National Conference with International Participation
ENGINEERING MECHANICS 2006
Svratka, Czech Republic, May 15 – 18, 2006
paper no. 107

PREDICTION MODELS FOR FATIGUE DAMAGE ACCUMULATION

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Summary: *The prediction model of fatigue crack propagation in a structural steel is given functioning to compare results of experiments on the phenomenon mentioned subject to simple loading sequences containing single and multiple tensile overloads applied periodically between smaller, constant amplitude cycles. Furthermore, a methodology to estimate the service life of metal constructional members under variable amplitudes is expanded inclusive of the random loading.*

1. The requirement for prediction models

The prediction of fatigue traits of construction and getting out of structural fatigue were stated being engineering challenges in the initial decades of the 20^{th} century. High stress concentrations were regarded as detrimental and should be obviating. The significance of stress concentration factors was known before 1950 and designers realized that the fatigue performance of a structure was dependent on improved detail design. The term "Designing against fatigue" was characteristic for the engineering fatigue problem. Various models were developed for the prediction of notch and size effects. Initially, the aim was to derive fatigue properties of notched elements from fatigue properties on unnotched specimens. The proposed models included a good deal of empirism. One specific goal was to predict the fatigue limit, an important fatigue property for many product of the industry. In the 1960s and afterwards a need was also felt to predict fatigue crack growth, welded structures and pressure vessels in view of fail-safe properties, service inspections and safety in general.

Prediction models on crack growth were much stimulated by the introduction of the stress intensity factor. Still another fatigue problem was associated with load spectra containing load cycles of various magnitudes, or in other words, fatigue under variable-amplitude (VA) loading. If fatigue cycles above the fatigue limit occur, crack initiation can not be avoided and a finite life is possible. A need for predictions on fatigue under VA loading was present. Several prediction problems can thus be defined. A practical problem also associated with VA-loading was the question of how long old structures could still be used without running into fatigue problems. In the second half of the previous century, this question was raised for old bridges, quite often bridges built in the 19^{th} century. The question was whether fatigue problems should be anticipated or whether the bridge should be replaced by a new one. Bridges were frequently more hard loaded by heavy traffic than in origin expected in the design process at one time.

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Matters coming up from the above description are associated with the reliability and accuracy of prediction models and the physical concept of fatigue damage. Prediction problems can be defined in two categories. The similarity concept (sometimes called the similitude concept) is characteristic for the first category. Fatigue damage accumulation is the starting point of the second class.

2. Crack growth in steel subject to variable cycles

Steady safe functioning of construction under variable amplitude loading is dependent on knowledge of the characteristics of a fatigue crack in such circumstances. The VA load sequences may introduce load interaction effects raising either retardation or acceleration of fatigue crack propagation. Though responses of various materials to VA load histories have been widely investigated a long time, mechanisms accountable for the load interaction events have not been identified and comprehended in full. But still, induced plasticity crack closure has been extensively recognized being useful in qualitatively explaining crack growth transitions and empirical tendencies watched in fatigue experiments.

Evaluations of the performance of prediction models for VA fatigue crack propagation were hitherto confined to aircraft materials where as their suitability to other metals is less known. Some testing outcomes by Skorupa give us to understand that mechanisms governing the load interaction effects in the course of fatigue crack growth in non-aircraft structural steels may differ than in Al alloys.

In the paper, the simulation predictions are given, according to (Skorupa et al., 1999), for crack growth subject to simple sequences embracing single and multiple tension overloads acted periodically between smaller baseline constant amplitude cycles.

The material applied was Polish 18 G2A steel, analogous to Grade 11503, in accordance with the Czech denomination (and corresponding to the Grade S 355 conforming with standard CSN EN 10025 + A1).

3. Prediction model of crack propagation

Crack growth predictions were fulfilled employing a strip yield simulation grounded on plasticity-induced crack closure. The SY type simulations of PICC use the Dugdale strip yield model of crack tip plasticity adapted to depart a wedge of plastically stretched material on the fatigue crack surfaces. This is achieved by severing the strip material in the crack tip plastic zone over a distance relevant to the fatigue crack growth increase. That is why the plasticity elongated material is putting on the fatigue crack surfaces being indicated in Fig. 1.

Note in Fig. 1 that the Dugdale fictitious crack of length a_{fict} is extended compared to the physical crack of length a by the plastic zone size, r_p. In order to meet the compatibility between the elastic plate and the plastically deformed strip material, a traction must be applied on the fictitious crack surfaces, both in the plastic zone ($a \leq x \leq a_{fict}$) and over some distance in the crack wake ($0 \leq x \leq a$), where the plastic elongations of the strip, $L(x)$, exceed the fictitious crack opening displacements, $V(x)$ (Fig.). The compressive tractions applied in the crack wake to validate $L(x) \equiv V(x)$ are related to being the contact stresses. In PICC-grounded prediction approaches, the material's memory of the previous load history is accounted for by the distribution of residual plastic stretches. With the SY model, these stretches are related to the contact stresses through the compatibility condition referred to above. According to Newman, the contact stresses affect the crack opening stress, S_{op}, which can be determined using the super-position principle, as follows

$$K(S_{op}\text{-}S_{min}) - K(\text{contact stresses}) = 0 \tag{2}$$

Simultaneously, the stress intensity factors on the left side correspond, respectively, to the contact stresses and the applied stress increment from the minimum level, S_{min}, to S_{op}. Here, Eq. (2) states that as long as the crack is closed, there can be no singularity at the physical crack tip.

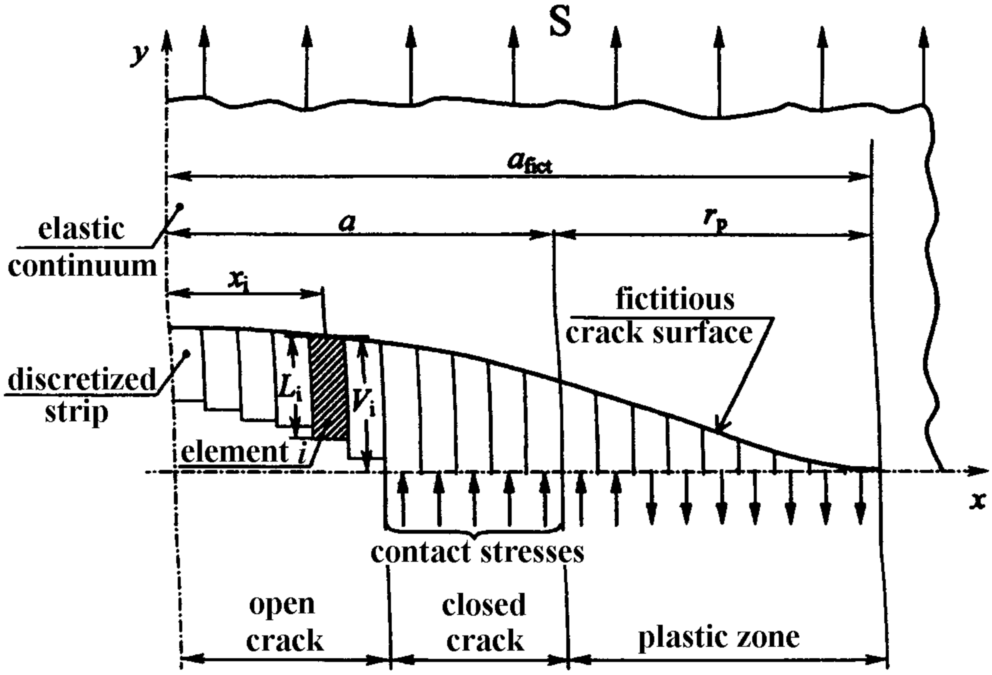

Fig. 1 Schematic illustrating the compatibility requirements in the strip yield model of crack closure

The PICC mechanism involves that the FCGR in a given cycle is controlled by the current level of the effective stress intensity factor range, ΔK_{eff}. In this manner

$$\mathrm{d}a / \mathrm{d}N = f\left(\Delta K_{\mathrm{eff}}\right) \tag{3}$$

where

$$\Delta K_{eff} = 0 \; if \;\; K_{op} \geq K_{\max}$$

$$\Delta K_{eff} = K_{\max} - K_{op} \;\; if \;\; K_{\min} < K_{op} < K_{\max}$$

$$\Delta K_{eff} = K_{\max} - K_{\min} \;\; if \;\; K_{op} \leq K_{\min}$$

and K_{min}, K_{op} and K_{max} are the minimum, crack opening and maximum levels of the stress intensity factor, respectively, and the function $f(\Delta K_{eff})$ is supposed to be the same for both CA and VA fatigue crack growth.

The SY model presentation developed by the authors (Skorupa et al., 1999) is similar to that proposed by (Newman, 1981). Particularly, the plastic strip discretization, the computation of S_{op} by Eq. (2), and the concept of a constraint factor on the tensile flow in the crack tip plastic zone to account for the three-dimensional nature of PICC have been adopted from Newman's approach. The numerical procedure, however, it distinct from that of Newman. The predictions have been made for a variable, so-called global constraint factor averaged over the yielded material ahead of the crack tip. The required inputs to the model are the material flow stress (defined as the mean of the yield stress and the tensile strength), the material da/dN versus ΔK data from CA tests, the specimen geometry and the load history.

To gain the da/dN x ΔK_{eff} relationship [Eq. (3)], the SY model was applied to determine the S_{op} stresses for selected specimens for which the fatigue crack growth rates were given by the Paris relation in the form

$$\mathrm{d}a / \mathrm{d}N = 1.1585 \times 10^{-8} - \left(\Delta K\right)^{2.76} \tag{4}$$

The equation of the regression line for the resulting da/dN versus ΔK_{eff} data was selected as

$$\mathrm{d}a / \mathrm{d}N = 2.051 \times 10^{-8} - \left(\Delta K_{eff}\right)^{2.91} \tag{5}$$

with da/dN in mm/cycle and ΔK_{eff} in $\mathrm{MPa}\sqrt{\mathrm{m}}$.

4. Random loading

In service, many structural constituents are subject to a zone of stress cycles that is more variable in amplitude than a series of repeated blocks, see (Frost et al., 1999).

Some improvement in simulating these loading histories can be achieved using testing machines having several different load channels controlled by punched tape, using short blocks, and randomizing the order of blocks. Such tests give results approximating to random loading tests. In many cases, however, it may be preferable to make use of an actual random process, such as the output of a white noise generator, being the basic element for the modelling at the application this outlet to a proper vibrator. It should be remarked that the term random loading as formally used to fatigue tests is inexact. Any testing machine must have a finite applicable load, and therefore the amplitude of any stress peak is only random between zero and some upper limit. In general, the order of application of the stress amplitudes does approximate to a random distribution but is influenced to some extent, sometimes considerably, by the inertia of the moving parts of the testing machine. In regular block tests, either at zero mean stress or with a fixed superimposed mean stress, the alternating stress varies sinusoidally about a known mean level (that is, the ratio of the number of zero crossings to the number of peaks is always unity), but this need not be the case in a random loading, because now both the stress range and mean stress may vary every cycle. The analysis of a random stress-time waveform thus presents some difficulties, whereas in a multi-level block test the cycle ratio *n/N* is always known whatever the number or order of application of blocks of cycles, so that it is relatively easy to sum the *n/N* values in a given test. In fundamental work, therefore, signals used in random loading tests are often passed through an appropriate filter to provide a narrow-band frequency distribution, that is, the ratio of the number of zero crossings to the number of peaks (the *irregularity factor*) approaches unity. Tests can then be carried out at either zero mean stress or any given mean stress, and the concept of cycles to failure is retained. The amplitude of the random stress waveform is defined by its root square mean value σ.

We can regard probability distributions of random waveforms as the probability density function *p(S/σ)*, which is the probability that the magnitude of the stress *S* will lie between two given values (this quantity is usually normalized by dividing it by the r.m.s. value *σ*) or, more usually, in terms of the cumulative distribution function *P(S/σ)*, which is the probability that the magnitude of *S* will exceed a certain value. It can be seen therefore that *P(S/σ)* is equal to the area under the curve of *p(S/σ)* versus *S/σ* obtained by the integration from *S/σ* to ∞. The two distributions usually faced in fatigue operation are the Gaussian or Normal distribution and the Rayleigh distribution. Gaussian describes a random function whose instantaneous value is described by the Gaussian probability density function as indicated in Fig. 2 and expressed by the relation

$$p\left(\frac{S}{\sigma}\right)=\frac{1}{\sqrt{(2\pi)}}\exp\left(\frac{-S^2}{2\sigma^2}\right) \tag{6}$$

Through integration and by supposing positive and negative values, the cumulative distribution function (Fig. 2) reads

$$P\left(\frac{S}{\sigma}\right)=\frac{2}{\sqrt{(2\pi)}}\int_{S/\sigma}^{\infty}\exp\left(\frac{-S^2}{2\sigma^2}\right)d\left(\frac{S}{\sigma}\right) \tag{7}$$

This distribution adopts to the instantaneous values of both broad-band (that is, the irregularity factor is less than unity) and narrow-band (that is, the irregularity factor approaches unity) random oscillation.

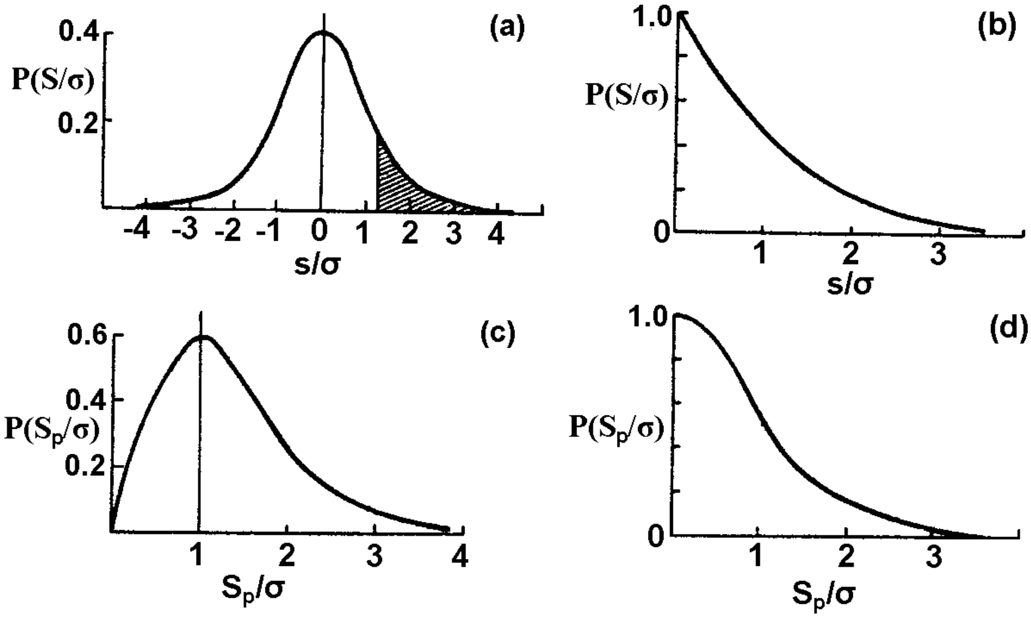

Fig. 2 Probability distributions. (a) Gaussian probability density. (b) Gaussian cumulative distribution function. (c) Rayleigh probability density. (d) Rayleigh cumulative distribution function

5. Discussion

The engineer asks for, when designing at the preliminary phase, some systems of estimating the characteristics of a structural member subject to varying stress amplitudes. This he may do by means of the Palmgren-Miner rule or by some more complex method. The latter methods are either too cumbersome for ready use, restricted to certain conditions or materials, or, while giving more accurate predictions (for example, the hypothetical *S/N* curve methods), require considerable experience to be used with confidence. The Palmgren-Miner rule is widely considered as inaccurate and, in many situations, dangerous. However, when considering cumulative-damage rules, it is important to consider what is meant by damage in a cumulative-damage test. The life of any specimen, subjected to any type of loading cycle, is the sum of the number of cycles required to initiate and develop a surface microcrack to the necessary depth for it to grow as a macrocrack and the number of cycles required for the macrocrack to grow across the specimen cross-section. In a plain specimen, there is no metallographic evidence to suggest that a surface microcrack progresses linearly with number of cycles or, even if it does at one stress level, that it will continue doing so at another stress level, as is assumed by the linear damage law. Indeed, it is known that the scatter in lives of specimens subjected to the same constant stress amplitude is associated with the initiation and development of microcracks, and therefore it is not unreasonable to suppose that this scatter will be increased under conditions of varying stress amplitudes.

In addition, if the specimens are subjected to numerous low-stress cycles, coaxing phenomena which occur in some materials will tend to retard the development of a microcrack at a higher stress level. However, with sharply notched specimens in which the life is spent in growing a macrocrack, damage assessed in terms of crack length or depth which increases with increasing number of cycles can be considered a physical reality. Indeed, it has been shown that, if no residual stresses are induced in the material ahead of the crack tip, the life of a specimen, spent wholly in propagating a macrocrack under a varying stress cycle, can be predicted from the linear damage law. However, it is known that, when the stress level is reduced from a higher value, residual compressive stresses are induced in the material ahead of the crack tip which will retard the subsequent growth rate. Depending on the change in stress level, the material, and whether or not the residual stresses are relaxed at the lower level, the crack growth rate at this latter stress may be abnormally slow or may even be reduced to zero, so resulting in the high values of $\Sigma(n/N)$ often found when sharply notched specimens are tested in simple high-low two-level tests. In multi-level block or random loading tests, the change in stress level is often not so drastic as in a two-level test, and residual stresses may not only be of smaller magnitude but may be more easily relaxed, thus giving values of $\Sigma(n/N)$ more nearly equal to unity. However, except in simple block tests in which the stress levels are not widely different (that is, there is little deviation from a constant stress amplitude test), the use of the linear damage law to predict the behaviour of plain specimens in a cumulative damage experiment in the majority of cases may result in hazardous predictions.

6. Conclusion

The strip yield simulation is able to give an account qualitatively of the experimental trends in crack growth due to the loading variables. The transient crack growth behaviour predicted by the model is in agreement with the experimental observations. In terms of crack growth lives, its accuracy is comparable to the precision reported for non-ferrous alloys under similar loading histories. Improvements of the SY model for its application to steel requires a more accurate description of the material stress-strain response. A low-carbon steel demonstrates noted load interaction influences in the course of fatigue crack growth under load sequences with periodically applied single and multiple overload cycles.

The quantity of crack growth retardation under periodic single overloads increase with an enlargement in the overload level. For multiple periodic overloads, increasing the size of the overload block enhances retardation.

7. Acknowledgements

The author gratefully acknowledges the financial support of the presented research by the Grant Agency of the Czech Republic (project No. 103/03/0655).

8. References

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