



## VIBRATION AND NOISE ANALYSIS OF THE GEARBOX

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**Summary:** *Gearboxes and similar machines containing rotating parts are complex systems with complicated structure and couplings. Generally they can be decomposed into the more simple subsystems. These subsystems are usually rotating shafts with gears joined by gear couplings and housing coupled with rotating shafts by bearings. The contribution is aimed at the mathematical modelling of gearboxes considered including their interior rotating shaft system and housing. The used bearing model respects real number of rolling elements and roller contact forces acting between the journals and the outer housing. The model of a complete gearbox is created using the modal synthesis method. The kinematic transmission errors in gear couplings are viewed as sources of excitation. Vibration and noise analysis of the gearbox housing is performed by means of the created model and possible analysis procedures are discussed. The presented methodology is applied to the simple test-gearbox.*

### 1. Introduction

Various sounds and noises can be easily perceived by humans in their environment. Noise is usually felt as an undesirable and troublesome phenomenon and therefore it is important to reduce existing or possible sources of noise. One class of the noise sources is characterized by vibrating structures surrounded by fluid (mostly by air) that radiate acoustic power to the environment and produce noise. Typical representants of this class are machines containing moving parts performing reciprocating or rotating motion. Their noise can arise from certain impacts and frictions of the moving parts or it can be the result of dynamic forces in couplings between the rotating or moving and non-rotating (stator) parts. This is so called structure borne noise. Second class of the noise sources, that aren't mentioned in this paper, are aerodynamic sources (Nový, 1995) caused by air movement or fluid flow.

Gearboxes and similar machines containing rotating parts coupled with some stator are significant noise sources if the coupling forces transmit the vibrations to the flexible housing. There must exist certain tools and analysis methodology before their noise could be understood and reduced. This contribution is aimed at the vibration modelling and acoustic analysis methodology of the large rotating systems with consideration of the flexible housing. The kinematic transmission errors in the gear couplings are viewed as the main sources of vibrations that are

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transmitted to the housing. Contrary on the previous papers, e.g. (Zeman et al., 2004), this contribution is goaled rather on the vibration and acoustic analysis of the housing than on the shafts vibration and analysis of the coupling forces and deformations in gearings. The new original approach to the modelling of bearing couplings (Zeman & Hajžman, 2005) is used for the connection of the rotor and stator parts. The second part of the contribution describes and documents some numerical experiments with the simple test-gearbox.

## 2. Linearized mathematical model of a gearbox

The gearbox can be generally decomposed into  $S$  subsystems divided into shafts with gears (subsystems  $s = 1, 2, \dots, S - 1$ ) rotating with angular velocity  $\omega_s$  and a housing (subsystem  $s = S$ ). Let the vibrations are small about the static equilibrium position.

The mathematical model of the shaft subsystem  $s$  can be written (Hajžman, 2004) in matrix form

$$\mathbf{M}_s \ddot{\mathbf{q}}_s(t) + (\mathbf{B}_s + \omega_s \mathbf{G}_s) \dot{\mathbf{q}}_s(t) + \mathbf{K}_s \mathbf{q}_s(t) = \mathbf{f}_s^E(t) + \mathbf{f}_s^G + \mathbf{f}_s^B, \quad s = 1, 2, \dots, S - 1, \quad (1)$$

where  $\mathbf{M}_s$ ,  $\mathbf{B}_s$  and  $\mathbf{K}_s$  are symmetrical mass, damping and stiffness matrices of the uncoupled subsystems of order  $n_s$  and  $\mathbf{G}_s$  is skew symmetrical matrix of the gyroscopic effects of the same order. These matrices are usually created by means of finite element method combined with discrete parameters representing masses of rigid gear discs. External forced excitation is described by vector  $\mathbf{f}_s^E(t)$ . Vector  $\mathbf{f}_s^G$  represents the forces in spur helical gear couplings and vector  $\mathbf{f}_s^B$  expresses the coupling forces in rolling-element bearings. Both vectors are acting on the subsystem  $s$ .

Similarly the mathematical model of the flexible stator (subsystem  $S$ ) has the form

$$\mathbf{M}_S \ddot{\mathbf{q}}_S(t) + \mathbf{B}_S \dot{\mathbf{q}}_S(t) + \mathbf{K}_S \mathbf{q}_S(t) = \mathbf{f}_S^E(t) + \mathbf{f}_S^B. \quad (2)$$

Mass, damping and stiffness matrices  $\mathbf{M}_S$ ,  $\mathbf{B}_S$ ,  $\mathbf{K}_S$  of order  $n_S$  are generated after discretization by finite element method. Vector  $\mathbf{f}_S^E(t)$  is possible external excitation and force effect of bearing couplings is expressed by vector  $\mathbf{f}_S^B$ .

If we assume the constant gear mesh and relatively small vibrations, the global gear coupling vector, in the general coordinate space

$$\mathbf{q}(t) = [\mathbf{q}_1^T(t), \mathbf{q}_2^T(t), \dots, \mathbf{q}_S^T(t)]^T, \quad (3)$$

can be written as

$$\mathbf{f}^G = \begin{bmatrix} \mathbf{f}_1^G \\ \mathbf{f}_2^G \\ \vdots \\ \mathbf{f}_{S-1}^G \\ \mathbf{0} \end{bmatrix} = -\mathbf{K}_G \mathbf{q}(t) - \mathbf{B}_G \dot{\mathbf{q}}(t) + \mathbf{f}_G(t). \quad (4)$$

It holds

$$K_G = \sum_z k_z C_z, \quad B_G = \sum_z b_z C_z, \quad C_z = \left[ \begin{array}{ccc|c} \vdots & \vdots & \vdots & \vdots \\ \cdots & \delta_{z,i} \delta_{z,i}^T & \cdots & -\delta_{z,i} \delta_{z,j}^T \cdots \\ \vdots & \vdots & \vdots & \vdots \\ \cdots & -\delta_{z,j} \delta_{z,i}^T & \cdots & \delta_{z,j} \delta_{z,j}^T \cdots \\ \vdots & \vdots & \vdots & \vdots \\ \hline & 0 & & 0 \end{array} \right],$$

for the stiffness and damping matrices of the gear couplings. Vectors  $\delta_{z,i}$  and  $\delta_{z,j}$  of dimension 6 are created using geometrical parameters of the gears fixed in nodal point  $i$  of the drive shaft and in nodal point  $j$  of the driven shaft. More details can be found e.g. in (Hajžman, 2004). Quantities  $k_z$  and  $b_z$  are stiffness and damping coefficients of the gear coupling  $z$ . Vector of internal kinematic excitation generated in gear meshings is

$$\mathbf{f}_G(t) = \sum_z \left( k_z \Delta_z(t) + b_z \dot{\Delta}_z(t) \right) \left[ \cdots -\delta_{z,i}^T \cdots \delta_{z,j}^T \cdots \mid \mathbf{0}^T \right]^T, \quad (5)$$

where  $\Delta_z(t)$  are kinematic transmission errors.

The linearized form of the bearing couplings force vector is

$$\mathbf{f}^B = \begin{bmatrix} \mathbf{f}_1^B \\ \mathbf{f}_2^B \\ \vdots \\ \mathbf{f}_S^B \end{bmatrix} = -\mathbf{K}_B \mathbf{q}(t) - \mathbf{B}_B \dot{\mathbf{q}}(t). \quad (6)$$

We assume that small vibrations around the static equilibrium position occur and that linearized bearing stiffnesses for given static loading are used, similarly as in the case of the gear couplings. Additionally internal excitation in bearings is neglected. The global bearing stiffness

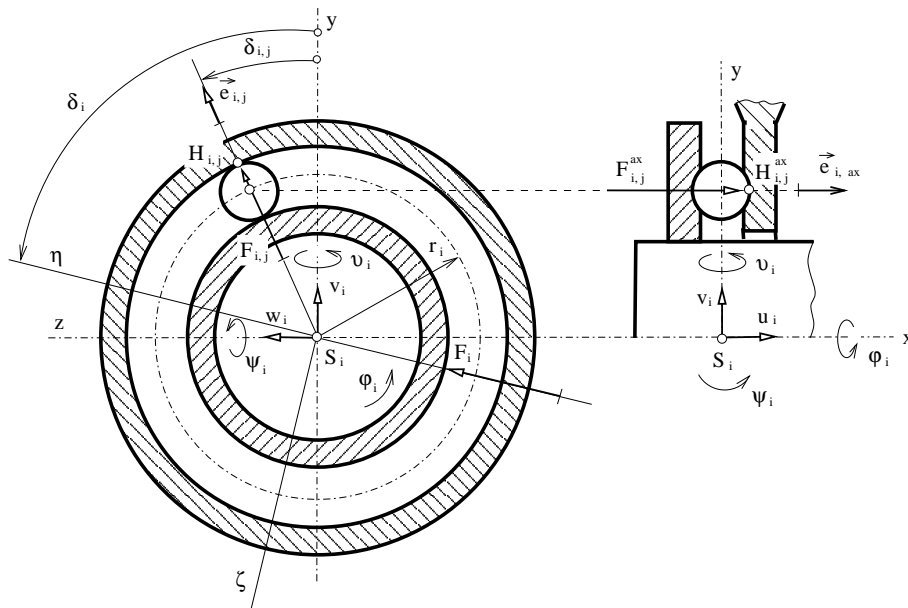


Fig. 1 Scheme of a bearing.

matrix is then given (Zeman & Hajžman, 2005) by

$$\mathbf{K}_B = \sum_i \sum_j \begin{bmatrix} \vdots & & \vdots \\ \cdots & (k_{i,j} \mathbf{t}_{i,j} \mathbf{t}_{i,j}^T + k_{i,j}^{ax} \mathbf{t}_{i,j \text{ ax}} \mathbf{t}_{i,j \text{ ax}}^T) & \cdots & -(k_{i,j} \mathbf{t}_{i,j} \mathbf{e}_{i,j}^T + k_{i,j}^{ax} \mathbf{t}_{i,j \text{ ax}} \mathbf{e}_{i,j \text{ ax}}^T) & \cdots \\ \vdots & & \vdots \\ \cdots & -(k_{i,j} \mathbf{e}_{i,j} \mathbf{t}_{i,j}^T + k_{i,j}^{ax} \mathbf{e}_{i,j \text{ ax}} \mathbf{t}_{i,j \text{ ax}}^T) & \cdots & (k_{i,j} \mathbf{e}_{i,j} \mathbf{e}_{i,j}^T + k_{i,j}^{ax} \mathbf{e}_{i,j \text{ ax}} \mathbf{e}_{i,j \text{ ax}}^T) & \cdots \\ \vdots & & \vdots \end{bmatrix},$$

This bearing model represents real number of rolling elements (index  $j$ ) in a bearing (index  $i$ ) and respects real number of contact forces acting between the journals and the outer housing. Linearized radial  $k_{i,j}$  and axial  $k_{i,j}^{ax}$  stiffnesses are computed for each contact point  $H_{i,j}$  (see Fig. 1). Vectors  $\mathbf{t}_{i,j}$  and  $\mathbf{e}_{i,j}$ , respectively  $\mathbf{t}_{i,j \text{ ax}}$  and  $\mathbf{e}_{i,j \text{ ax}}$ , are made using the geometrical parameters of radial, respectively axial, contact points. The nonzero elements are placed in matrix  $\mathbf{K}_B$  according to the position of the shaft journal general coordinates and to the position of the housing contact points general coordinates in the global vector  $\mathbf{q}(t)$  (3). The global damping bearing matrix is consider as proportional to the stiffness matrix

$$\mathbf{B}_B = \beta_B \mathbf{K}_B.$$

It is advantageous to assemble condensed mathematical model of the system with reduced degrees of freedom (DOF) number, because mainly the housing subsystem could have large DOF number and this can hinder from consecutive performing of various dynamical analysis and optimization. The modal transformations

$$\mathbf{q}_s(t) = {}^m \mathbf{V}_s \mathbf{x}_s(t), \quad s = 1, 2, \dots, S, \quad (7)$$

are introduced for this purpose. Matrices  ${}^m \mathbf{V}_s \in \mathbb{R}^{n_s, m_s}$  are modal submatrices obtained from modal analysis of the mutually uncoupled, undamped and non-rotating subsystems, whereas  $m_s$  ( $m_s \leq n_s$ ) is the number of the chosen master modes of vibration. The new configuration space of the dimension  $m$  is defined by vector

$$\mathbf{x}(t) = [\mathbf{x}_1^T(t), \mathbf{x}_2^T(t), \dots, \mathbf{x}_S^T(t)]^T, \quad m = \sum_{s=1}^S m_s. \quad (8)$$

The models (1) and (2) can be then rewritten in the global condensed form

$$\begin{aligned} \ddot{\mathbf{x}}(t) + \left( \mathbf{B} + \omega_0 \mathbf{G} + \mathbf{V}^T (\mathbf{B}_B + \mathbf{B}_G) \mathbf{V} \right) \dot{\mathbf{x}}(t) + \\ + \left( \mathbf{\Lambda} + \mathbf{V}^T (\mathbf{K}_B + \mathbf{K}_G) \mathbf{V} \right) \mathbf{x}(t) = \mathbf{V}^T \left( \mathbf{f}_E(t) + \mathbf{f}_G(t) \right), \end{aligned} \quad (9)$$

where  $\mathbf{f}_E(t) = [(\mathbf{f}_1^E(t))^T, (\mathbf{f}_2^E(t))^T, \dots, (\mathbf{f}_S^E(t))^T]^T$  is the global vector of external excitation,

$$\mathbf{B} = \text{diag} \left( {}^m \mathbf{V}_s^T \mathbf{B}_s {}^m \mathbf{V}_s \right), \quad \mathbf{G} = \text{diag} \left( \frac{\omega_s}{\omega_0} {}^m \mathbf{V}_s^T \mathbf{G}_s {}^m \mathbf{V}_s \right), \quad \mathbf{V} = \text{diag} \left( {}^m \mathbf{V}_s \right)$$

are block diagonal matrices ( $\omega_s = 0$  holds for the stator subsystem) and  $\mathbf{\Lambda} = \text{diag} \left( {}^m \mathbf{\Lambda}_s \right)$  is diagonal matrix composed from spectral submatrices  ${}^m \mathbf{\Lambda}_s \in \mathbb{R}^{m_s, m_s}$  of the subsystems.



### 3. Modal analysis and steady state dynamic response of a gearbox

The basic and one of the most important analysis is modal analysis of the conservative undamped condensed model (see equation (9))

$$\ddot{\mathbf{x}}(t) + \left( \mathbf{A} + \mathbf{V}^T (\mathbf{K}_B + \mathbf{K}_G) \mathbf{V} \right) \mathbf{x}(t) = \mathbf{0}. \quad (10)$$

This leads to the eigenvalue problem in the form

$$\left( \mathbf{A} + \mathbf{V}^T (\mathbf{K}_B + \mathbf{K}_G) \mathbf{V} - \Omega_\nu^2 \mathbf{E} \right) \mathbf{x}_\nu = \mathbf{0}, \quad \nu = 1, 2, \dots, m, \quad (11)$$

where  $\mathbf{E}$  is a square identity matrix of order  $m$ ,  $\mathbf{x}_\nu$  are eigenvectors in the new configuration space (8) and real positive  $\Omega_\nu$  [rad/s] are eigenfrequencies. Eigenvectors  $\mathbf{q}_\nu$  in the original configuration space (3) can be obtained using the modal transformations (7). As the basic analysis it can be sufficient to compare the uncoupled subsystems eigenmodes  $\mathbf{v}_\nu^{(s)}$  and the eigenmodes  $\mathbf{q}_\nu^{(s)}$  of the subsystems considered as a part of the whole system.

The main aim of this paper is to study the radiated noise by the housing, that is the effect of the steady state vibrations excited by internal excitation in gear couplings. The kinematic transmission error in the form of Fourier series

$$\Delta_z(t) = \sum_{k=1}^K \left( \Delta_{z,k}^C \cos k\omega_z t + \Delta_{z,k}^S \sin k\omega_z t \right) \quad (12)$$

can be rewritten in the complex form

$$\tilde{\Delta}_z(t) = \sum_{k=1}^K \Delta_{z,k} e^{ik\omega_z t}, \quad \Delta_{z,k} = \Delta_{z,k}^C - i\Delta_{z,k}^S, \quad (13)$$

whereas meshing frequencies  $\omega_z = \frac{\pi n}{30} p_z$  are functions of operation speed  $n$  [rpm] and ratio  $p_z = \frac{\omega_z}{\omega_0}$ . Error amplitudes  $\Delta_{z,k}^C$  and  $\Delta_{z,k}^S$  are real coefficients of Fourier series. The steady dynamic response of the whole condensed model (9) has the form

$$\tilde{\mathbf{x}}(t) = \sum_{z=1}^Z \sum_{k=1}^K \mathbf{x}_{z,k} e^{ik\omega_z t}, \quad (14)$$

with complex amplitudes  $\mathbf{x}_{z,k}$

$$\mathbf{x}_{z,k}(n) = \mathbf{Z}_{z,k}^{-1}(n) \mathbf{f}_{z,k}(n), \quad (15)$$

where

$$\mathbf{Z}_{z,k} = -k^2 \omega_z^2 \mathbf{E} + ik\omega_z \left( \mathbf{D} + \omega_0 \mathbf{G} + \mathbf{V}^T (\mathbf{B}_B + \mathbf{B}_G) \mathbf{V} \right) + \mathbf{A} + \mathbf{V}^T (\mathbf{K}_B + \mathbf{K}_G) \mathbf{V} \quad (16)$$

is the dynamical compliance matrix and

$$\mathbf{f}_{z,k} = \mathbf{V}^T \mathbf{c}_z (k_z + ik\omega_z b_z) \Delta_{z,k}. \quad (17)$$

The complex amplitudes  $\mathbf{q}_{z,k}$  of the steady state dynamic response in original generalized coordinate space can be obtained by transformation (7)  $\mathbf{q}_{z,k} = \mathbf{V}\mathbf{x}_{z,k}$ . After differentiation of expression (14) the complex amplitudes of generalized velocities can be expressed as

$$\dot{\mathbf{x}}_{z,k} = ikp_z \frac{\pi n}{30} \mathbf{x}_{z,k}, \quad \dot{\mathbf{q}}_{z,k} = \mathbf{V}\dot{\mathbf{x}}_{z,k}. \quad (18)$$

These quantities are sufficient for the analysis of radiated as will be shown in the next paragraph. It is convenient to introduce the upper effective estimates of the complex amplitudes

$$\hat{\mathbf{q}}(n) = \sqrt{\sum_z \sum_k |\dot{\mathbf{q}}_{z,k}|^2} \quad (19)$$

to describe the level of the overall polyharmonic dynamic response.

#### 4. Noise radiated by a gearbox housing

Computations and measurements of acoustical quantities belongs to standalone branches of science. Comprehensive noise and sound analysis requires knowledge of acoustic sources for particular cases and solving general wave equation that describes sound dispersion. The used quantities in acoustics are mainly acoustic pressure and velocity, acoustic energy, power and intensity. From the viewpoint of the gearbox noise radiated to the environment the most interesting are quantities, that can be easily measured and compared with computed ones. The acoustic pressure is the most suitable quantity for the measurement and it is also possible to measure acoustic intensity (Smetana et al., 1994). If the effective values of pressure  $p_{ef}(\mathbf{r}_i)$  have been measured in the enough points  $\mathbf{r}_i$  of the space around the object (the noise source), it would be possible to express acoustic power of the source by relation

$$P = \int_{\Omega} \frac{p_{ef}^2}{\rho c} d\Omega \approx \frac{1}{\rho c} \sum_i p_{ef}^2(\mathbf{r}_i) \Omega_i. \quad (20)$$

Air density is denoted as  $\rho$ ,  $c$  is the sound velocity in air and  $\Omega_i$  is the basic surface belongs to the point  $\mathbf{r}_i$ .

For the purpose of the gearbox radiated sound computation the simplest way is to compute effective normal velocities  $v_e$  of the gearbox housing surface. According (Nový, 1995) the radiated acoustic power density [W/m<sup>2</sup>] can be then written as

$$w_e = v_e^2 \rho c s, \quad (21)$$

where  $s$  is the sound efficiency. The sound efficiency is the most problematic factor and we couldn't determine its value by a simple computation. It depends especially on some critical frequency and characteristic dimensions of the sound source. Therefore in the next parts the sound efficiency  $s$  is considered to be equal to 1. Overall acoustic power can be computed using

$$P = \sum_e w_e S_e, \quad (22)$$

after expressing of the acoustic power densities  $w_e$  for all surface elements with area  $S_e$ .

## 5. Numerical experiments with the test-gearbox

The presented methodology was verified using the simple test-gearbox (Fig. 2). The gearbox was decomposed into two rotating shafts with gears ( $s = 1, 2$ ) and into the housing ( $s = 3$ ). Shafts were discretized using shaft finite elements (Hajžman, 2004) and gears were modelled using their discrete parameters (mass and moments of inertia). The shaft subsystem models were created and their modal analyses were performed in MATLAB code. The housing was modelled as 3D continuum using FEM in ANSYS system. The necessary housing modal values (eigenfrequencies and chosen eigenvectors) were exported from ANSYS to MATLAB. The condensed model of the whole system was assembled in MATLAB code on the basis of the presented methodology. MATLAB system was also used for the computation of the eigenvalues and steady state dynamic response. The original nonreduced models of subsystems had together over 15 000 DOF and the reduced model of the system had 580 DOF ( $m_1 = m_2 = 90$ ,  $m_3 = 400$ ). The shaft system is included by means of flexible torsional couplings into a drive system. It is supposed constant angular speeds of the driving and driven parts of the system.

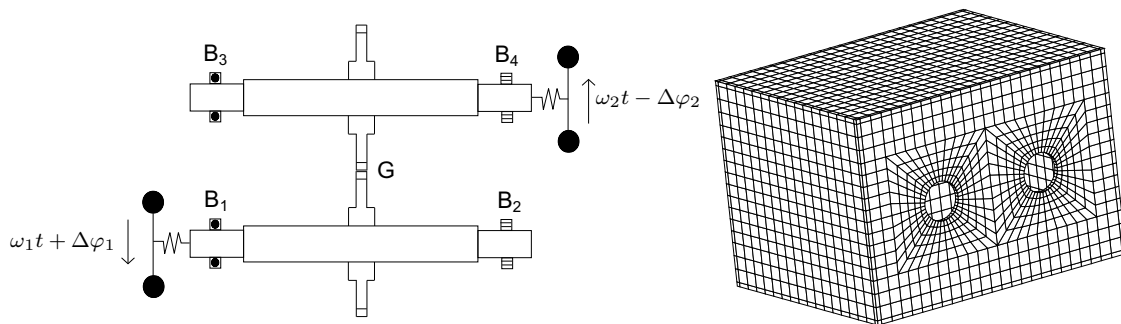


Fig. 2 Scheme of the test-gearbox.

Tab. 1 shows chosen eigenfrequencies  $f_\nu$  [Hz] of the interior shaft system fixed to a rigid frame, eigenfrequencies of the housing (stator subsystem) and eigenfrequencies of the whole gearbox. It can be noted, that the housing is relatively compliant and due to this fact the first eigenfrequencies of the whole gearbox are relatively low.

The radiated sound power by the housing surface to the environment was computed also in MATLAB code. Exported finite element mesh datas (i.e. elements table and nodal coordinates) from ANSYS were processed and the outer faces of elements were found. Densities (21) of

Tab. 1 Chosen eigenfrequencies  $f_\nu$  [Hz] of various models.

$\nu$	Shaft system to rigid frame	Housing	Whole gearbox
1	104	335	104
2	395	463	193
3	426	482	310
4	429	518	382
5	455	583	402
6	456	671	454
7	463	741	455
8	1689	819	462
9	1737	861	483
10	1821	996	515

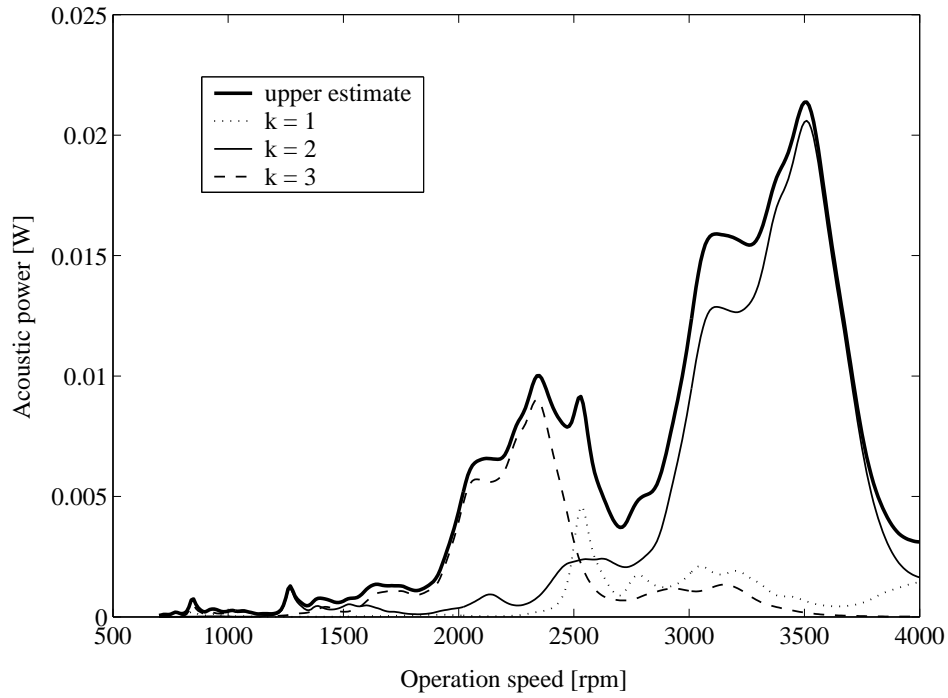


Fig. 3 Overall gearbox acoustic power in dependence of the operation speed.

the acoustic power were computed for each element face using normal velocities of the housing nodes. The overall acoustic power  $P$  (22) were calculated in dependence of operation speed  $n$  [rpm]. Kinematic transmission error in gear coupling ( $z = 1$ ) was approached by Fourier series with three harmonic components

$$\Delta_{1,1}^S = 5 \cdot 10^{-6} \text{ m}, \quad \Delta_{1,2}^S = \frac{\Delta_{1,1}^S}{2}, \quad \Delta_{1,3}^S = \frac{\Delta_{1,1}^S}{3}, \quad \Delta_{1,1}^C = \Delta_{1,2}^C = \Delta_{1,3}^C = 0.$$

The acoustic power as a function of the operation speed for upper effective estimates of the velocities and its particular harmonic components is illustrated in Fig. 3. The distributions of the acoustic power density for chosen resonant state  $n = 2350$  rpm and for upper effective estimates and chosen harmonic components of velocities are shown in Fig. 4 to Fig. 6.

## 6. Conclusion

The modelling methodology of the large rotating systems considering flexible stator is presented. The new bearing model based on the respecting real number of rolling elements and roller contact forces acting between the journals and the outer housing is used for coupling the shafts and the housing subsystems. The radiated sound analysis method is proposed on the basis of the effective normal velocities and consequently the acoustic power densities computation. The overall acoustic power radiated by the gearbox surface in dependence of the operation speed can be plotted. This quantity is suitable for objective function formulation from the viewpoint of the minimal radiated sound.

## 7. Acknowledgement

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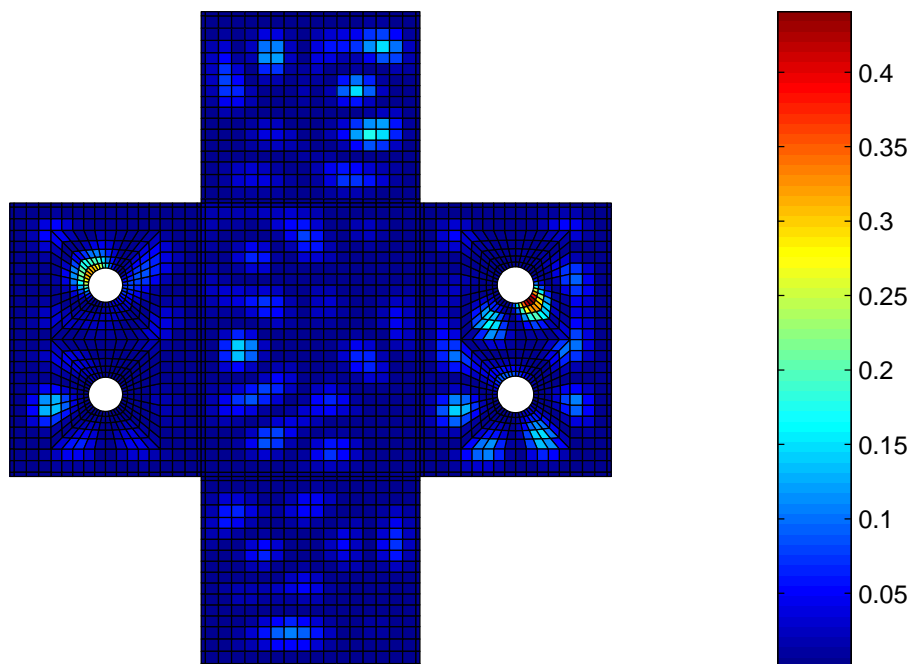


Fig. 4 Acoustic power density [ $\text{W/m}^2$ ] radiated by test-gearbox for chosen resonant state  $n = 2350$  rpm calculated by means of velocity upper effective estimates.

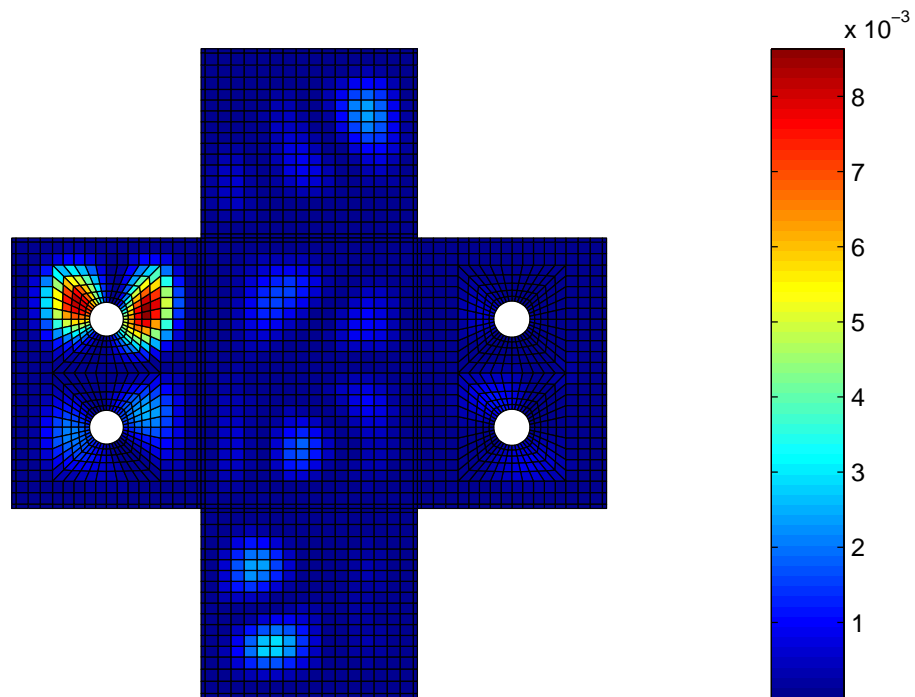


Fig. 5 Acoustic power density [ $\text{W}/\text{m}^2$ ] radiated by test-gearbox for chosen resonant state  $n = 2350$  rpm calculated by means of the first harmonic components of velocities.

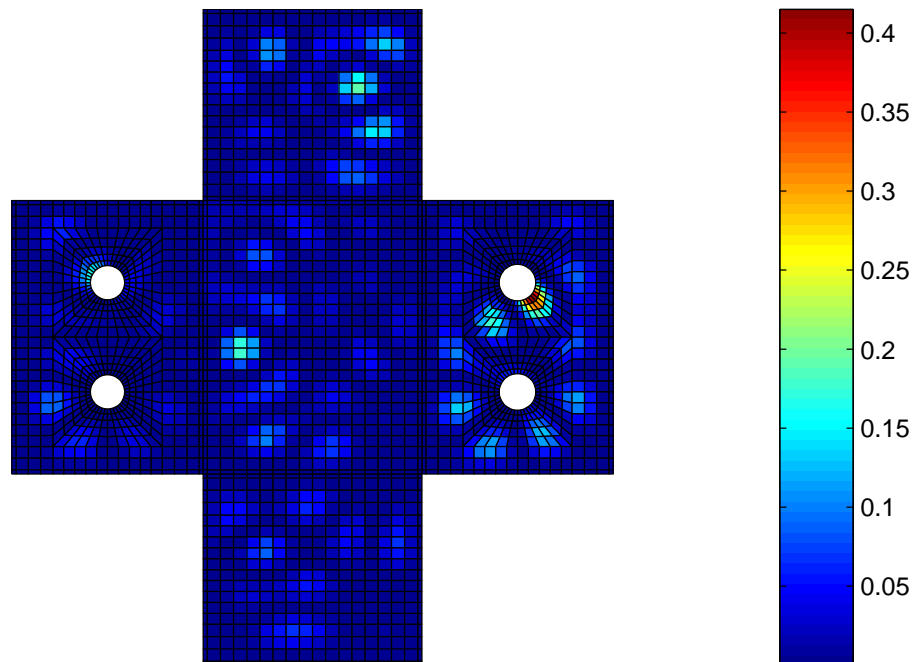


Fig. 6 Acoustic power density [ $\text{W}/\text{m}^2$ ] radiated by test-gearbox for chosen resonant state  $n = 2350$  rpm calculated by means of the third harmonic components of velocities.