is possible simulating with accuracy temperatures and stress behavior inside the structure.

The governing equation of the heat transfer is derived from Fourier's law of heat conduction, $g_{x_i} = -k_{x_i} \cdot \frac{\partial T}{\partial x_i}$, where g_{x_i} is the heat flow conducted per unit area in the direction of the axis x_i. Assuming the heat flow equilibrium in the analyzed area, we obtain

$$\frac{\partial}{\partial x}\left(k_x \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z \frac{\partial T}{\partial z}\right) + f = \rho c \frac{\partial T}{\partial t}, \tag{2}$$

where f is the rate of heat generated per unit volume, ρ is the mass density and c is the heat capacity. The boundary condition may be defined in the terms of either Dirichlet type T = g, where g is a function prescribing temperature on the boundary, or the Neumann type $\frac{dT}{dn}k = j_q$, which defines the amount of heat transferred through the boundary. To obtain the governing equation for FEM we use the Garlekin method, which yields the governing equation in the form

$$C\dot{r} + Kr = h + b \tag{3}$$

where C is the capacity matrix, K is the conductivity matrix, r is the vector of nodal values, h is the vector of internal heat supplies and b is the vector describing the boundary conditions. This equation describes a general non-stationary distribution of temperature T(x,y,z,t). In the case of non-stationary heat flow, the initial condition, which prescribes temperature distribution in the time t_0, is introduced

$$T(x,y,z,t_0) = T(x,y,z) \tag{4}$$

STATIONARY FLOW: If $\frac{\partial T}{\partial t} = 0$, thus the temperature distribution is independent on time, the relation (3) is reduced to equation $Kr = h + b$, which describes the steady heat flow. This equation is a lineal algebraic equation system.

NON-STATIONARY FLOW: The equation (3) is used for solving the non-stationary case. The function h and b are now functions of time. Therefore, the initial condition (4) is considered. Assume that the solution r_{i-1} at time t_{i-1} is known. Then, consider a time step $\Delta t = t_i - t_{i-1}$ at which the vector of unknowns is approximated linearly

$$r(t) = \psi r_i + (1-\psi) r_{i-1},$$

where $\psi = (t - t_{i-1})/\Delta t$. Similarly, the right hand side vector is approximated

$$q(t) = \psi q_i + (1-\psi) q_{i-1}.$$

The numerical time derivate is obtained from the relation

$$\frac{dr}{dt} = \frac{1}{\Delta t}(r_i - r_{i-1}).$$

Substituting these relations in the equation (3) we obtain the basic equation for non-stationary heat flow

$$\left[K\psi + \frac{C}{\Delta t}\right] r_i = q_{i-1}(1-\psi) + q_i \psi + \left[\frac{C}{\Delta t} - K(1-\psi)\right] r_{i-1}$$

It is desirable to choose ψ from the interval $1/2 \geq \psi \geq 1$. If ψ is a fixed value, then this relation is a linear algebraic equation system for the unknowns, r_i.

4. Modified Chen model of plasticity

The Chen model of plasticity is a three-parameter model for concrete displaying isotropic hardening. This model expresses the elasto-plastic behavior of concrete. The typical behavior of concrete is varying stress-strain characteristic under tension and compression. Two different, but similar, functions were proposed for each of the loading surfaces, in the compression-compression region and in the tension-tension or tension-compression regions. In our example, compression loading is considered hence only the equations for the compression-compression region are in the paper introduced.

The failure surface is assumed in the compression-compression region as

$$f_u^c(\sigma,h)=J_2+\frac{A_u(h)}{3}I_1-\tau_u^2(h)=0\,.$$

The initial yield surface in the compression-compression region is given by

$$f_0^c(\sigma,h)=J_2+\frac{A_0(h)}{3}I_1-\tau_0^2(h)=0\,,$$

where $A_0(h)$, $\tau_0(h), A_u(h)$ and $\tau_u(h)$ are material constants which can be determined from simple tests. They are determined as functions of the ultimate stresses under uniaxial compression, $f_c(h)$, and under equal biaxial compression, $f_{yc}(h)$ and $f_{ybc}(h)$. With increasing strength of concrete the loading surfaces are expanding. More can be found in the paper Šípalová et al. (2005).

5. Application to bridge construction

To illustrate the applicability of the presented approach, a real structure was considered. The Border bridge is a part of the newly constructed D8 highway connecting Prague, CZ, and Dresden, DE. This composite bridge is about 500 meters long and overpasses a deep valley. The intermediate columns are circa 50 meters tall, which prohibits pumping concrete directly from the bottom of the valley to the bridge deck, which is designed as reinforced concrete slab. Therefore, the concrete needs to be transported to the location of placement across the already finished reinforced concrete deck. As falling behind the schedule is a very possible threat, especially in this case, when the construction site is located in a mountainous area, where it is a subject to unfavorable weather conditions, a tool for estimation of the earliest possible entrance to the newly concreted section of the deck is desirable, moreover, when the tool also provides some information on the possible damage caused by premature loading.

A section of the reinforced concrete deck of the composite bridge was analyzed by means of uniaxial problem, because in the analysis of thermal fields in a bridge deck the direction normal to the deck plane is the influencing direction, the analysis can be considered as a uniaxial problem. The following expression was used:

$$b\frac{\partial^2 T}{\partial x^2}+\rho_c\dot{H}(t)=\rho C\frac{\partial T}{\partial t}, \tag{5}$$

where T is temperature, t is time, x_i is a coordinate, b is thermal conductivity, C is specific heat, ρ is mass density of concrete, ρ_c is cement mass per unit volume of concrete and H is a function of heat generation per unit mass of cement. This function (H) was derived from Eq.(1), where the scaling parameter a5 was taken as 8,423 so that the ultimate heat generated was equal to 407 kJ. The prescription of *H* can also be taken from, e.g. JSCE (2002). The effect of temperature increase due to the hydration reaction was not considered here.

This thermal analysis is centered in the study of concrete behavior in a deck of composite bridge with 0.3 meters of thickness. The finite element model of the deck is shown in Fig.1.

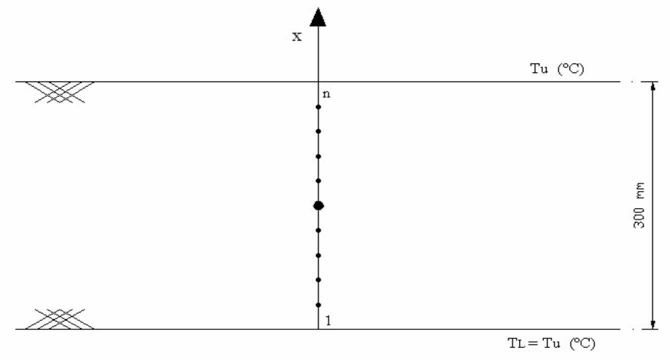

Fig. 1. Uniaxial model of thick concrete deck.

The thermal properties used in Eq.(5) are listed in Table 1

b (J/m/h/ºC)	**c** (J/ºC/Kg)	ρ (kg/m^3)	ρ_c (kg/m^3)	H_∞ (KJ/Kg)
2.7	1150	2500	490	407

Table 1. Material parameters of concrete for thermal analysis.

The temperature history in the center of the bridge deck is plotted in the following figures. In Fig.2., the ambient temperature, set as the boundary conditions, was kept constant at 20°C. The Fig.3. shows the temperature history when the concreting starts at 8 p.m. and the boundary conditions depend on the concrete age.

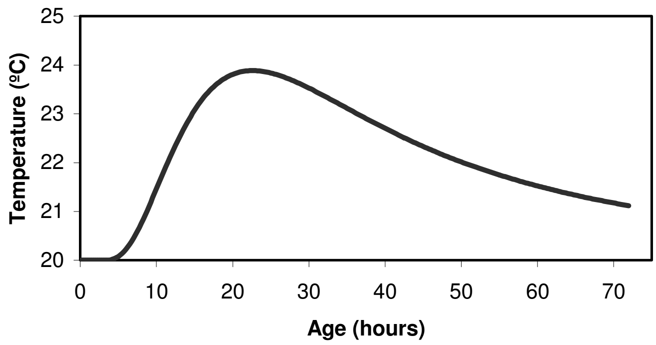

Fig.2. Temperature history in center of bridge deck.

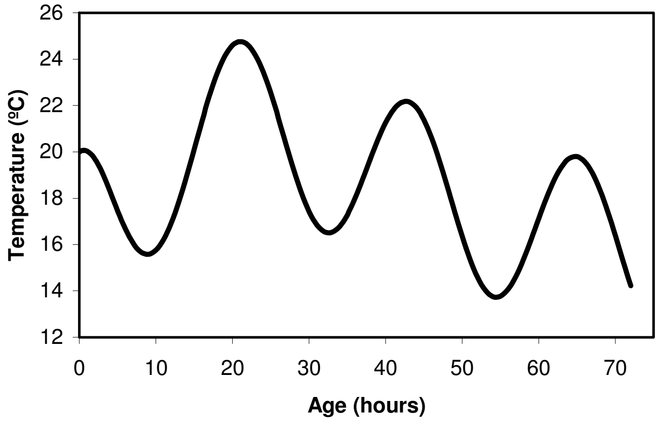

Fig.3. Temperature history in center of bridge deck including effect of ambient temperature.

For studying the deformation of the section of the deck bridge, Chen model of plasticity was used. This one was modified so that all its parameters were the function of the degree of hydration. These parameters were approximated scaling Eq.1.

$$f_y = S_{f_y} \cdot h(t, T, w/c) \tag{6}$$

$$f_c = S_{f_c} \cdot h(t, T, w/c) = \frac{S_{f_y}}{0.6} \cdot h(t, T, w/c) \tag{7}$$

$$E = S_E \cdot h(t, T, w/c) \tag{8}$$

$$\varepsilon_u = \frac{S_{\varepsilon_u}}{h(t, T, w/c)} \tag{9}$$

S_{f_y} (MPa)	S_{f_c} (MPa)	S_E (GPa)	S_{ε_u} (MPa)
0,37	0,617	0,52	0,01

Table 2. Scaling parameters of concrete for Chen Model of Plasticity.

The main object in derivation of these expressions was the simplicity, which is demonstrated by introduction of only one scaling parameter for derivation of all mechanical parameters used in the analysis from the evolutionary function, Eq.1. As a result, the yield strain, ε_y, is constant as it is obtained from the Hook's law, $\varepsilon_y = f_y/E$, where both the denominator and the numeration a scaled from the Eq.1. This drawback needs to rectified, however, it does not prohibit the used of the presented model, as the plastic behavior is dominant in the extremely early ages.

The Fig.4. shows the deformation that the deck bridge suffers (using the Eq.7). In this graph, it is observed that from seven hours, the behavior of concrete is elastic under the truck tire, which is represented by a local force of 45 kN. Until this time, a plastic behavior is presented also. The relation between stress (in our case 0.75 MPa) and f_y (Eq.5) is plotted in Fig.5.

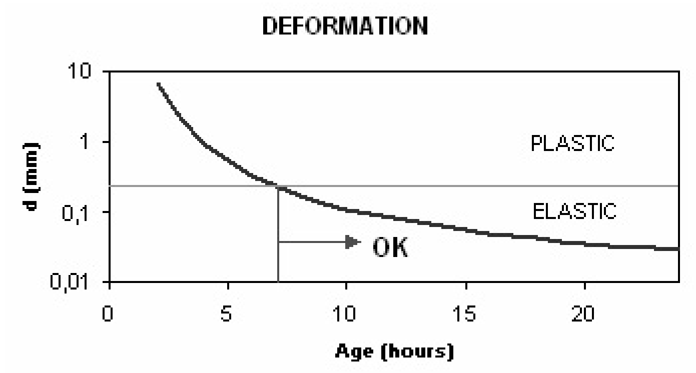

Fig.4. Deformation of bridge deck.

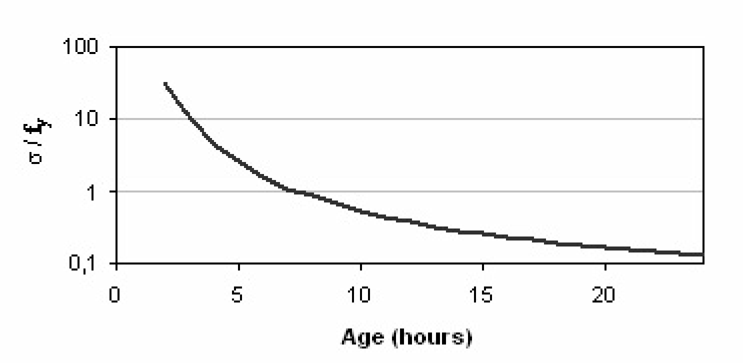

Fig.5. Relation between actual stress (under tire) and yield stress (f_y).

6. Conclusions

In this paper, it was attempted to estimate the possible deformation of hardening concrete slab when a truck enters it. For the analysis it was necessary define a function expressing the evolution of the microstructure. It was decided that the concept of the degree of hydration was the most suitable as it is acceptable to assume that all mechanical parameters are a function of the degree of hydration. Therefore, a five-parameter hyperbolic function was derived, which depended on the water-cement ratio, temperature and time. For the sake of simplicity, all mechanical parameters are obtained by scaling of the degree of hydration.

The material model used in this analysis was the Chen model of plasticity which was modified so that all its parameters were the function of the degree of hydration. The deformation of the concrete slab was investigated for various ages and it was established that from the age of seven hours the concrete is only in the elastic region when subjected to the load. Also, for further extension of the model, a finite element tool was programmed for calculating the thermal distribution, which took into account the change in the ambient temperature.

7. Acknowledgement

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