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RHEOLOGIC MODEL OF MECHANICAL PROPERTIES OF POLYURETHANE FOAM

D. Cirkl [1]

Summary: *Paper deals with definition of polyurethane (PU) foam simulation rheologic model with concentrated parameters. The model includes elastic forces, viscoelastic components and frictional damping. It is successfully verified with real data measured on PU foam specimen kinematically excited with triangle and harmonical course of displacement.*

1. Introduction

Polyurethane foam has become a traditional and very extensively used material in many branches It is predominantly used in driver seat production. For computer simulated statical or dynamical comfort of seating it is necessary to describe its properties in sufficient degree which this paper deals with.

Properties of polyurethane foam were investigated on opened cells PU foam specimen of cubic shape with size (100×100×50) mm, density 55 kg/m^3, made from material TDI.

The specimen was inserted into two paralel rigid plates and deformed by means of hydraulic actuator. Kinematic displacement excitation $x(t)$ has a triangle shape with constant amplitude $A = 19.5$ mm, constant mean value $A_0 = A$ =19.5 mm and frequency varied in range $f = (0.01 \div 1.28)$ Hz. It is given by equation (1), where $T = \frac{1}{f}$ is period of excitation and $n = 0, 1, 2, \ldots, \infty$ means number of periods. Corresponding figure of specimen excitation is represented in fig. 1.

$$x(t) = \begin{cases} \frac{4A}{T}(t - nT) & \text{pro } t \in \langle nT; nT + T/2) \\ -\frac{4A}{T}(t - (n+1)T) & \text{pro } t \in \langle nT + T/2; (n+1)T \rangle \end{cases} . \tag{1}$$

[1] Ing. David Cirkl, Ph.D., Technical University of Liberec, Komenského 2, 460 17, Liberec, tel.: +420 485 354 150, e-mail: david.cirkl@tul.cz

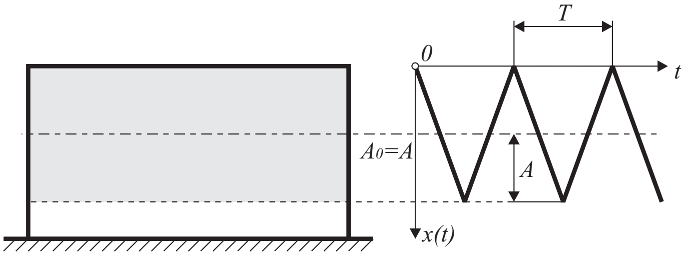

Figure 1: Triangle kinematic excitation (equation (1)) of PU foam specimen

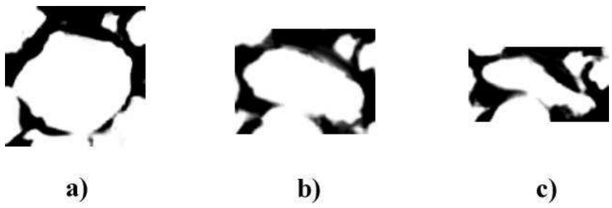

Figure 2: Gradual deformation of loaded cell (enlarged 10×)

2. Definition of PU foam model

2.1. Elastic force

Pores of PU foam create a typical material structure which is to a certain degree able to resist to pressure loading due to its buckling strength. In fig. 2 there is a graduating deformation of loaded cell. Undeformed shape of cell is in fig. 2a), partially deformed in fig. 2b), and fig. 2c) corresponds with state where cells are deformed in high rate and their buckling strength marked F_{b0} has already been overcome and further stops to influents total elastic force F_e. This phase is approximated by degressive function (2). F_b is a force evoked by buckling strength of foam cells, c_b is a coefficient of the structure buckling strength.

$$F_b = F_{b0}(1 - \exp^{-c_b x}). \tag{2}$$

After the initial cells crush with increasing compression there comes to contact between cell-walls. The characteristics of this phase is very similar to course of force during compression of ideal gas in closed vessel. This is described by progressive polytropic function (3). S_p, p_p, h_p, n_p are constants of model, where h_p means the vertical asymptote position (in fig. 4). Total elastic force is given by equation (4).

$$F_p = p_p S_p \left[\left(\frac{h_p}{h_p - x} \right)^{n_p} - \left(\frac{h_p}{h_p + x} \right)^{n_p} \right]. \tag{3}$$

$$F_e = F_b + F_p. \tag{4}$$

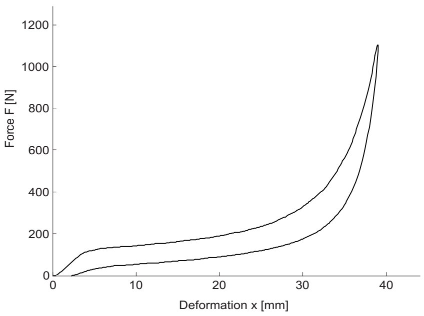

Figure 3: Force response to triangl kinematic excitation with velocity approximately 150 mm/s

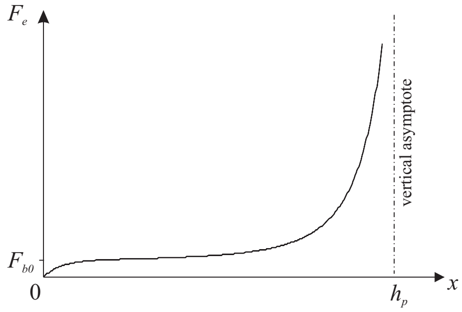

Figure 4: Course of elastic force F_e defined by (4)

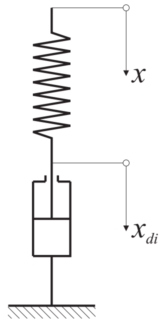

Figure 5: Maxwell's component

2.2. Viscoelastic damping

Damping of the matrix material is described by Maxwell's viscoelastic component with non-linear spring with polytropical characteristics (5) with constants $S_{0i}, p_{0i}, h_i, n_{0i}$, and nonlinear damper with constant of damping b_i and exponent n_i defined by (6) where m is a number of used Maxwell's components.

$$F_{di} = p_{0i} S_{0i} \left[\left(\frac{h_i}{h_i - (x - x_{di})} \right)^{n_{0i}} - \left(\frac{h_i}{h_i + (x - x_{di})} \right)^{n_{0i}} \right]. \tag{5}$$

$$F_{di} = b_i v_{di}^{n_i}, \quad v_{di} = \dot{x}_{di}, \quad i = 1 \ldots m. \tag{6}$$

Differential equation of this element is given by equality of forces in serially added components:

$$p_{0i} S_{0i} \left[\left(\frac{h_i}{h_i - (x - x_{di})} \right)^{n_{0i}} - \left(\frac{h_i}{h_i + (x - x_{di})} \right)^{n_{0i}} \right] = b_i v_{di}^{n_i}. \tag{7}$$

After variables separation the differential increment of damper deformation x_{di} is expressed by the equation (8) which is ready for numerical solution:

$$dx_{di} = \left\{ \frac{p_{0i} S_{0i}}{b_i} \left[\left(\frac{h_i}{h_i - (x - x_{di})} \right)^{n_{0i}} - \left(\frac{h_i}{h_i + (x - x_{di})} \right)^{n_{0i}} \right] \right\}^{\frac{1}{n_i}} dt. \tag{8}$$

2.3. Friction damping

With regard to the contact of cell-walls and struts of cells during compression and their mutual slipping there is logical assumption that also friction participates in PU damping. Friction is included in model with course of friction coefficient f_f defined in dependence on velocity $v = \dot{x}$ by function $\arctan$ in combination with power function in equation (9) which is presented in fig. 6.

$$f_f = \frac{2 f_{f0}}{\pi} \arctan(c_1 v) + c_2 |v|^{c_3} \operatorname{sign}(v). \tag{9}$$

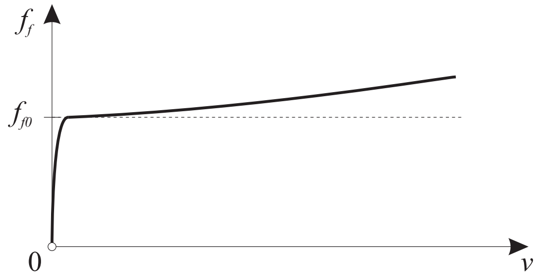

Figure 6: Dependence of friction coefficient f_f on velocity v

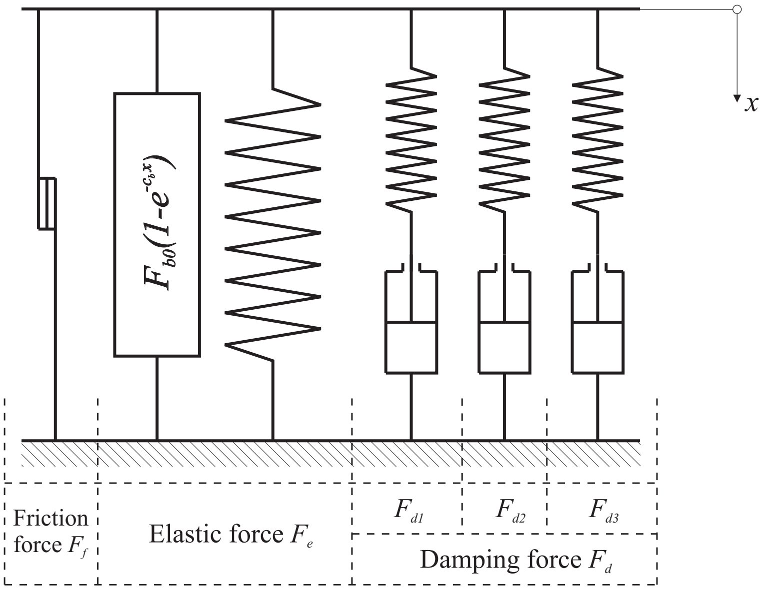

Figure 7: Scheme of PU material model for m=3

The base value for friction force calculation is sum of elastic force F_e and force of Maxwells viscoelastic components $\sum F_{di}$:

$$F_{ed} = F_e + \sum_{i=1}^{m} F_{di}. \tag{10}$$

Friction force then is

$$F_f = f_f F_{ed}. \tag{11}$$

Total force response of PU foam model as is presented in fig. 7 is:

$$F = F_{ed} + F_f. \tag{12}$$

3. Model verification

Model with parameters mentioned in tab. 1 has been verified on the same courses of excitation signals as it was experimentally tested. It means triangle course of displacement $x(t)$ given by equation (1) with frequencies in range f=(0.01÷1.28)Hz. In fig. 8 and fig. 9 there is a

Simulated part of force	Parameter	Physical unit	Value		
F_b	F_{b0}	[N]	80		
	c_b	[N/m]	600		
F_p	S_p	[m^2]	0.0095		
	p_p	[Pa]	100		
	n_p	[1]	6.2		
	h_p	[m]	0.06		
F_{di}			$i=1$	$i=2$	$i=3$
	S_{0i}	[m^2]	1.2	0.03	0.8
	p_{0i}	[Pa]	100		
	n_{0i}	[1]	3	3.5	2
	h_i	[m]	0.05	0.05	0.18
	b_i	[-]	50	300	300
	n_i	[1]	1/5	1/5	1/3
F_f	f_{f0}	[1]	0.05		
	c_1	[s/m]	5000		
	c_2	[-]	0.2		
	c_3	[1]	1		

Table 1: Parameters of PU foam model for force response simulation in case triangle kinematic excitation

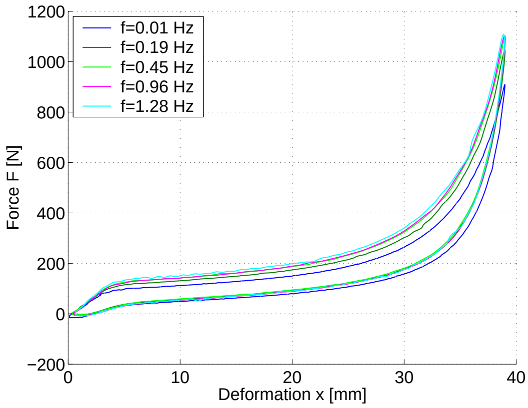

Figure 8: Measured total force F, A=19.5mm, f=(0.01÷1.28)Hz, A_0=19.5mm

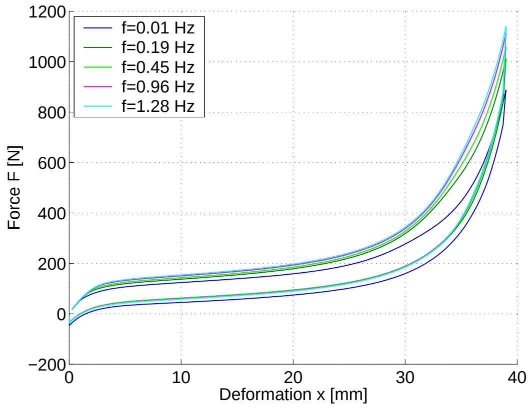

Figure 9: Simulated total force F, A=19.5mm, f=(0.01÷1.28)Hz, A_0=19.5mm

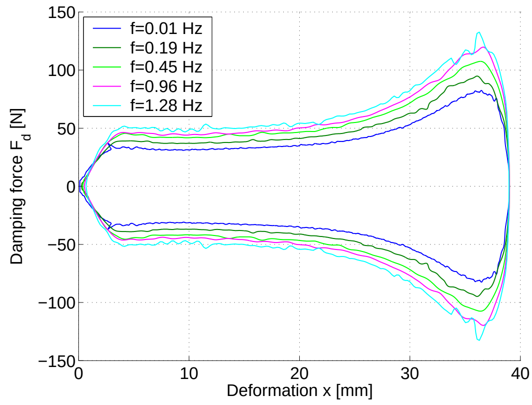

Figure 10: Measured damping force F_d, A=19.5mm, f=(0.01÷1.28)Hz, A_0=19.5mm

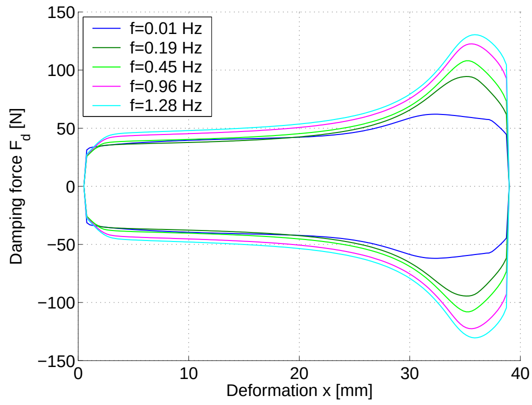

Figure 11: Simulated damping force F_d, A=19.5mm, f=(0.01÷1.28)Hz, A_0=19.5mm

comparison of experimentally measured total force of loaded PU foam specimen and force response of simulating model. Is possible to say that total force is simulated with very satisfying accuracy.

In fig. 10 and fig. 11 there is measured damping force and simulated one. Also courses of damping force separated from total force are simulated in very good precision. However in case of lowest frequency f=0.01 Hz simulated damping force does not reach desirable values for high rates of compression.

4. Conclusion

Rheologic model of polyurethane foam was developed in this article. How verification shows this kind of model definition leads to satisfactory results. In case of verification for triangle course of displacement with little variation of exciting signal parameters (only exciting frequency was varied) the achieved results were very accurate. From the verification in case of harmonic exciting signal with highly varied signal parameters (mean value, amplitude, frequency) follows that for satisfactory accuracy of simulation in all range of parameters variation it is necessary to set some model constants different from those in triangle excitation (tab. 1). This mean that the desired accuracy of this model is possible to guarantee in predetermined type and scope of excitation.

5. Acknowledgment

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6. References

[1] Cirkl D.: Mechanical properties of polyurethane foam. *Ph.D. thesis* (in Czech), Technical university of Liberec, Liberec 2005.