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MODELLING OF THE RATE-DEPENDENT BEHAVIOUR OF FILLED RUBBERS

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Summary: *The paper presents an application of a phenomenological material model for a viscoelastic stress response at large strains. The model is used for the simulation of carbon-black filled rubber in monotonic and cyclic deformation processes under isothermal conditions. The material stress response is decomposed into two constitutive parts which act in parallel: an elastic equilibrium stress response and a rate-dependent viscoelastic overstress response. The response of a particular filled rubber in the cyclic and relaxation tests was measured experimentally. The parameters of the constitutive functions are determined from the experimental data by an identification process using nonlinear optimization methods. The paper concludes with a simulation by FEM of the cyclic loading of a simple rubber specimen.*

1. Introduction

Rubber materials are applied in various branches of mechanical engineering because of their damping properties. One of such applications is the cushion of tram-wheels by rubber segments manufactured by Bonatrans Bohumin. During the operation the segments are under the temporally constant compressive preload due to shrinkage between the corpus of the wheel and the hoop and under the dynamic compressive and shear loads due to the transfer of the vehicle weight during the wheel rotation and the transfer of torque. The static preload leads to compressive permanent set of segment and the periodic deformation leads to hysteresis behaviour and heat generation which considerably affects properties of rubber. The modelling and FEM calculation of the structural response requires a constitutive model which captures the complex material behaviour.

The constitutive theory of finite linear viscoelasticity is a major foundation for modelling rate-dependent filled-rubber behaviour based on the phenomenological approach. This general theory is formulated using functionals with fading memory properties. The stress is decomposed into an equilibrium stress that corresponds to the stress response at an infinite slow rate of deformation and a viscosity-induced overstress. The overstress is expressed as an integral over the deformation history and a relaxation function is specified as a measure for the material memory (Simo, 1987; Holzapfel and Simo, 1996; Holzapfel, 1996; Kaliske and Rothert, 1997). The thermodynamic consistency requires the relaxation function to be positive with negative slope and to possess a positive curvature (Haupt and Lion, 2002). Within this restriction certain number of decreasing exponentials can be superimposed, referred as a so-called Prony series. In this approach, a suitable hyperelasticity model is employed to

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reproduce the elastic responses represented by the springs, while the dashpot represents the inelastic or the so-called internal strain. This process may invite a large number of material parameters in the model that are difficult to estimate. Another innovative approach (Haupt, 2002, Haupt and Lion, 2002) uses compact relaxation function based on power laws, the Mittag–Leffler function (Lion and Kardelky, 2004), in describing Payne effect, and involves only a very few number of material parameters.

There exists another possibility to establish finite strain models of viscoelasticity by considering the multiplicative decomposition of the deformation gradient into elastic and inelastic parts (Lion 1997, Reese & Govindjee 1998, Bonet 2001, Laiarinandrasana 2003, Reese 2003, Bergstrom & Boyce,1998). The temporal behavior is determined by an evolution equation that is consistent with the second law of thermodynamics.

2. Model for finite viscoelasticity

The origin of the material model of finite strain viscoelasticity used in our work is the concept of Simo (1987) and Govindjee & Simo (1992). The finite element formulation of the model was elaborated by Holzapfel (1996) and used by Holzapfel & Gasser (2000) to calculate the viscoelastic deformation of fibre reinforced composite material undergoing finite strains. The model was incorporated into the new version of ANSYS 10.

The model is based on the theory of compressible hyperelasticity with the decoupled representation of the Helmholtz free energy function with the internal variables (Holzapfel, 2000, p. 283) :

$$\Psi(\boldsymbol{C},\boldsymbol{\Gamma}_1,.....,\boldsymbol{\Gamma}_m)=\Psi^{\infty}_{VOL}(J)+\Psi^{\infty}_{ISO}(\overline{\boldsymbol{C}})+\sum_{\alpha=1}^{m}\Upsilon_{\alpha}(\overline{\boldsymbol{C}},\boldsymbol{\Gamma}_{\alpha}), \quad \overline{\boldsymbol{C}}=J^{-2/3}\boldsymbol{C}. \tag{1}$$

The first two terms in (1) characterize the equilibrium state and describe the volumetric elastic response and the isochoric elastic response as $t\to\infty$, respectively. The third term is the dissipative potential responsible for the viscoelastic contribution. The derivation of the 2[nd] Piola-Kirchhoff stress with volumetric and isochoric terms:

$$\boldsymbol{S}=2\frac{\partial\Psi(\boldsymbol{C},\boldsymbol{\Gamma}_1,.....,\boldsymbol{\Gamma}_m)}{\partial\boldsymbol{C}}=\boldsymbol{S}^{\infty}_{VOL}+\boldsymbol{S}^{\infty}_{ISO}+\sum_{\alpha=1}^{m}\boldsymbol{Q}_{\alpha} \tag{2}$$

where S^{∞}_{VOL} and S^{∞}_{ISO} is the volumetric and the isochoric stress response respectively and the overstress $\boldsymbol{Q}_{\alpha}$ is stress of 2[nd] Piola-Kirchhoff type.

$$\boldsymbol{S}^{\infty}_{VOL}=J\frac{d\Psi^{\infty}_{VOL}(J)}{dJ}\boldsymbol{C}^{-1}, \quad \boldsymbol{S}^{\infty}_{ISO}=J^{-2/3}Dev[2\frac{\partial\Psi^{\infty}_{ISO}(\overline{\boldsymbol{C}})}{\partial\overline{\boldsymbol{C}}}] \tag{3}$$

$$Q_{\alpha}=J^{-2/3}Dev[2\frac{\partial\Upsilon_{\alpha}(\overline{\boldsymbol{C}},\boldsymbol{\Gamma}_{\alpha})}{\partial\overline{\boldsymbol{C}}}], \tag{4}$$

$$Dev(\boldsymbol{.})=(\boldsymbol{.})-1/3[(\boldsymbol{.}):\boldsymbol{C}]\boldsymbol{C}^{-1}. \tag{5}$$

where $Dev(.)$ is the deviatoric operator in the Lagrangean description. Motivated by the generalized Maxwell rheological model (Fig. 1), the evolution equation for the internal variable $\boldsymbol{Q}_{\alpha}$ takes on the form (6).

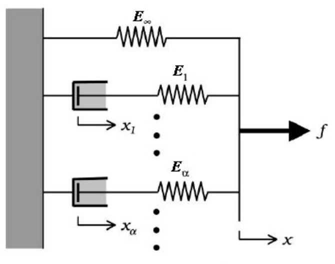

Fig. 1. Maxwell reological model

$$\dot{\boldsymbol{Q}}_{\alpha}+\frac{\boldsymbol{Q}_{\alpha}}{\tau_{\alpha}}=\dot{\boldsymbol{S}}_{ISO\alpha}, \quad (6)$$

$$\boldsymbol{S}_{ISO\alpha}=J^{-2/3}Dev[2\frac{\partial\Psi_{ISO\alpha}(\overline{\boldsymbol{C}})}{\partial\overline{\boldsymbol{C}}}], \quad (7)$$

$$\Psi_{ISO\alpha}(\overline{\boldsymbol{C}})=\beta_{\alpha}^{\infty}\Psi_{ISO}^{\infty}(\overline{\boldsymbol{C}}), \quad (8)$$

$$\boldsymbol{S}_{ISO\alpha}=\beta_{\alpha}^{\infty}\boldsymbol{S}_{ISO}^{\infty}(\overline{\boldsymbol{C}}). \quad (9)$$

$\beta_{\alpha}^{\infty}\in(0,\infty)$ in the expressions (8) and (9) are the non-dimensional strain energy factors (Simo, 1987 ; Govindjee & Simo, 1992) and τ_{α} are the relaxation times which must be determined from experiments. The closed form solution of the linear evolution equation is given by the convolution integral and the recurrence updated formula (Holzapfel, 1996) for the internal stress:

$$\begin{gathered}\boldsymbol{Q}_{\alpha}=\exp(-T/\tau_{\alpha})\boldsymbol{Q}_{\alpha 0}+\int_{0}^{T}\exp(-(T-t)/\tau_{\alpha})\beta_{\alpha}^{\infty}\dot{\boldsymbol{S}}_{ISO}^{\infty}(\overline{\boldsymbol{C}})\,dt,\\(\boldsymbol{Q}_{\alpha})_{n+1}=\exp(2\xi_{\alpha})(\boldsymbol{Q}_{\alpha})_{n}+\exp(\xi_{\alpha})\beta_{\alpha}^{\infty}[(\boldsymbol{S}_{ISO}^{\infty})_{n+1}-(\boldsymbol{S}_{ISO}^{\infty})_{n}],\ \ \xi_{\alpha}=-\frac{\Delta t}{2\tau_{\alpha}}.\end{gathered} \quad (10)$$

The material is assumed to be slightly compressible, the volumetric and isochoric parts of Helmholtz free energy function were chosen in the form:

$$\Psi_{VOL}^{\infty}(J)=\frac{1}{d}(J-1)^{2},\quad \Psi_{ISO}^{\infty}(\overline{\boldsymbol{C}})=c_{1}(\overline{I}_{1}-3)+c_{2}(\overline{I}_{2}-3), \quad (11)$$

where the parameters $c_{1,}c_{2}$ and d are to be determined from experiments. The viscoelastic behaviour is modelled by use of $\alpha=2$ relaxation processes with the corresponding relaxation times τ_{α} and free energy factors β_{α}^{∞} . The second Piola–Kirchhoff stress and the stretch in the loading direction of test specimens were determined from experimental results. The seven material parameters were calculated by non-linear optimization methods in Matlab.

3. Relaxation tests

The relaxation behaviour at different strain levels is examined in detail through multi-step relaxation test. In the compression tests, a strain rate of 0,05mm/s was applied during the loading path. The stress relaxation was recorded for 1200 s. Fig. 2 shows the time histories of force at different strain levels in compression regime. All curves reveal the existence of a very fast stress relaxation during the first 10 seconds followed by a very slow rate of relaxation that continues in an asymptotic sense. This conforms with observations reported by Haupt and Sedlan (2000). Comparing the results obtained at different strain levels, it can be seen that relaxation tests carried out at higher strain levels possess larger over-stresses and subsequently show a faster stress relaxation than those at lower strain levels with lower over-stresses as reported also by Amin (2005). In the classical approach, equilibrium states are reached if the duration of the relaxation periods is infinitely long. Thus, the stresses measured at the termination points of the relaxation periods are approximate values of the equilibrium stress. The difference between the current stress and the equilibrium stress is the so-called overstress.

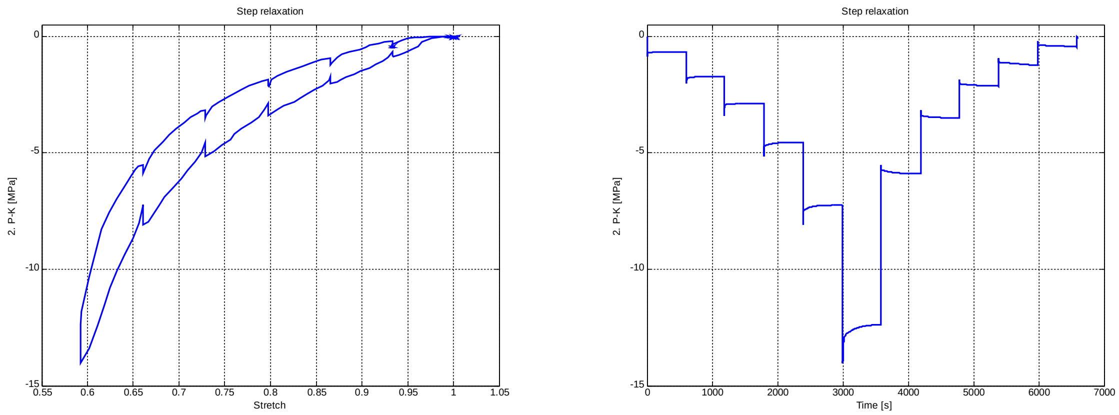

Fig. 2 Multi-step relaxation experiment

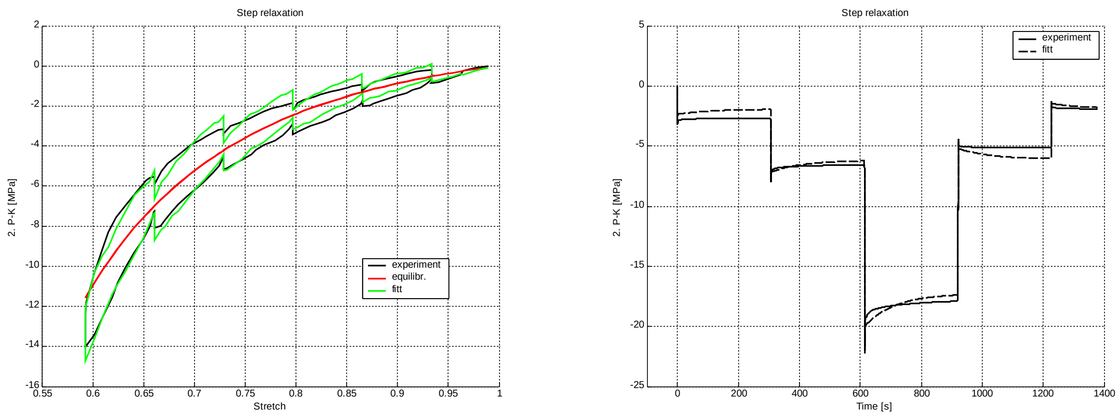

Fig. 3 Multi-step relaxation experiment and fitting

Fig.3 compares the experimental data and the curves fitted to the proposed material model by nonlinear least squares method. The red curve approximates the equilibrium stress $\boldsymbol{S}^{\infty}_{VOL}+\boldsymbol{S}^{\infty}_{ISO}$. The good performance of the model in capturing the main material features is obvious.

To characterize better the viscosity properties, a series of simple relaxation tests at different stretch levels were carried out. In this course, a stretch rate of 0.05/s followed by a hold time of 20 min was used in the tests. The results are at Figs. 4 and 5.

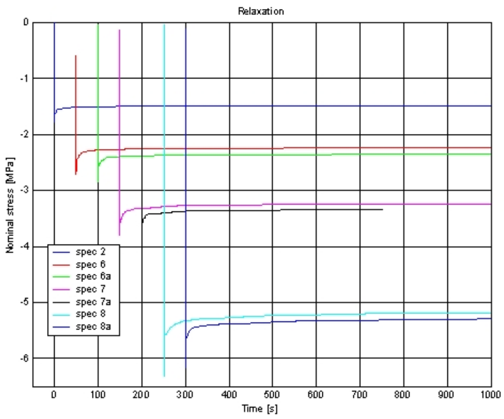

Fig. 4 Single relaxation tests – experiment

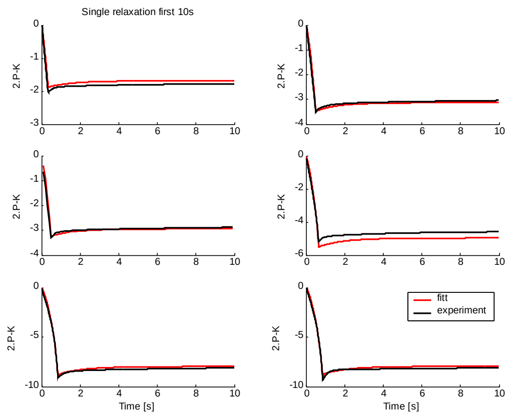

Fig. 5 Single relaxation - first 10 s

4. Finite element simulation

The cyclic loading test was simulated by FEM with the viscoelastic material at finite strains in ANSYS 10 modelled as the combination HYPER and PRONY options. This ANSYS implementation was allegedly inspired by Holzapfel viscoelastic model (Holzapfel,1996) described above. The parameters of the model were determined by non-linear least squares from the results of the cyclic experiments. The capability of the material model to simulate the rate-dependent monotonic response of rubber is examined in Fig. 7 by comparing the stress calculated by FEM with experimental data. The comparison shows an excellent correlation between simulation and experiment for slow strain rates in compression. In general, the accuracy in predicting the experimental response was found to be better for slow strain rates and in the unloading stage of compression.

The improvement is achievable in monotonic response prediction by considering non-linear viscosity phenomena in the constitutive model (Amin, 2005) and to introduce the history dependence of viscosities (Lion,1997) which leads to non-linearly coupled equations which cannot be solved analytically. The effective relaxation times depending on amplitude and temperature are usually applied (Nemeth, 2005).

5.Conclusion

Step-strain relaxation and single relaxation of a filled rubber were modelled with viscoelastic theory. The parameters of the model were determined from relaxation data by employing a nonlinear least-squares method. The proposed model is then compared with experimental data for filled rubber subjected to different loading histories. It is shown that the model gives good quantitative agreement for different relaxational behaviour. An intensive research of the mechanical behaviour of carbon black filled rubbers is currently in progress and will be the topic of a later report.

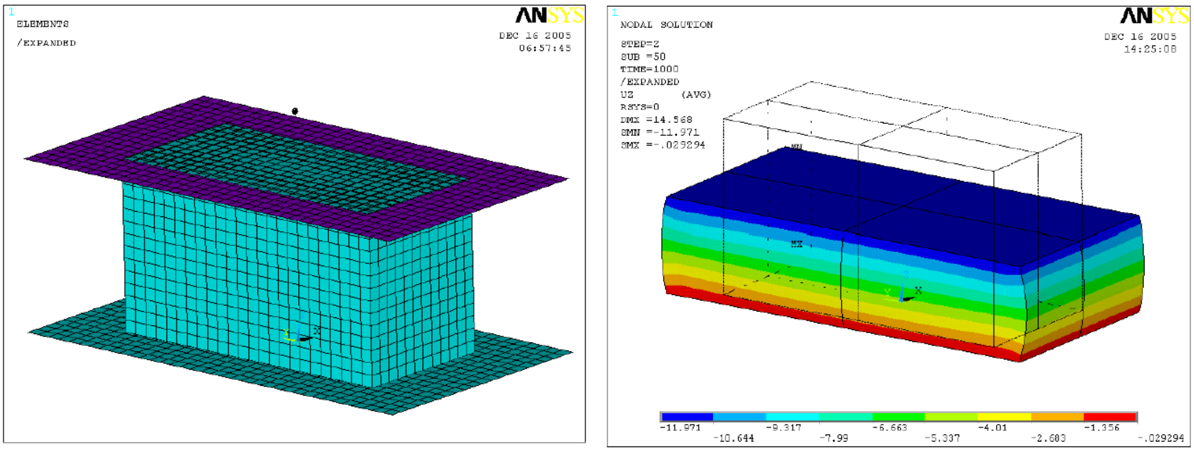

Fig. 6 Model of the experimental specimen in ANSYS (contact coefficient of friction f = 0.2)

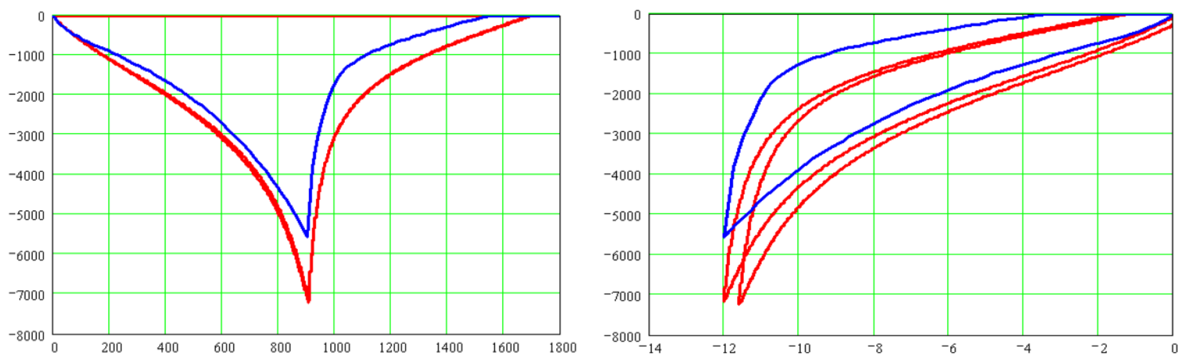

Fig. 7 Simulation of cyclic compression test in ANSYS, red=experiment, blue = simulation

6.Acknowledgements

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