IMPLEMENTATION OF MICROPLANE MODEL IN ANALYSIS OF RC FRAMES

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Summary: *During the last decades, the finite element method has become the most powerful structural analysis tool and it allows using of more and more sophisticated material models. These describe real material very correctly, but the computational cost of an analysis with such models is overly high. Good example of this fact is concrete - one of the most widespread building materials. In this paper, authors focused on the microplane model. This model describes rather nonlinear behaviour of concrete very truly, the computational cost, however, is insofar high making wider usage in practical engineering not possible. That is why we proposed an adaptive analysis of RC concrete frames with microplane joints. Initially, a frame is composed of only beam elements with nonlinear behaviour. In the course of computation, parts which exceeded suitable criterion are replaced by three-dimensional microplane model. At the moment of collapse, the microplane model is used only in some sections of the frame (especially close to joints), so computational effort is smaller compared to frame modelled completely by 3D elements.*

1.   Introduction

In this paper the use of the microplane model in an analysis of reinforced concrete frames is discussed. The microplane model is generally a constitutive, fully three-dimensional model capable to describe concrete in its very complex behaviour. Thanks to its qualities, the microplane model is convenient for simulations of experimental results [1] and comprehensive analysis of various structures from concrete. In particular, we focused on RC frames where we can analyse ultimate displacement, creation of plastic hinges etc. However, wider usage of this model is not possible because of high computational cost. That is why we proposed below discussed adaptive analysis.

It is based on a geometrical model of frame compound of beams and 3D elements (bricks). The microplane model is applied only for bricks, which will be used for segments of the frame with significant nonlinear strains. The rest of the frame will be modelled by beam elements with fibered cross-section, which partly allows simulation of nonlinear behaviour of material.

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Due to nonlinearity of the microplane model, the loading is applied in an incremental manner. In the first loading steps, the whole frame is formed only by beam elements. In next steps, cross-sections of all beams are monitored and when strain state of some concrete fibres leaves elastic part of strain-stress diagram, those beam elements are replaced by fully three-dimensional model of beam with microplane material model. This approach enables to achieve required loading level with less number of elements thus solving of the problem takes less time. However, the solution time of nonlinear static analysis is still too large, so for large problems we have to use explicit dynamic solver. Finally, we conclude our paper by numerical example.

2.   Microplane Model

The classical constitutive modelling approach is based on a direct relationship between strain and stress tensors and their invariants. Contrary to it, constitutive relations of the microplane model for concrete are formulated in terms of strain and stress components on planes of arbitrary spatial orientation, so-called microplanes. This approach excels in conceptual simplicity and allows straightforward modelling of anisotropy and other features related to planes of different orientation. The penalty to be paid is the great increase of computational effort. Although the microplane theory was originally proposed for plastic behaviour of metals, it can be generally used for any type of material including concrete [2, 3]. The relationship between micro and macro level is obtained by projecting strain tensor to the particular microplanes (so-called kinematic constraint) or by projecting stress tensor (static constraint). Then constitutive relations between microstrains and corresponding microstresses are evaluated (the constitutive relations can be defined in different ways, depending to it we distinguish four models M1, M2, M3 and M4). The missing link (between micro and macro stresses for kinematic constraint and between micro and macro strain for static constraint, respectively) is obtained by application of the principle of virtual work. Such kind of a material model is capable to describe the triaxial nonlinear behaviour of concrete including tensional and compressive softening, damage of the material, different types of loading, unloading or cyclic loading.

The microplane model is numerically extremely demanding - computation of the stress tensor in a single integration point involves the strain projection to microplanes, the evaluation of local microplane constitutive laws (which may lead to iteration) on each microplane and the homogenization procedure for computing the overall stress tensor. Moreover, the tangent stiffness matrix can be hardly obtained. For some microplane formulations, there is no direct formula and the only possibility is to construct stiffness from its definition. But this is a very expensive procedure. Due to the lack of tangent stiffness one can use the initial elastic matrix for the whole analysis, but this will lead to a very poor convergence. Therefore, the use of implicit methods, which require the stiffness matrix, is cumbersome, due to an extremely slow iteration process.

That is why dynamic relaxation method based on explicit time integration is more efficient. In addition, for nonlinear problems it is convenient to use optimum or quasi-optimum loading minimising inertia forces.

3. Computational Scheme

1. Brief review of the central difference explicit integration method This method is based on the direct integration of the governing equilibrium equation. The equation in time t can be written

$$M\ddot{r}_t + C\dot{r}_t + F_t(r_t) = R_t \ , \tag{1}$$

where M and C represent the mass and damping matrices of the discretized system, respectively. R_t is the load vector and F_t is the vector of real internal forces at time t. The F_t vector is evaluated using constitutive relations, the microplane model in our case. The new values of displacement, velocities and accelerations at time $t + \Delta t$ are computed from discretized equilibrium equation at time t using the known values of displacements r, velocities $\dot{r}$ and accelerations $\ddot{r}$ at time t. As assumption about the evolution of these characteristics in interval $\langle t, t + \Delta t \rangle$, a simple differential scheme is used

$$\dot{r}_t = (r_{t-\Delta t} + r_{t-\Delta t})/(2\Delta t) \ , \ \ddot{r}_t = (r_{t+\Delta t} - 2r_t + r_{t-\Delta t})/\Delta t^2 \ . \tag{2}$$

Substituting Eqs (2) into Eq. (1) and expressing it in an incremental form, we obtain the following relation

$$\left(M \frac{1}{\Delta t^2} + C \frac{1}{2\Delta t} \right) \Delta r_t = R_t - F_t + \left(M \frac{1}{\Delta t^2} - C \frac{1}{2\Delta t} \right) \Delta r_{t-\Delta t} \ , \tag{3}$$

where $\Delta r_t = r_{t+\Delta t} - r_t$. From this equation, the unknown displacement increment Δr_t can be computed. The corresponding vectors of velocities and accelerations can be easily computed using

$$\dot{r}_t = (\Delta r_t + \Delta r_{t-\Delta t})/(2\Delta t) \ , \ \ddot{r}_t = (\Delta r_t - \Delta r_{t-\Delta t})/\Delta t^2 \ . \tag{4}$$

Particularly, the use of diagonal mass matrix M is assumed. If dumping matrix C is expressed in the special form of Rayleigh dumping $C = \alpha M$ ($\alpha \in \Re$), one does not need the stiffness matrix. Typically, the nonequilibrated internal forces are applied as loading in the next time step. These choices significantly simplify the solution of the problem and lead to a very efficient computational scheme, which can be parallelized in a straightforward way.

2. Dynamic relaxation using optimum load time history for nonlinear analysis Dynamic relaxation is used in dynamic structural analysis programs based on explicit time integration schemes to make static analysis possible. The solution of the static problem is found as the ultimate settled state of the associated transient dynamic problem. By applying dynamic relaxation we obtain efficient algorithm. Nevertheless, there are drawbacks obstructing the efficient use of this method in nonlinear problems. Conventional dynamic relaxation employs the step function for the load time history. Inertia forces play a significant role in the initial phase of motion before they decay due to damping. In problems involving nonlinear path dependent materials, the final settled state may then differ from the state that would be reached if proportional static loading were applied. Both drawbacks can be eliminated if a slow continuous loading is used instead of the step load, in order to keep the inertia and damping forces small from the very beginning. If a fixed time interval and an ultimate load level are given, the inertia and damping forces depend on the load time history.

Required formula for calculation of the load time history was proposed in [4]. Author minimised here the vector of inertia and damping forces

$$\boldsymbol{y}_t = \boldsymbol{M}\ddot{\boldsymbol{r}}_t + \boldsymbol{C}\dot{\boldsymbol{r}}_t = p_t\boldsymbol{P} - \boldsymbol{F}_t \ . \tag{5}$$

This equation follows directly from Eq. (1). The vector $\boldsymbol{P}$ determines the space distribution of the loading (known) and $p(t)$ is a time-dependent factor of loading representing wanted load time history. If multi degree of freedom nonlinear undamped systems are considered the following equation is obtained

$$p_t = \frac{1}{\boldsymbol{P}^T\boldsymbol{M}^{-1}\boldsymbol{P}}\boldsymbol{F}_t^T\boldsymbol{M}^{-1}\boldsymbol{P} + c\frac{\tau - t}{\tau} \ , \tag{6}$$

where τ is total time and c is a parameter which determines the rate of the loading process and which is evaluated from

$$\boldsymbol{P}^T\boldsymbol{r}_\tau = \frac{c\tau^2}{3}\boldsymbol{P}^T\boldsymbol{M}^{-1}\boldsymbol{P} \ . \tag{7}$$

The vector $\boldsymbol{r}_\tau$ or the scalar $\boldsymbol{P}^T\boldsymbol{r}_\tau$ has to be guessed.

4. Geometrical Model

Because a complicated finite element mesh is required for discussed model, we had to create special preprocessor capable to generate mesh consisting of beam elements, bricks and trusses. In addition we have to connect these elements into one compact unit.

1. Mesh of reinforcement In order to create a correct and flexible model of 3D parts of a frame, it has to be compound of two independent meshes. The primary mesh, built by 3D elements (here hexahedrons $\sim$ bricks are used), represents concrete part of frame. Because microplane material constants have to be fitted according to experiments for one specific element size, all bricks have to be identical in size. The secondary mesh is built by truss elements and represents the reinforcement.

To guarantee bond between concrete and reinforcement, both meshes have to be interconnected. For usual models this problem is solved during the creation of the primary mesh. It is generated so the primary mesh is intersected by reinforcement exactly in vertices of bricks and nodes of the secondary mesh can be identified with the nodes of the primary one (see Fig. 1).
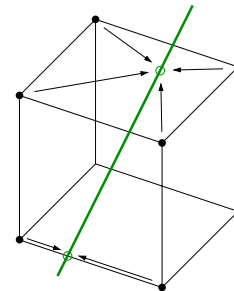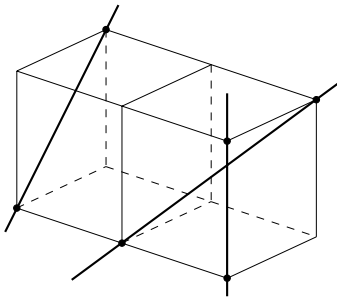

Figure 1: Brick and truss elements with shared nodes.

Figure 2: ● - master node, ○ - hanging node.

In regular mesh, 3D elements are mostly intersected besides of vertices. In such case, interaction between reinforcement bars and concrete bricks is ensured by hanging-nodes. It means that behaviour of a node of the secondary mesh is subordinate to behaviour of several nodes of the primary mesh. Practically we have to find all intersection points of a reinforcement bar and sides (edges) of bricks. Hanging nodes lie at these points and master nodes are identical with vertices of corresponding sides (edges) (see Fig. 2).

At first, the preprocessor finds all hanging nodes and divides rods of the secondary mesh, which are given as line segments or polygons, into elements. For each hanging node it finds its master nodes and computes natural coordinates inside corresponding brick, side or edge. In the case of a large amount of elements, it would be too slow to find hanging nodes going over all elements and finding possible intersection with each reinforcement polygon. That is why the preprocessor firstly maps complete connectivity of the primary mesh. Next it goes over each element and finds a master element for the first node of the polygon. Now the following intersection is searched always on elements adjacent to the last intersected (master) element only. In this manner it continues along the polygon to its end.

2. Hanging nodes When given element (side or edge of element) and node interacts (they have identical displacement), but the node cannot be identified with an existing node on the element, we can treat this node as a hanging node and nodes of the element are called *master nodes*. Displacement of master nodes $\boldsymbol{u}_p$ is obtained from the solution of governing equations and displacement of hanging node $\boldsymbol{u}_{hn}$ is computed from element interpolation functions $\boldsymbol{N}$ and corresponding nodal displacement vector $\boldsymbol{u}_p$ by

$$\boldsymbol{u}_{hn} = \boldsymbol{N}\boldsymbol{u}_p \ . \tag{8}$$

Analogically we can handle other quantities (loading etc.).

3. Rigid arm Connection between a beam element and a part of the frame modelled by bricks is formed by rigid arms. A node at the end of the beam is called master node and nodes on corresponding surface of the 3D part of the frame are slaves. Displacement vector of master node $\boldsymbol{u}_m$ is obtained from the solution of governing equations and displacement vector of the slave node $\boldsymbol{u}_s$ is computed from $\boldsymbol{u}_m$, transformation matrix and the distance between the master and the slave node.

5. 3D Beam Element

In those parts of a frame where behaviour of concrete in pressure is only slightly nonlinear beam elements with *fibred cross-section* are used. This cross-section is divided into a number of fibres. Each fibre can adopt different material model and they can be in different stress-strain state, so internal forces and geometrical characteristics of cross-section have to be integrated after each change of loading. For fibres representing rods of reinforcement common elasto-plastic material model is used. For fibres representing concrete we had to create a special material model. In pressure it is elastic with secant elastic modulus $E_b = 0.85\ E_{in}$, where E_{in} is the initial elastic modulus. In tension the material is elasto-plastics with softening - it simulates real behaviour of concrete (see Fig. 3).

After each loading step a check is performed to detect beam elements with fibres in considerably nonlinear state. There are only two nonzero stress components σ_x and τ_{xy} at an integration
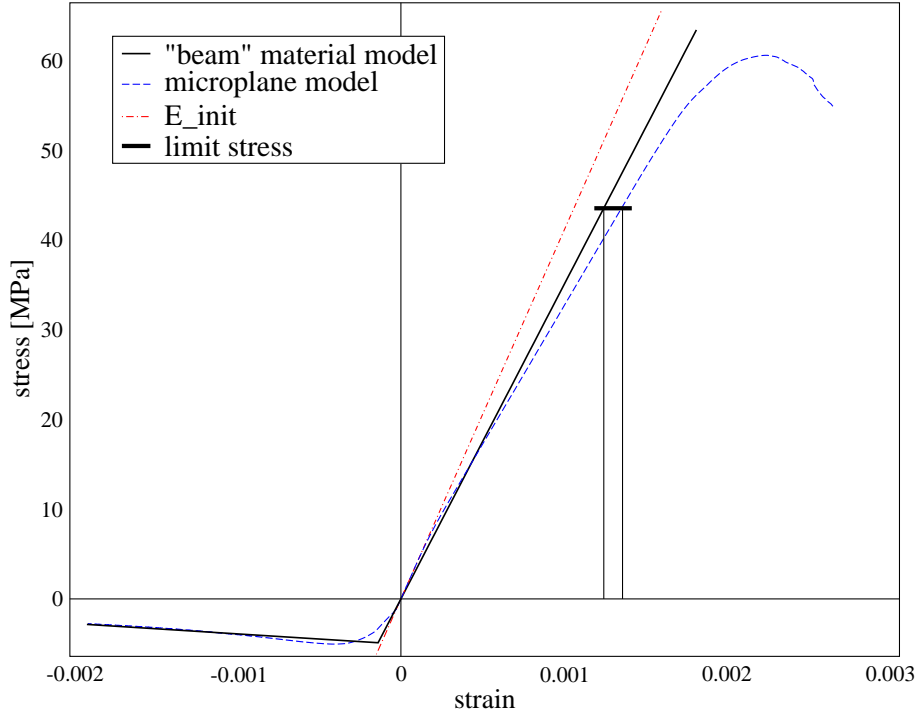
Figure 3: Stress-strain diagram.

point of a fibre. Several stress-strain diagrams were obtained for various ratios σ_x/τ_{xy} using a single brick model with microplane material and a single fibre model with "beam" material model. For each ratio we can find limit stress σ_x^{lim} (τ_x^{lim}) which can be considered as slightly nonlinear. The limit is approximately found as stress for which prescribed percentual energy error e is reached:

$$e = 100 W_b/W_m. \tag{9}$$

where W_b and W_m are carried out works for fibre and brick, respectively, with same value of stress. Set of limit stresses forms boundary curve.

6. Example

For numerical verification of the proposed method a two story frame loaded by both vertical and lateral uniformly distributed load was used. The frame geometry is shown in Fig. 4. The employed constitutive model is based on microplane model M4. The constitutive properties for brick of size 60x60x60 mm are following: density $\rho = 2500$ kg/m^3, dumping coefficient $\alpha = 0$, Young modulus $E = 41039$ GPa, Poisson coefficient $\nu = 0.18$, M4 parameters $k_1 = 0.000228$, $k_2 = 500$, $k_3 = 15$, $k_4 = 150$. The solution started with the initial mesh 1, which was five times adapted (see Figs 5-10). After each modification the computation was started from the beginning again. Because of low number of elements of primary meshes, using nonlinear static analysis in first steps was more efficient than using dynamic relaxation. To keep solution consistency we employed nonlinear static analysis for all steps of the solution, although for final mesh it was time demanding. Dynamic relaxation analysis was employed only for the last
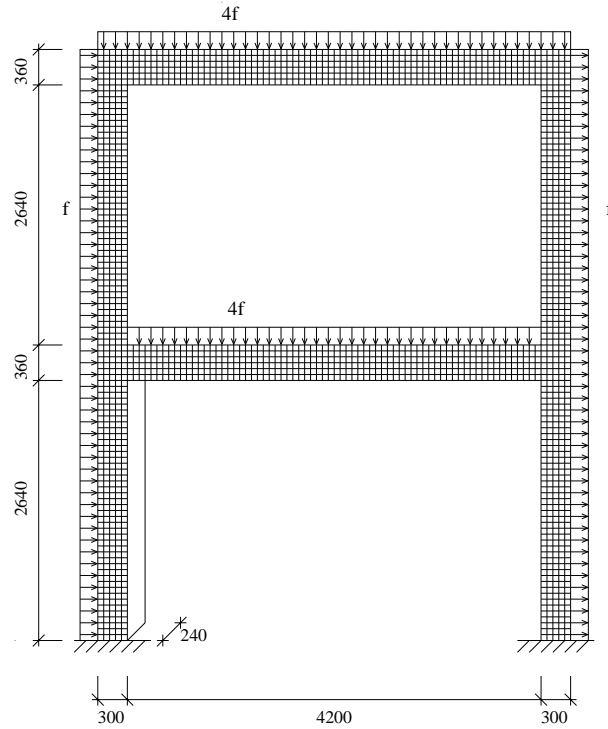
Figure 4: Frame geometry, loading f=0.01MN/m, full mesh - 0 beams, 7360 bricks, 1760 truss elements.

mesh. Considering that the dynamic solution is significantly faster than static one, we can recommend nonlinear static solution for research of initial behaviour of frames. For investigation of structures subjected to ultimate load we recommend the dynamic solution.

7. Conclusion

Microplane model is computationally very demanding. RC frame structures formed only by 3D elements with microplane model put high requirements even on today's powerful computers. The proposed method implemented into an existing object oriented finite element environment is able to reduce number of 3D elements with demanding material model and at the same time retains computational accuracy of the microplane model.

8. Acknowledgements
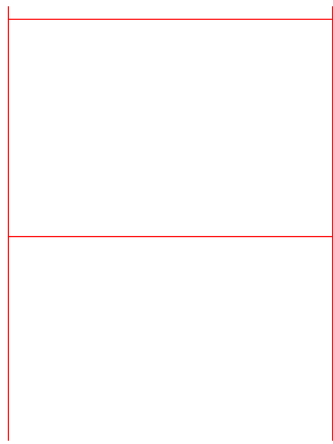
Figure 5: Mesh 1, 352 beams.

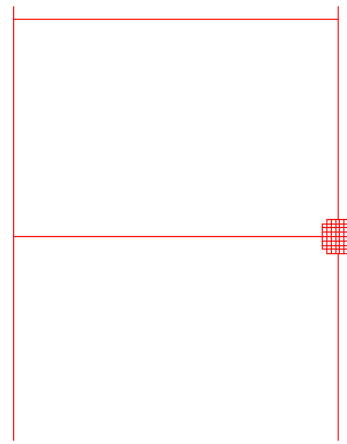Figure 6: Mesh 2, 340 beams 184 bricks, 68 truss elements.

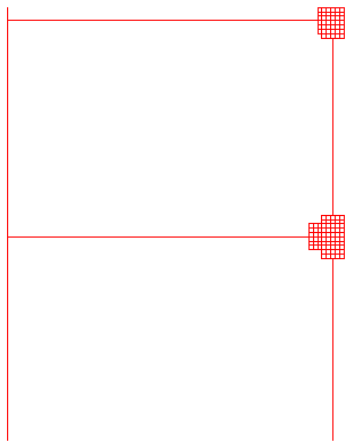

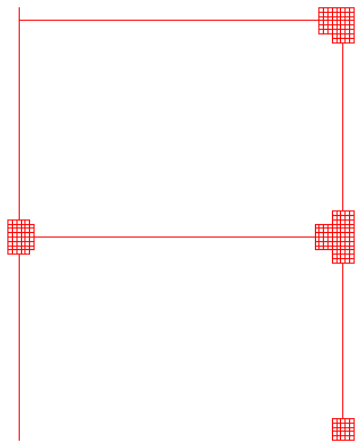Figure 7: Mesh 3, 325 beams 436 bricks, 152 truss elements.

Figure 8: Mesh 4, 302 beams 852 bricks, 270 truss elements.
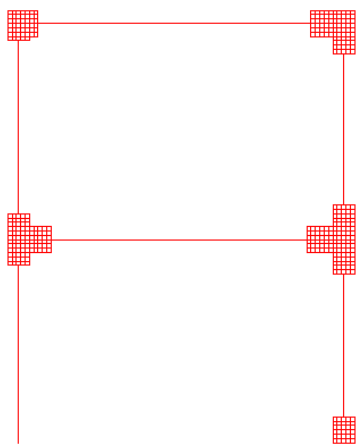

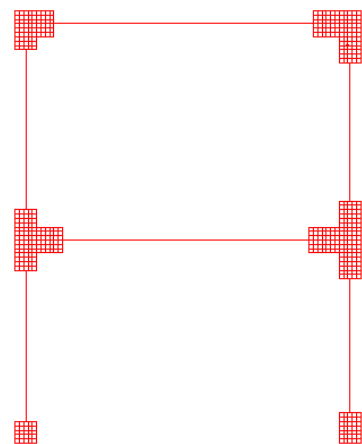Figure 9: Mesh 5, 271 beams 1452 bricks, 432 truss elements.

Figure 10: Mesh 6, 252 beams 1852 bricks, 518 truss elements.

9. References

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