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VISCOELASTIC MODELLING OF CORD-REINFORCED RUBBER COMPOSITES

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Summary: *A viscoelastic model for the fully three-dimensional stress and deformation response of cord-reinforced rubber composites at finite strains requires the knowledge of material parameters of their components. The relaxation response of each compound of the composite is modelled separately and the global response is obtained by an assembly of all contributions. This study summarizes the investigation of the non-linear performance of polyamide cords, when undergoing static tensile tests, stress relaxation tests and dynamical tensile loading. The description of the time-dependent performance is based on a two-component model, which comprises elastic and viscous contribution. An appropriate relaxation function is introduced that is based on the Prony series approximation of the hereditary integral model of the material tested. The experimental procedures and the analysis of the experimental data are described.*

1. Introduction

One of applications of orthotropic cord-reinforced rubber composite in engineering practice is the sheat of air-springs used for the vibro-isolation of driver seats. Hyperelastic models of such composite material at finite strains were developed in our laboratory and used successfully for FEM simulation of deformations of a cylindrical air-spring (Tran Huu Nam, 2004; Urban, R., 2004). Nowadays we are interested in the dissipative behaviour of cord-reinforced rubber composites, in their viscoelastic properties which determine their damping function. Cords are often made of material which shows noticeable viscoelastic or plastic deformation. The properties of the cords determine the overall behaviour of the material such that the correct modelling of the orthotropy of the composite is very important.

The constitutive theory and numerics for anisotropic viscoelastic materials at finite strains is far from completion and the list of publications in that field is quite short. A fully three-dimensional finite strain viscoelastic model which is not restricted to isotropy was proposed in the paper by Simo (1987). A model which captures the hyperelastic material behaviour of pneumatic membranes reinforced with roven-woven fibres is described by Reese(2000) and extended by her to large elastoplastic deformation (Reese, 2003). The reader is referred to Tran Huu Nam (2004) for exhaustive list of papers dealing with models for elastomeric fibre-reinforced composites.

We use the model of Holzapfel and Gasser (2000) which employs Simo's constitutive framework. A particular anisotropic Helmholtz free-energy function allows to model a

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composite in which a matrix material is reinforced by two families of fibres, and the mechanical properties of the composites depend on two preferred fibre directions.

2. Model for finite anisotropic viscoelasticity

The postulated Helmholtz free-energy function (Holzapfel, 2000; Holzapfel and Gasser, 2000) describes separately volumetric and isochoric contributions:

$$\Psi(\boldsymbol{C}, \boldsymbol{A}_0, \boldsymbol{B}_0, \boldsymbol{\Gamma}_1, \dots, \boldsymbol{\Gamma}_m) = \Psi_{VOL}^{\infty}(J) + \Psi_{ISO}(\overline{\boldsymbol{C}}, \boldsymbol{A}_0, \boldsymbol{B}_0, \overline{\boldsymbol{\Gamma}_1}, \dots, \overline{\boldsymbol{\Gamma}_m}), \quad \overline{\boldsymbol{C}} = J^{-2/3}\boldsymbol{C}, \tag{1}$$

where $\boldsymbol{A}_0, \boldsymbol{B}_0$ are structural tensors of order two associated with the directions of fibres, and $\boldsymbol{\Gamma}_1, \dots, \boldsymbol{\Gamma}_m$ are the internal second order tensorial variables akin to the strain measure $\boldsymbol{C}$.

The isochoric part of free energy is further split into equilibrium and viscoelastic contributions:

$$\Psi_{ISO} = \Psi_{ISO}^{\infty}(\overline{I}_1, \overline{I}_2, \overline{I}_4, \dots, \overline{I}_8) + \sum_{\alpha=1}^{m} \Upsilon_\alpha(\overline{\boldsymbol{C}}, \boldsymbol{A}_0, \boldsymbol{B}_0, \overline{\boldsymbol{\Gamma}_\alpha}), \tag{2}$$

where the invariants $\overline{I}_4, \dots, \overline{I}_8$ of $\boldsymbol{C}$ **,** $\boldsymbol{A}_0$ and $\boldsymbol{B}_0$ are associated with anisotropy generated by the two families of fibres (Holzapfel, 2000, pp.275). The stress response is obtained from the Helmholz free-energy function as

$$\boldsymbol{S} = 2\frac{\partial \Psi(\boldsymbol{C}, \boldsymbol{A}_0, \boldsymbol{B}_0, \boldsymbol{\Gamma}_1, \dots, \boldsymbol{\Gamma}_m)}{\partial \boldsymbol{C}} = \boldsymbol{S}_{VOL}^{\infty} + \boldsymbol{S}_{ISO}^{\infty} + \sum_{\alpha=1}^{m} \boldsymbol{Q}_\alpha \tag{3}$$

where S_{VOL}^{∞} and S_{ISO}^{∞} is the volumetric and the isochoric stress response respectively and the overstress $\boldsymbol{Q}_\alpha$ is stress of 2nd Piola-Kirchhoff type. The evolution equations is formulated separately for each isochoric stress contribution:

$$\dot{\boldsymbol{Q}}_{\alpha a} + \frac{\boldsymbol{Q}_{\alpha a}}{\tau_{\alpha a}} = \beta_{\alpha a}^{\infty} \boldsymbol{S}_{ISO\alpha a}^{\infty}, \tag{4}$$

where $\alpha = 1, 2, \dots, m$ and a = 1,2 4,...,8. The constants $\beta_{\alpha a}^{\infty}$ are non-dimensional free-energy factors, which are associated with the relaxation times $\tau_{\alpha a}$. Closed-form solutions of the linear equations (4) may be represented by the simple convolution integrals:

$$\boldsymbol{Q}_{\alpha a} = \exp(-T/\tau_{\alpha a})\boldsymbol{Q}_{\alpha a 0} + \int_0^T \exp(-(T-t)/\tau_{\alpha a})\beta_{\alpha a}^{\infty} \dot{\boldsymbol{S}}_{ISOa}^{\infty}\, dt, \tag{5}$$

The material parameters $\beta_{\alpha a}^{\infty}$ and $\tau_{\alpha a}$ have to be determined from experiments.

3. Experiment

The material investigated is polyamide cord, PA6 140 1x2 390/390, supplied by Kordarna Velka nad Velickou . According to the manufacturer, there is certain impregnation of cords by some unspecified acrylate dispersion. The cords are destined for textile reinforcement of composite materials with rubber matrix used in air-springs.

Tensile and relaxation tests were performed using an Instron 4411 machine on Dpt. of Textile Structures TUL. The crosshead displacement was used to measure the stretch of specimens. The specimen length was 250 and 500 mm. The load and extension data were electronically captured. The specimens were not subjected to preconditioning before each series of tests. Dynamical loading tests were performed using the vibration testing bench of Weaving laboratory TUL. The specimen length was 2200mm. The setup of tests and results are apparent in Figures 2 and 3.

Five different deformation modes were used in the experiments:

- standard tensile tests at the crosshead speed (50,100,150 and 200)mm/min
- one-step relaxation tests – specimens were stretched to stretch $\lambda = 1.04$ at crosshead speed 100mm/min, then allowed to relax at constant stretch for a period of 120 s
- cyclic uploading-unloading tests at crosshead speed 100mm/min – specimens were subjected to five subsequent cycles between the two stretches $\lambda_{max} = 1.06$ and $\lambda_{min} = 1.02$
- nine-step relaxation tests at uploading and unloading crosshead speed 100mm/min and 60s relaxation stage between every loading
- dynamical tensile loading tests at the frequencies of (10, 12,14,16,18 and 20)Hz

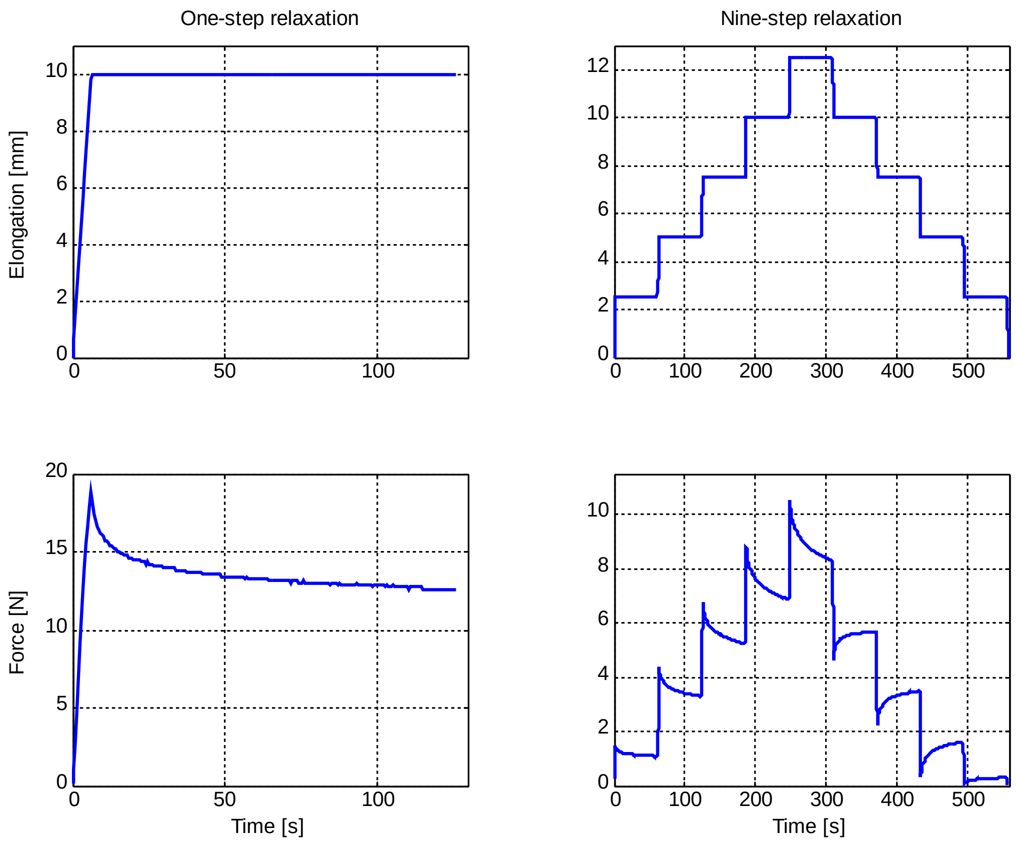

Fig.1 Relaxation tests.

4. Results and discussion

The nonlinear regression was performed by the optimization toolbox of MATLAB and the coefficients of the Prony series were determined by the method of nonlinear least squares. Since the percentage of the number of data points at the beginning of the relaxation period is small, the error function is dominated by a long uniform tail region of the relaxation period. To reduce the error and improve the fit at the beginning of the relaxation, we augmented the

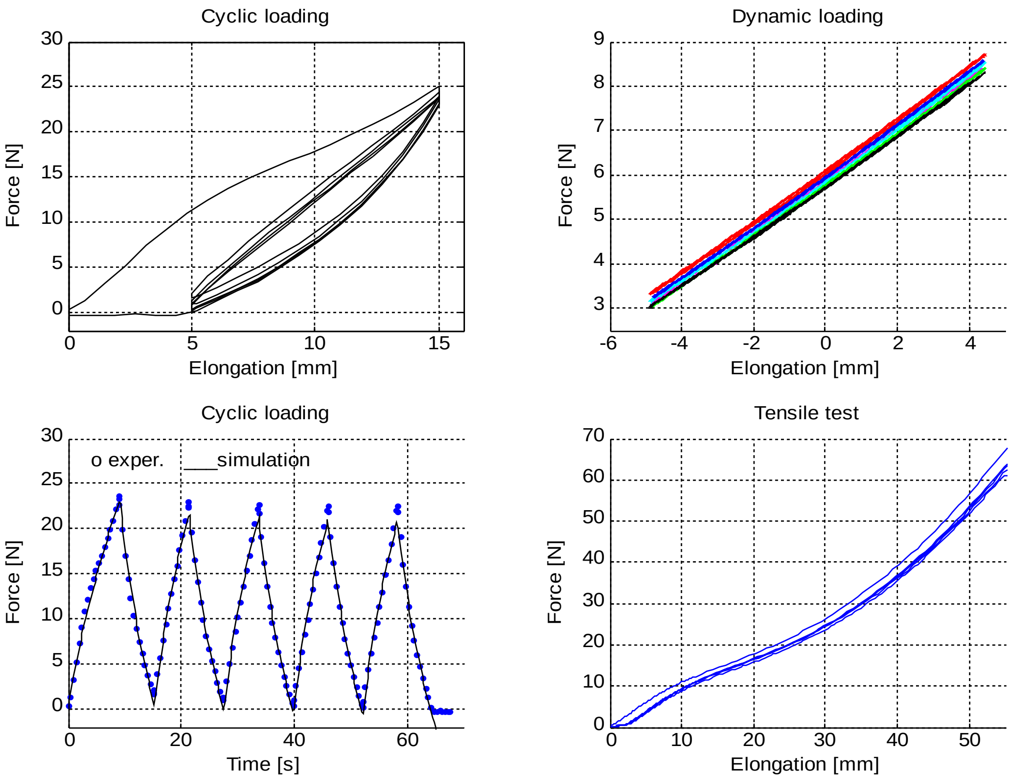

Fig.2 Cyclic, dynamic and tensile tests

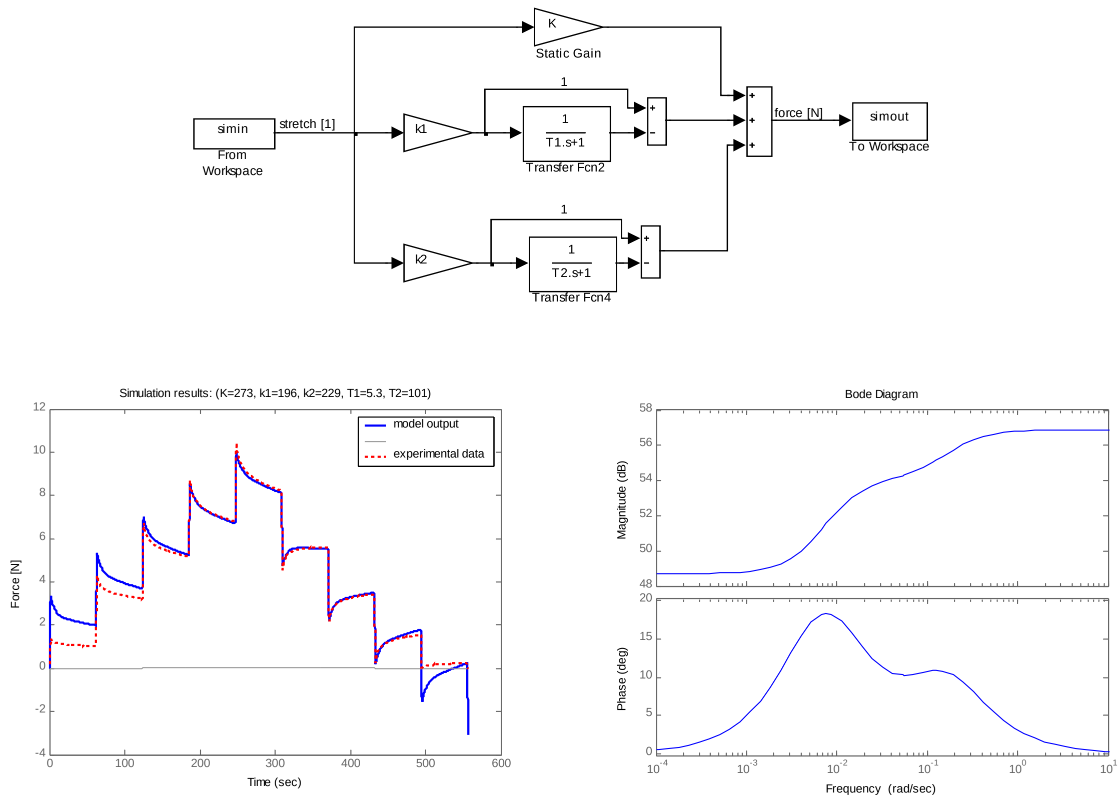

Fig.3 Multiple relaxation model, simulation results and Bode diagram

number of data points in this period by interpolation. Another solution of this problem could be a weighted nonlinear regression. The relaxation time of the first term in Prony series was about 120 s.

Besides the stress-relaxation tests, we performed other types of tests to evaluate viscoelastic materials. One of them was the quasi-static loading test in which the strain was gradually ramped up at the constant strain-rate, and ramped down after reaching its peak. In response to this type of loading, a different stress response resulted. Figure 2 left shows the stress-strain curves and the fit obtained for the three-element model.

We performed a series of dynamic tests, where sinusoidal tensile strains with the range of frequencies between 10 and 20 Hz were applied to the cords, and the stress response was measured at each frequency. These frequencies proved to be too high so that the anelastic properties of cords could take effect. Referring to the generalized Maxwell model, when tension is applied at a relatively fast rate, dashpots does not have time to respond and springs stretch very little. In this case, the system stiffness is essentially only that of springs. The dynamic strain-stress curve did not exhibit the hysteresis and was steeper than the static one, as shown in Figure 2 right. The frequency range of tests was dictated by the possibilities of the experimental device. The working frequencies of the cords are lower, usually between 0.1 and 10 Hz, where the graphs of load versus deformation sinusoidal in time is approximately elliptic in shape showing the characteristic hysteresis which provides the measure for energy dissipation and damping.

The multiple relaxation experiment data were first fitted by nonlinear least squares. These fits were not convenient enough. Then the simulation model was built-up in control system toolbox of MATLAB using tf-mfile. Figure 3 shows the simulation scheme and the results of simulation. The simulation is quite satisfactory for the range between 3rd and 7th step.

We used different strain rates in the tensile tests to characterize the supposed rate dependence of the yield limit of the cord material. In the narrow range of rates used we did not prove any significant yield limit shift.

4. Conclusion

We believe we have demonstrated that the current model is qualitatively capable of representing the major features observed in the time dependent deformation of polymer cords in the range of the moderate strains. Response above the yield limit as well as effects associated with finite strains, permanent set and nonlinearity are left to future investigations and to further refinement of the model.

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