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MATHEMATICAL MODELING AND EXPERIMENTAL INVESTIGATION OF FRICTIONAL PROPERTIES OF VULCANIZED RUBBER

J. Slabeycius, R. Bartko, A. Beláková*

Summary: *Knowledge of the physical properties of materials is basis for design and mathematical modeling of complex systems, where the materials are used. Frictional properties of rubber belong to the most important physical properties of rubber. Specific sliding motion of two surfaces, which are in contact, is stick-slip. This motion consists of two phases: stick and slip, which alternate more or less periodically. In spite of fact, that stick-slip has great practical importance; only a few works have been done on this problem. Mathematical modeling of frictional properties of vulcanized rubber is shown in this paper. Results obtained from models are compared with experimental results.*

1. Introduction

The friction between rubbery polymers and hard substrate has a quite other character than that between two solid surfaces. When rubber is brought into contact with hard surface, it deforms elastically and the real contact area will increase with increasing normal load and consequently the friction coefficient will decrease with increasing normal load. The rougher is the solid surface the stronger is the effect. The specific case of sliding motion is stick-slip, when sliding body moves irregularly. The stick-slip consists to two phases: stick and slip, which alternate more or less periodically.

The friction force between rubber and a rough solid surface has two contributions: hysteretic and adhesion components [4-6]. The hysteresis component result from the internal friction of the rubber and it relates with its viscoelastic properties; the adhesion component is important for very clean rubber surfaces. The first mechanism is connected with the dissipation of energy in the small volume of rubber close to contact area – it is the bulk effect, while the second one can be assumed as a surface effect. Local adhesion bonds between hard surface and the polymer chains cause adhesive force. These bonds are periodically elongated and abrupt through motion of rubber.

The friction coefficient of rubber may vary considerably with sliding velocity. Therefore it is necessary to measure friction over the range of velocities of interest. Friction is also dependent on temperature, which can lead to inaccuracies at high velocities because of heat

* **Prof. RNDr. Juraj Slabeycius, PhD., Ing. Róbert Bartko, PhD., Ing Andrea Beláková** : Fakulta priemyselných technológií, TnU AD; T. Vansovej 1054/45; 020 32 Púchov; e-mail: slabeycius@fpt.tnuni.sk,

built-up at the contacting surfaces. On the other hand, applying the principle of time-temperature equivalence [7], we can construct the dependence of friction coefficient to temperature from the velocities measurements and vice versa [8].

2. Description of method for friction measurement

The principal scheme of the experiment is shown on the Fig.1. Tested sample of rubber was fixed on the bottom of the holder (massive steel block) and drawn down the plane surface of glass or steel by a cable. The cable fasten on the side of the holder was connected through pulley with the upper head grips of the INSTRON 4302 tearing machine. In the experiment the upper head moved with constant velocity v_s and the drawing force versus displacement of head x was recorded.

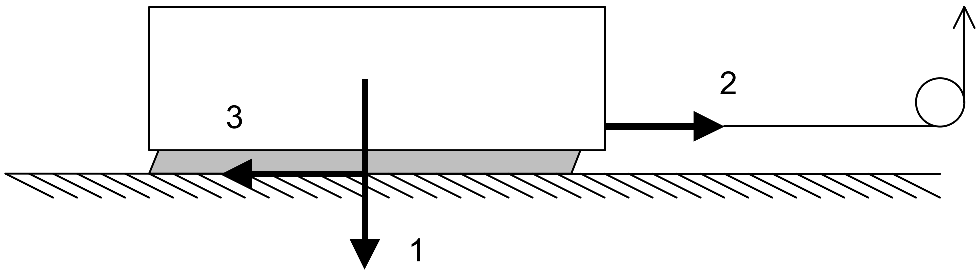

Fig. 1. Principal scheme of the experiment

The normal force (1) was created by weight of steel block. The friction force (3) was generated on the interface rubber glass as a result of relative motion. The rubber sample had the shape of a round plate of vulcanized rubber with diameter of 67 mm and thickness 6 mm. It was fastened at the bottom of massive cylindrical steel holder (diameter of 81 mm; thickness 20 mm). The holder with rubber was pressed to the surface investigated by supplement weight (steel cylinders ∅ 81 mm, h = 15 mm).

3. Mathematical model

The equation of motion for the block is, see Fig.2

$$m\frac{d^2x}{dt^2} = k(v_s t - x) - F_f(\dot{x}) \tag{1}$$

where m is the mass of the steel block (inclusive rubber layer), v_s is velocity of other end of cable (connected to upper head of INSTRON). The elastic properties of cable are described by stiffness k. The friction force F_f is unknown function of velocity $v = \dot{x} = dx/dt$ and the normal force.

Starting with steady sliding motion and reducing the spring velocity v_s to the critical velocity v_c, steady sliding is replaced by stick-slip motion. The critical velocity can be determined using stability analysis. During steady sliding motion

$$x(t) = x_0 + v_s t \tag{2}$$

which satisfies (1) if

$$kx_0 - F_f(v_s) = 0 \tag{3}$$

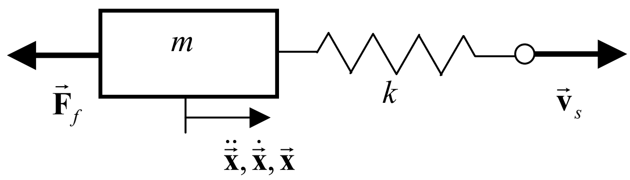

Fig. 2 Model of experiment

To determine when the steady sliding motion becomes unstable, add a small perturbation $\xi(t)$

$$x(t) = x_0 + v_s t + \xi(t) \tag{4}$$

and substitute it in (1). Expanding $F_f(\dot{x})$

$$F_f(\dot{x}) \approx F_f(v_s) + F_f'(v_s)\dot{\xi} \tag{5}$$

gives new equation

$$m\,\ddot{\xi} + F_f'(v_s)\dot{\xi} + k\xi = 0 \tag{6}$$

Assume solution as

$$\xi(t) = Ae^{\kappa t} \tag{7}$$

and substituting it to (6) gives

$$\kappa^2 + \frac{F'(v_s)}{m}\kappa + \frac{k}{m} = 0 \tag{8}$$

The equation has two roots. If the real part of roots is negative, then steady sliding state is stable with respect to small perturbation. If the real part of roots is positive, steady motion is unstable. Thus condition that the real part of roots is zero is condition for separating steady sliding motion from stick-slip motion

$$F'(v_s) = 0 \tag{9}$$

This condition is independent on *k*, what is contrary to experimental observations, where it is found that stick-slip motion can always be eliminated by using enough stiff spring *k*.

4. Results from experimental measurements

The measurements were carried out for normal load from 2 kPa to 10 kPa and for the sliding velocity from 1 mm/s to 6 mm/s. The air humidity and the temperature were constant during the test (T = 23°C, φ = 55%). The length of the test varied from 0,4 m to 0,1 m. The tested surfaces were glass and steel.

The main results can be summarized as follows. The friction coefficient between rubber and glass depends markedly on sliding velocity and normal pressure. The friction between rubber and glass is depicted on Fig. 3. As can be seen, the dependence of friction coefficient to normal load shows maximum value for normal load about 5 kPa.

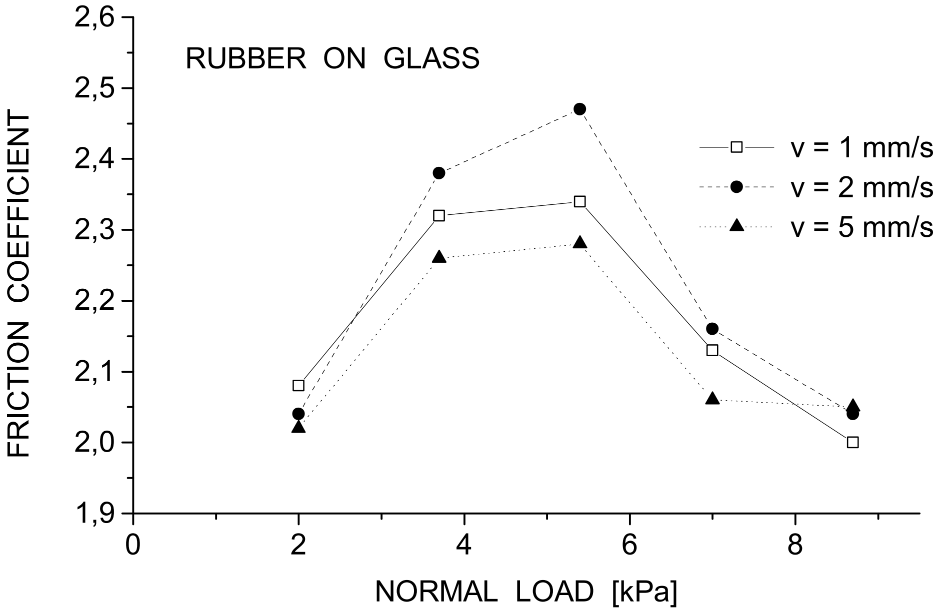

Fig. 3. The dependence of friction coefficient between rubber and glass to normal load for sliding velocity of 1 mm/s —□—, 2 mm/s ---●--- and 5 mm/s ····▲····

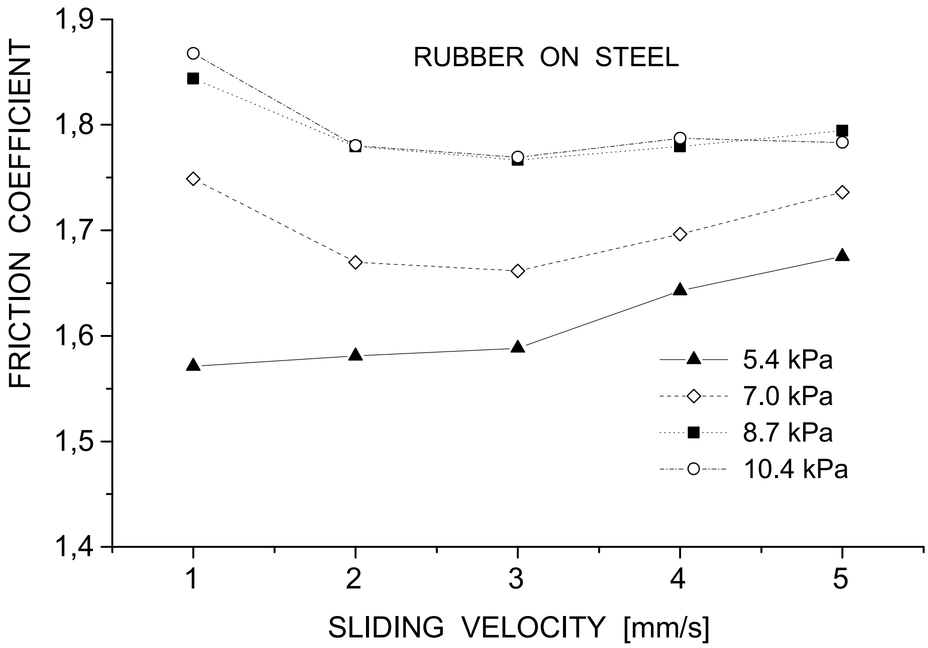

Fig. 4. The dependence of friction coefficient between rubber and steel to sliding velocity for normal load of 5.4 kPa —▲—, 7.0 kPa ---◊---, 8.7 kPa ····■···· and 10.4 kPa -·-□-·-

The next tested surface was steel plate. The surface was polished and afterwards coated by special anticorrosive layer. The Fig.4 shows the results for rubber on steel friction measurements. The dependence of friction coefficient on sliding velocity is not so strong as for glass. It decreases slightly for low velocity, but then it is constant or even increase. Note should be taken that the friction coefficient increases monotonously with sliding velocity for low normal load (5.4 kPa).

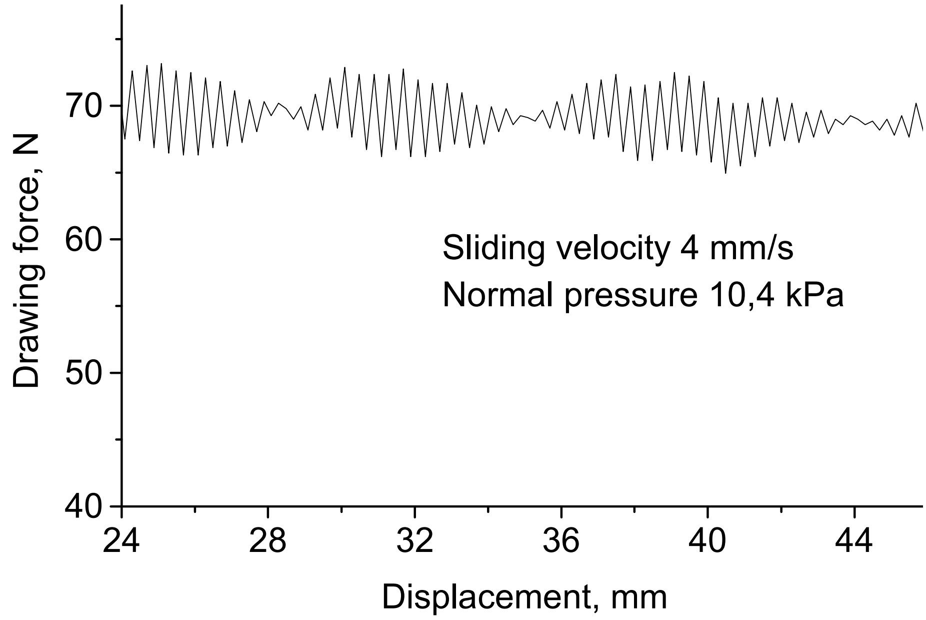

Fig. 5. Rubber on glass friction: The double periodical stick-slip at $v_s = 4$ mm/s and $p_N = 10,4$ kPa

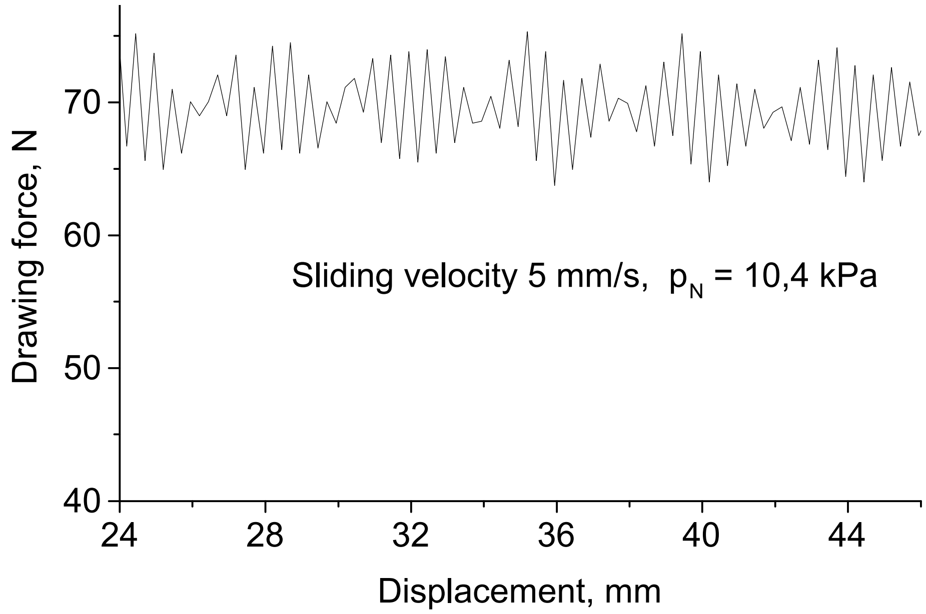

Fig. 6. Rubber on glass friction: The double periodical stick-slip at $v_s = 5$ mm/s and $p_N = 10,4$ kPa

The measurements showed that the sliding motion of rubber on glass was stationary for low normal load and low sliding velocity. For example, for $v_s = 4$ mm/s the stick-slip appeared at $p_N > 10$ kPa, while for $v_s = 8$ mm/s already at $p_N > 6$ kPa. The character of stick-slip varied from quasi periodical at low sliding velocity (Fig. 5, Fig. 6) to strongly non-stationary at high sliding velocity (Fig. 7, Fig. 8). At normal pressure $p_N = 10{,}4$ kPa and sliding velocity $v_s = 4$ mm/s was the frequency of stick-slip about 10 Hz and its amplitude about 3 N. The amplitude was modulated with frequency about 0,5 Hz (Fig. 5). For sliding velocity $v_s = 5$ mm/s was frequency the same, while the amplitude slightly increased (5 N). The modulatory frequency was about 1 Hz.

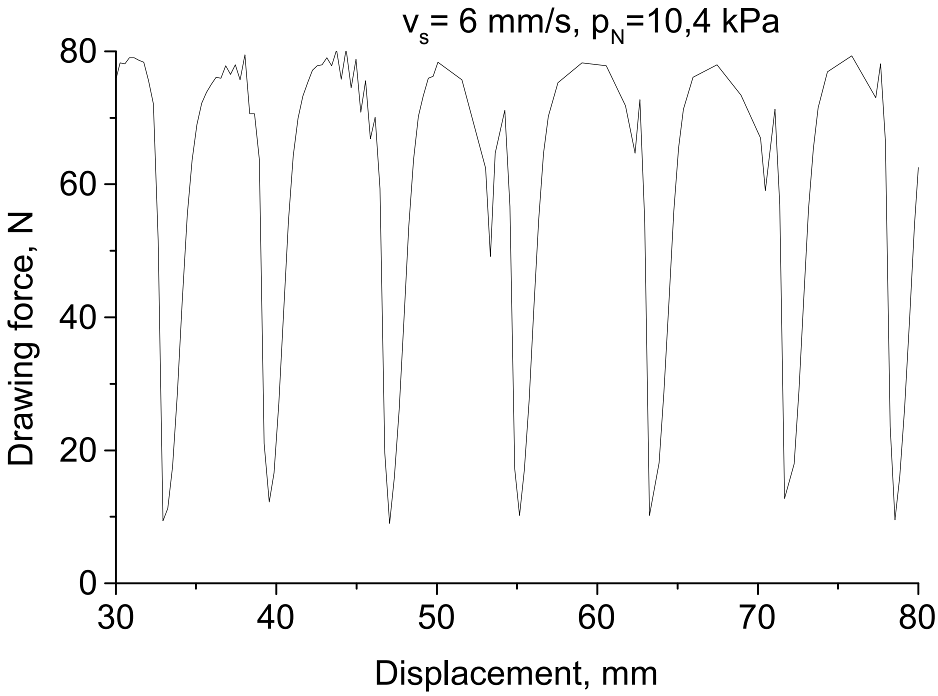

Fig. 7 Rubber on glass friction: The strongly non-stationary stick-slip at $v_s = 6$ mm/s and $p_N = 10{,}4$ kPa

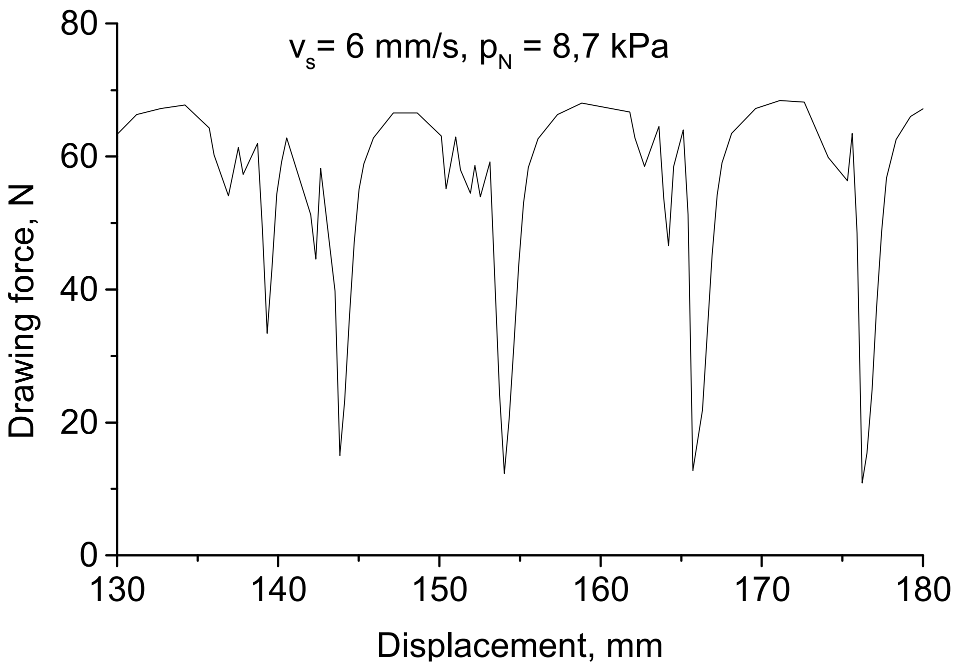

Fig. 8 Rubber on glass friction: The strongly non-stationary stick-slip at $v_s = 6$ mm/s and $p_N = 8{,}7$ kPa

At the high sliding velocity ($v_s \geq 6$ mm/s) can be observed quite other character of non-stationary motion. While friction force is about 80 N for $p_N = 10{,}4$ kPa (63 N for $p_N = 8{,}7$ kPa respectively), the drawing force falls periodically nearly to 10 N, and then again jump up to previous value. The period of jumping is about 1,3 s for at $v_s = 5$ mm/s and $p_N = 10{,}4$ kPa and 1,7 s for $v_s = 6$ mm/s and $p_N = 8{,}7$ kPa. It indicates that the block moves so irregularly, that cable is from time to time completely released.

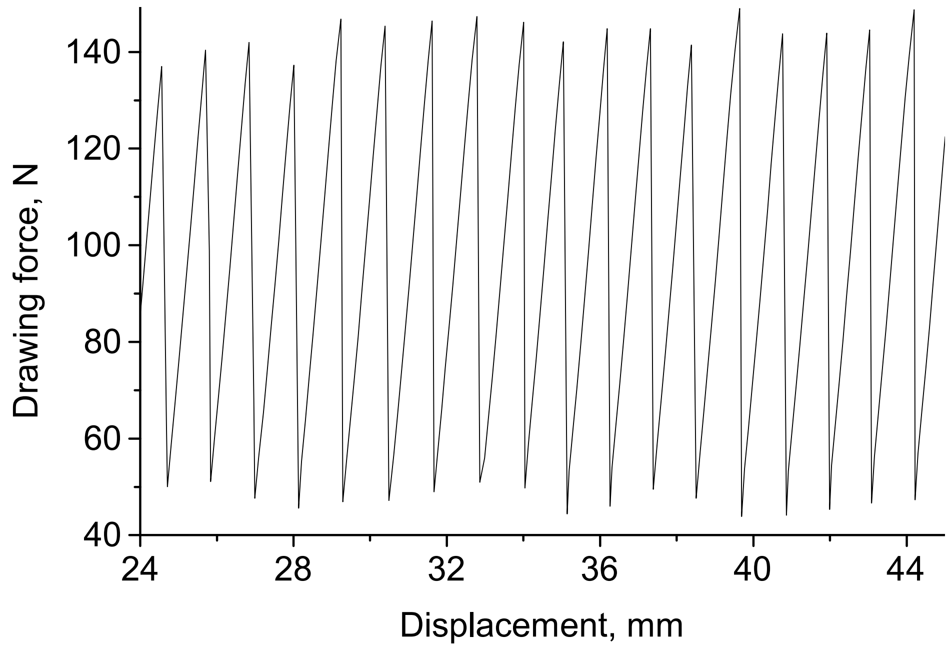

Fig. 9 Rubber on steel friction: The periodical stick-slip at $v_s = 0{,}17$ mm/s and $p_N = 5{,}4$ kPa

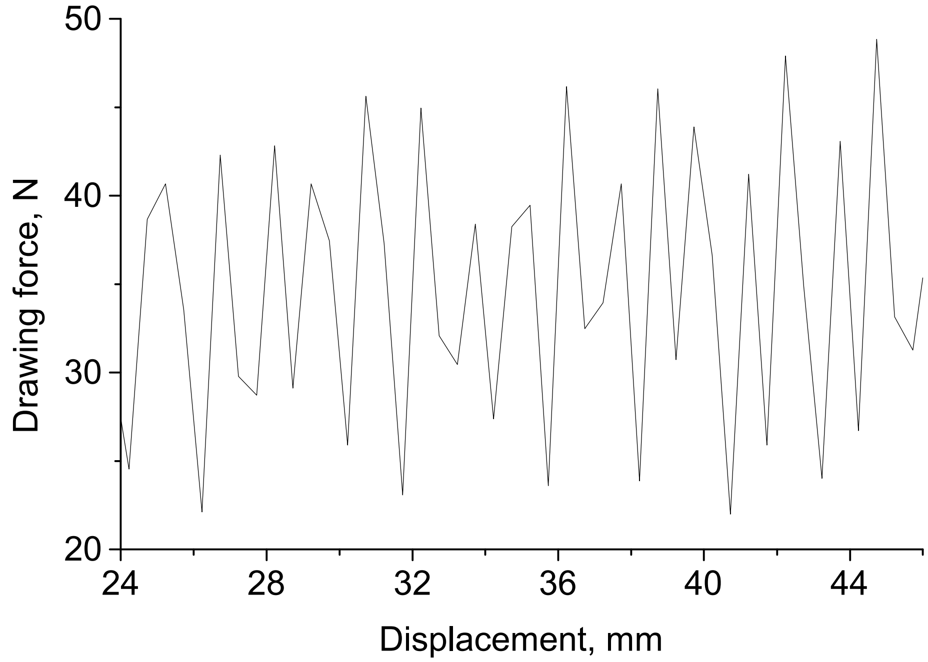

Fig. 10 Rubber on steel friction: The chaotic stick-slip at $v_s = 3{,}1$ mm/s and $p_N = 2{,}0$ kPa

The second tested surface (steel with anticorrosive layer) showed very strong stick-slip almost at all values of normal load and sliding velocity. The amplitude of stick-slip was very high (up to 40 N) and character depends on sliding velocity.

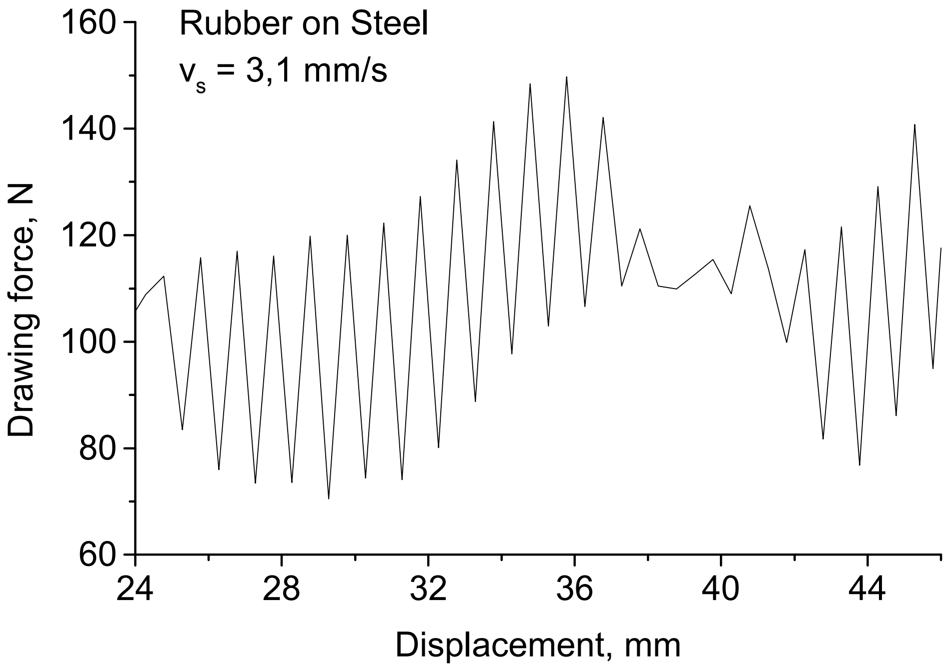

Fig. 11 Rubber on steel friction: The double periodical stick-slip at v_s = 3,1 mm/s and p_N = 5,4 kPa

Fig. 9 shows almost ideally periodical motion at very low velocity (v_s = 0,17 mm/s) and normal pressure p_N = 5,4 kPa with amplitude 46 N and frequency about 0,14 Hz. Next figure (Fig. 10) depicted chaotic stick-slip at sliding velocity v_s = 3 mm/s and normal load p_N = 2,0 kPa. We have observed also quasi-double periodical motion – for example at v_s = 3,1 mm/s and p_N = 5,4 kPa (Fig. 11).

5. Results from mathematical modeling

Mathematical model was done in program MATLAB/Simulink in accordance equation (1). The model is in shape of nonlinear differential equation of second order. The model was used to study influence of particular elements on behavior of whole system and compare results obtained from mathematical model with results obtained from experiments.

As a solver ode23s method was used, which is based on a modified Rosenbrock formula of order 2. Because it is a one-step solver, it can be more efficient than ode15s at crude tolerances. It can solve some kinds of stiff problems for which ode15s is not effective. For a stiff problem, solutions can change on a time scale that is very short compared to the interval of integration, but the solution of interest changes on a much longer time scale. Methods not designed for stiff problems are ineffective on intervals where the solution changes slowly because they use time steps small enough to resolve the fastest possible change.

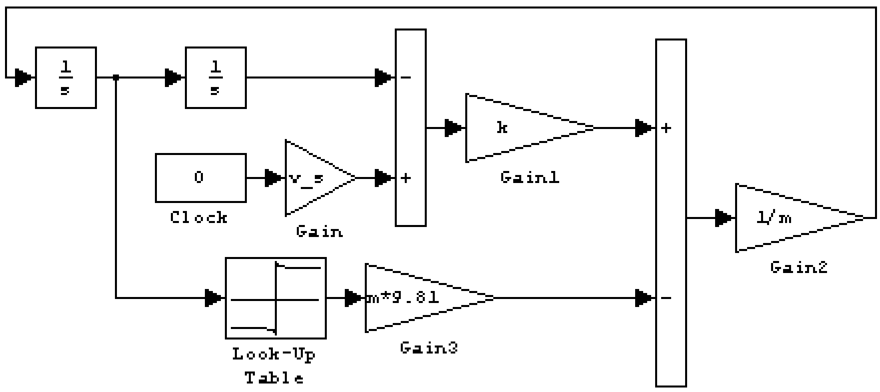

Fig. 12 Mathematical model in MATLAB/Simulink

Mathematical model was used for simulation with different values of velocity, mass and strength of spring. On Fig. 13 there are results, which demonstrate influence of mass and strength of spring on force in spring, and we can see cases, where slip-stick is changed to sliding motion. On Fig. 14 we can see the case when slip-stick disappears if velocity is higher than critical velocity.

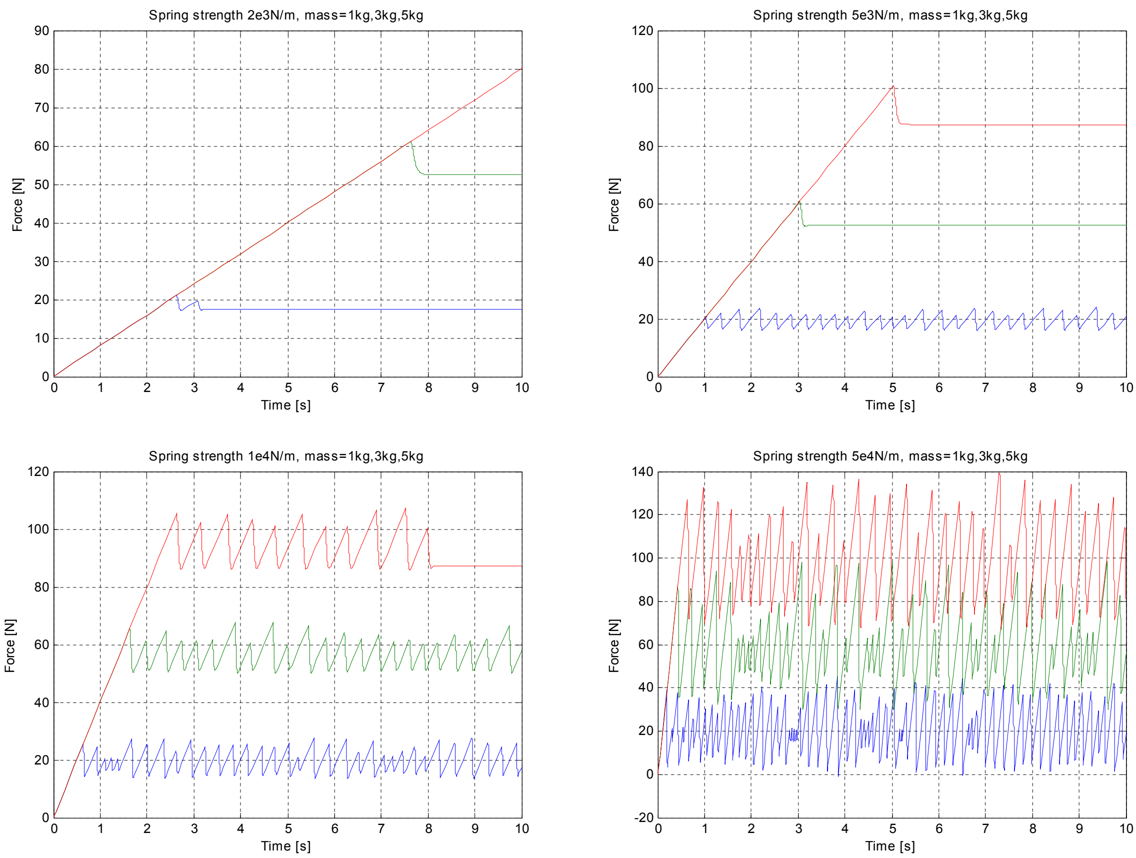

Fig. 13 Time history of force in spring for different values of mass and strength of spring

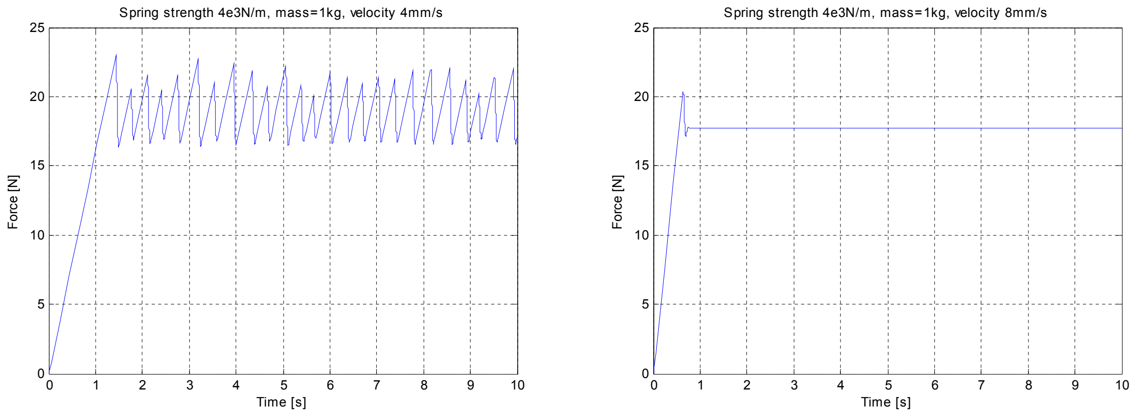

Fig. 14 Time history of force in spring for different values of velocity

6. Conclusions

In the paper are shown results from measurements of friction coefficient in various conditions with focus on slip-stick movement. Obtained results showed very broad types of stick-slip motion from full chaotic to almost periodical motion. The character of stick-slip depends not only on sliding velocity and normal load, but also on mechanical properties of all the mechanical system (holder, cable, geometry of experiment etc). The dependencies of friction coefficient on sliding velocity and normal load are in good agreement with the data published in literature.

Mathematical model was created and results for various values of spring strength, mass, velocity is shown. Some results obtained in experiments were not found using mathematical model. So we should find better model of friction. Function friction coefficient on velocity may be insufficient.

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