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INCORPORATION OF A VAPOUR CAVITATION IN SHORT LEMON HYDRODYNAMICAL BEARINGS WITH A DEEP CENTRAL GROOVE INTO COMPUTATIONAL MODELS OF ROTOR SYSTEMS - RESPONSE ON UNBALANCE EXCITATION

J. Zapoměl

Summary : *A vapour cavitation in hydrodynamical bearings is a significant factor that influences dynamical properties of the total rotor system. Its substanace consists in boiling the oil if the pressure in the bearing gap drops below a critical value. It always arrives at formation of a two-phase medium and at rupture of the oil film. Incorporation of a vapour cavitation into the computational models of fluid film bearings assumes that two kinds of areas exist in the bearing clearance. In noncavitated regions a hydrodynamical effect is developed and then the pressure distribution is governed by a Reynolds equation. In cavitated areas the pressure remains constant. To be kept the continuity of flow between these regions the pressure gradient on both sides of their common border must be zero. If geometry and design parameters of the bearing with a deep central groove make possible to consider it as short, then solution of the Reynolds equation (including the case of bearings of non-circular cross section) can be expressed in a closed form. If the pressure magnitude should drop below a critical value a cavitation takes place. Solution of the Reynolds equation is then divided into two parts between which an interval of constant pressure exists. Components of the hydraulical force are calculated by integration of the pressure distribution along the lenght and circumference of the bearing. For solution of the equation of motion a modified Newmark method has been adopted. Applicability of the developed approach has been tested by means of computer simulations.*

1. Introduction

A cavitation is an important factor that influences properties and behaviour of hydrodynamical bearings. It represents a complex of physical processes that take place in the bearing gap due to sucking the air from the ambient space, boiling the oil, and liberation of gases dissolved in it if the pressure in the bearing drops below a critical level. The cavitation always arrives at occurance of a two-phase medium and at rupture of the oil film.

Doc. Ing. Jaroslav Zapoměl, DrSc., VŠB-Technical University of Ostrava, Department of Mechanics, 17.listopadu 15, Ostrava-Poruba, 708 33,
tel.: 59 732 3267, fax.: 59 732 1287, e-mail : jaroslav.zapomel@vsb.cz

The experimental work of Cole and Huges [1] confirmed formation of a cavitated region in a dynamicaly loaded bearing that moved together with the shaft journal. White [2] found out existence of cavitation bubbles also in the area of high pressure if the relative eccentricity of the journal was greater than 0.3. Hibner and Bansal [3] were explaining the discrepancy between the results obtained by utilizing the classical theory of fluid film bearings and the observations by compressibility of the oil caused by gases liberated from it. Feng and Hahn [4] published an approach to determination of the density and viscosity of a two-phase homogeneous medium.

The kind and substance of the cavitation in hydrodynamical bearings and squeeze film dampers was experimentally studied by Zeidan and Vance [5]. They revealed five cavitation regimes. The vapour cavitation takes place due to the oil boiling if pressure at some location of the bearing gap drops below the critical value. Their observations showed that pressure of the medium in cavitated regions remained approximately constant. After increasing the pressure the vapour bubbles collapsed and the gas phase was immediately changed into the liquid one. Results of the research carried out by both authors implied that the vapour cavitation reduced the load capacity of the bearings. On the other hand the cavitation is not always undesirable because it also decreases friction in the bearing gap, energy losses, resistance against the rotor rotation, and heating of the lubricant.

2. Calculation of the pressure distribution in the bearing gap

The fluid film bearings are usually incoporated into the computational models by means of nonlinear force couplings. To determine components of the hydraulical force through which the layer of lubricant acts on the rotor journal and bearing shell it is necessary to know a pressure distribution in the bearing gap. According to the classical theory of hydrodynamical lubrication it is assumed that (i) the bearing surfaces are absolutely rigid and smooth, (ii) the cross section in the bearing shell is constant in the axial direction, (iii) the oil film thickness is small compared to the journal radius, (iv) the lubricant is incompressible Newtonian liquid of constant viscosity and adheres perfectly to the bearing surfaces, (v) the inertia effects of the lubricant are negligibly, (vi) the flow is laminar and isotermic, (vii) the pressure of the lubricant is constant in the radial direction, and (viii) the velocity gradient in the radial direction is large in relation to those in the tangential and axial ones.

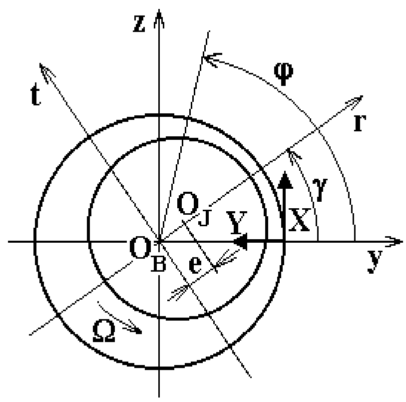

Fig.1 Scheme of a fluid film bearing

Incorporation of a vapour cavitation into the computational models of fluid film bearings assumes that in regions where the pressure is higher than the critical value a hydrodynamical effect is developed and that the flow of the lubricant is caused by the pressure gradient. In the areas where the pressure should drop below the critical one a cavitation occurs. The pressure remains constant and the lubricant flows only due to its adherance to the bearing surfaces. It is expected that pressure in the cavitated regions is equal to the pressure of saturated oil vapour at given temperature. To be satisfied the continuity of flow between the cavitated and noncavitated areas the pressure gradient on both sides of their common border must be zero.

In noncavitated regions the pressure distribution is governed by a Reynolds equation

$$\frac{1}{R^2}\frac{\partial}{\partial\varphi}\left(\frac{h^3}{\eta}\frac{\partial p}{\partial\varphi}\right)+\frac{\partial}{\partial Z}\left(\frac{h^3}{\eta}\frac{\partial p}{\partial Z}\right)=\frac{6}{R}\frac{\partial}{\partial\varphi}\left[h\left(u_1+u_2\right)\right]+6\frac{\partial}{\partial Z}\left[h\left(w_1+w_2\right)\right]+12\frac{\partial h}{\partial t} \qquad (1)$$

where

$$h = h_0 - e.\cos(\varphi-\gamma) \qquad (2)$$

φ, Z - circumferential, axial coordinates (Fig.1 - Z is perpendicular to X, Y),
e, γ - eccentricity of the rotor journal centre, position angle of the line of centres (Fig.1),
h_0, h - width of the gap at centric, eccentric position of the journal,
R, η, t - radius of the rotor journal, oil dynamical viscosity, time,
p - pressure, pressure function,
u_1, w_1 - circumferential, axial velocity components of the bearing shell surface,
u_2, w_2 - circumferential, axial velocity components of the rotor journal surface,

If the length to diameter ratio is small (less than approximately 0.25) and if sealing at the bearing faces is insufficient (O'cvirck bearing, short bearing), then it can be assumed that the pressure gradient in the axial direction is much greater than those in the circumferential one and therefore the first term on the left-hand side of the Reynolds equation (1) can be omitted. As the bearing does not move in the axial direction, the axial velocity components of the bearing shell and rotor journal surfaces are zero

$$w_1 = 0 \qquad , \qquad w_2 = 0 \qquad (3)$$

Taking into account the above considerations the Reynolds equation modified for the case of short bearings has the form [7], [8]

$$\frac{\partial}{\partial Z}\left(\frac{h^3}{\eta}\frac{\partial p}{\partial Z}\right)=\frac{6}{R}\frac{\partial}{\partial\varphi}\left[h\left(u_1+u_2\right)\right]+12\frac{\partial h}{\partial t} \qquad (4)$$

To solve the Reynolds equation (4) two additional conditions must be added. If it is satisfied that pressure at both faces of the bearing is the same, then the pressure boundary conditions for the bearing with a deep central groove are expressed

$$p = p_s \qquad \text{for} \qquad Z = \pm\frac{s}{2} \qquad (5)$$

$$p = p_a \qquad \text{for} \qquad Z = \pm\frac{L}{2} \qquad (6)$$

L, s - length of the bearing, width of the central groove.

After double integration of (4) the pressure distribution takes the form

$$p = \frac{1}{2} A Z^2 + C_1 Z + C_2 \tag{7}$$

where

$$A = \frac{6\eta}{Rh^3} \frac{\partial}{\partial \varphi}\left[h(u_1 + u_2)\right] + \frac{12\eta}{h^3}\frac{\partial h}{\partial t} \tag{8}$$

The integration constants C_1, C_2 are calculated from the boundary conditions (5) and (6). Then it holds

$$p = \frac{1}{2} AZ^2 - \left[\frac{2\,(p_s - p_a)}{L - s} + \frac{1}{4} A(L+s)\right] Z + \frac{p_s - p_a}{L - s} s + \frac{1}{8} ALs + p_s \qquad \text{for} \quad Z \geq +\frac{s}{2} \tag{9}$$

$$p = \frac{1}{2} AZ^2 + \left[\frac{2\,(p_s - p_a)}{L - s} + \frac{1}{4} A(L+s)\right] Z + \frac{p_s - p_a}{L - s} s + \frac{1}{8} ALs + p_s \qquad \text{for} \quad Z \leq -\frac{s}{2} \tag{10}$$

For calculation of the circumferential velocities the following relationships can be used with acceptional accuracy

$$u_1 = 0 \qquad , \qquad u_2 = R\,\Omega \tag{11}$$

The relations (9) and (10) show that the pressure profile is symmetric in the axial direction and therefore components of the bearing force can be calculated only from the pressure acting on one half of the bearing length.

If magnitude of the pressure given by (9), (10) drops below the critical value p_{cav} (pressure of saturated oil vapour), a cavitation appears. For $Z > 0$ it happens if the following conditions are satisfied

$$p_{min} < p_{cav} \qquad , \qquad \frac{s}{2} \leq Z_{min} \leq \frac{L}{2} \tag{12}$$

where

$$p_{min} = \frac{1}{2} AZ_{min}^2 + \left[\frac{2(p_a - p_s)}{L - s} - \frac{1}{4} A(L+s)\right] Z_{min} + \frac{1}{8} ALs - \frac{s}{L - s}(p_a - p_s) + p_s \tag{13}$$

$$Z_{min} = \frac{1}{4}(L+s) + \frac{2(p_s - p_a)}{A(L - s)} \tag{14}$$

p_{min} , p_{cav} - minimum pressure value, pressure in cavitated region (critical pressure value),
Z_{min} - coordinate of the location of the pressure minimum.

Solution of the Reynolds equation is then divided into two parts between which an interval of constant pressure exists. For $Z > 0$ the integration constants C_1 and C_2 in (7) and coordinates defining the extent of the cavitated area Z_{c1}, Z_{c2} are determined by application of the following boundary conditions

$$p = p_s \quad \text{for} \quad Z = \pm\frac{s}{2} \qquad \text{and} \qquad p = p_{cav} \;,\; \left[\frac{\partial p}{\partial Z}\right] = 0 \quad \text{for} \quad Z = Z_{C1} \tag{15}$$

$$p = p_a \quad \text{for} \quad Z = \pm\frac{L}{2} \qquad \text{and} \qquad p = p_{cav} \;,\; \left[\frac{\partial p}{\partial Z}\right] = 0 \quad \text{for} \quad Z = Z_{C2} \tag{16}$$

After performing appropriate manipulations the pressure distribution (for $Z > 0$) is expressed by the following relationships

$$p = p_s \qquad \text{for} \qquad 0 < Z < s/2 \tag{17}$$

$$p = \frac{1}{2}AZ^2 - AZ_{c1}Z - \frac{1}{8}As^2 + \frac{1}{2}AZ_{c1}s + p_s \qquad \text{for} \qquad s/2 \le Z \le Z_{c1} \tag{18}$$

$$p = p_{cav} \qquad \text{for} \qquad Z_{c1} < Z < Z_{c2} \tag{19}$$

$$p = \frac{1}{2}AZ^2 - AZ_{c2}Z + p_{cav} + \frac{1}{2}AZ_{c2}^2 \qquad \text{for} \qquad Z_{c2} \le Z \le L/2 \tag{20}$$

and for the extent of the cavitated area it holds

$$Z_{c1} = \frac{s}{2} + \sqrt{\frac{2}{A}(p_s - p_{cav})} \tag{21}$$

$$Z_{c2} = \frac{L}{2} - \sqrt{\frac{2}{A}(p_a - p_{cav})} \tag{22}$$

Components of the hydraulical force F_{hy}, F_{hz} are then given by integrals (23) and (24) whose calculation is performed by combination of the analytical and numerical approaches

$$F_{hy} = -2R\int_0^{2\pi}\int_0^{\frac{L}{2}} p(\varphi, Z)\cos\varphi \, dZ \, d\varphi \tag{23}$$

$$F_{hz} = -2R\int_0^{2\pi}\int_0^{\frac{L}{2}} p(\varphi, Z)\sin\varphi \, dZ \, d\varphi \tag{24}$$

F_{hy}, F_{hz} - components of the hydraulical force.

3. Solving the equation of motion

The assumed model rotor systems have the following properties : (i) the shaft is represented by a beam-like body that is discretized into finite elements, (ii) the stationary part is flexible, (iii) the discs are absolutely rigid, (iv) inertia and gyroscopic effects of the rotating parts are taken into account, (v) material damping of the shaft is viscous, other kinds of damping are considered to be linear, (vi) the rotor is coupled with the stationary part through fluid film bearings, (vii) the rotor rotates at constant angular speed, and (viii) is loaded by concentrated and distributed forces of general time histories.

Lateral vibration of such rotors is described by a nonlinear equation of motion and by the relationship for boundary conditions

$$\mathbf{M}.\ddot{\mathbf{x}} + (\mathbf{B} + \eta_V.\mathbf{K}_{SH} + \Omega.\mathbf{G}).\dot{\mathbf{x}} + (\mathbf{K} + \Omega.\mathbf{K}_C).\mathbf{x} = \mathbf{f}_A + \mathbf{f}_V + \mathbf{f}_H(\mathbf{x}, \dot{\mathbf{x}}) \tag{25}$$

$$\mathbf{x}_{BC} = \mathbf{x}_{BC}(t) \tag{26}$$

M, **G**, **K** - mass, gyroscopic, stiffness matrices of the rotor system,
B, $\mathbf{K}_C$ - (external) damping, circulation matrices of the rotor system,
$\mathbf{K}_{SH}$ - stiffness matrix of the shaft,
$\mathbf{f}_A$, $\mathbf{f}_V$, $\mathbf{f}_H$ - vectors of applied, constraint, hydraulical forces acting on the rotor system,
$\mathbf{x}, \dot{\mathbf{x}}, \ddot{\mathbf{x}}$ - vectors of generalized displacements, velocities, accelerations of the rotor system,
$\mathbf{x}_{BC}$ - vector of boundary conditions,
Ω - angular speed of the rotor rotation,
η_V - coefficient of viscous damping (material of the shaft).

Solution of the equation of motion (25) utilizing a Newmark method [6] arrives at each integration step at solving a set of algebraic equations that are nonlinear due to the bearing forces (elements of vector $\mathbf{f}_H$). To avoid this operation elements of the vector of hydraulical forces $\mathbf{f}_H$ at time t+Δt are determined by means of their expansion into a Taylor series in the neighbourhood of time t

$$\mathbf{f}_{H,t+\Delta t} = \mathbf{f}_{H,t} + \mathbf{D}_{B,t}.(\dot{\mathbf{x}}_{t+\Delta t} - \dot{\mathbf{x}}_t) + \mathbf{D}_{K,t}.(\mathbf{x}_{t+\Delta t} - \mathbf{x}_t) + \ldots \tag{27}$$

Substitution only of the linear portion of the Taylor series (27) into (25) results into a modified equation of motion related to the instant of time t+Δt

$$\begin{aligned} &\mathbf{M}.\ddot{\mathbf{x}}_{t+\Delta t} + (\mathbf{B} + \eta_V.\mathbf{K}_{SH} + \Omega.\mathbf{G}).\dot{\mathbf{x}}_{t+\Delta t} + (\mathbf{K} + \Omega.\mathbf{K}_C).\mathbf{x}_{t+\Delta t} = \\ &= \mathbf{f}_{\mathbf{A},t+\Delta t} + \mathbf{f}_{V,t+\Delta t} + \mathbf{f}_{H,t} + \mathbf{D}_{B,t}(\dot{\mathbf{x}}_{t+\Delta t} - \dot{\mathbf{x}}_t) + \mathbf{D}_{K,t}(\mathbf{x}_{t+\Delta t} - \mathbf{x}_t) \end{aligned} \tag{28}$$

Square matrices of partial derivatives $\mathbf{D}_{K,t}$ and $\mathbf{D}_{B,t}$

$$\mathbf{D}_{K,t} = \left[\frac{\partial \mathbf{f}_H(\mathbf{x}, \dot{\mathbf{x}})}{\partial \mathbf{x}} \right]_{\mathbf{x}=\mathbf{x}_t, \dot{\mathbf{x}}=\dot{\mathbf{x}}_t} \tag{29}$$

$$\mathbf{D}_{B,t} = \left[\frac{\partial \mathbf{f}_H(\mathbf{x}, \dot{\mathbf{x}})}{\partial \dot{\mathbf{x}}} \right]_{\mathbf{x}=\mathbf{x}_t, \dot{\mathbf{x}}=\dot{\mathbf{x}}_t} \tag{30}$$

are calculated at the point of time t for which all kinematic parameters of the rotor system are known (have been calculated at the previous integration step).

To be satisfied the boundary conditions at any moment of time the equation of motion (28) referred to time t+Δt is transformed to the form

$$\mathbf{A}_{2,t+\Delta t}.\ddot{\mathbf{y}}_{t+\Delta t} + \mathbf{A}_{1,t+\Delta t}.\dot{\mathbf{y}}_{t+\Delta t} + \mathbf{A}_{0,t+\Delta t}.\mathbf{y}_{t+\Delta t} = \mathbf{b}_{t+\Delta t} \tag{31}$$

Matrices $\mathbf{A}_{2,t+\Delta t}$, $\mathbf{A}_{1,t+\Delta t}$, $\mathbf{A}_{0,t+\Delta t}$ and vectors $\mathbf{b}_{t+\Delta t}$ and $\mathbf{y}_{t+\Delta t}$ are obtained from (32) - (35) and $\mathbf{x}_{t+\Delta t}$ respectively by omitting appropriate rows and columns corresponding to the degrees of freedom to which the boundary conditions are assigned

$$\mathbf{A}^*_{2,t+\Delta t} = \mathbf{M} \tag{32}$$

$$\mathbf{A}^*_{1,t+\Delta t} = \mathbf{B} + \eta_V.\mathbf{K}_{SH} + \Omega.\mathbf{G} - \mathbf{D}_{B,t} \tag{33}$$

$$\mathbf{A}^*_{0,t+\Delta t} = \mathbf{K} + \Omega.\mathbf{K}_C - \mathbf{D}_{K,t} \tag{34}$$

$$\mathbf{f}^*_{t+\Delta t} = \mathbf{f}_{A,t+\Delta t} + \mathbf{f}_{H,t} - \mathbf{D}_{B,t}.\dot{\mathbf{x}}_t - \mathbf{D}_{K,t}.\mathbf{x}_t - \mathbf{A}^*_{2,t+\Delta t}.\ddot{\mathbf{x}}_{BC,t+\Delta t} - \mathbf{A}^*_{1,t+\Delta t}.\dot{\mathbf{x}}_{BC,t+\Delta t} - \mathbf{A}^*_{0,t+\Delta t}.\mathbf{x}_{BC,t+\Delta t} \tag{35}$$

In addition the mentioned modification eliminates unknown values of the vector of constraint forces $\mathbf{f}_V$.

4. Example

Rotor of the investigated rotor system ROT7 (Fig.2) consists of a shaft (SH) and of two discs (D1, D2) that are mounted on its overhung end. The shaft is coupled with the stationary part (FP) through two lemon hydrodynamical bearings with one deep central groove and their parameters make possible to consider them as short. The rotor rotates at constant angular speed (250 rad/s) and is loaded by its weight and excited by centrifugal forces caused by the discs imbalance.

The task was to analyze the steady state component of the induced vibration.

In the computational model the shaft was represented by a beam-like body that was discretized into finite elements. Both discs were considered as thin and absolutely rigid. The imbalance loading was modelled by two pairs of mutually perpendicular concentrated harmonical forces whose time histories are shifted by the phase leg of $\pi/2$.

Results of the performed analysis are summarized in the following figures. Time history of displacement of the discs centres can be seen in Fig.3. If the gap between the discs and the stationary part is narrow, these results can be used to judgement of the rotor from the point of view of the limit state of deformation. The forms of orbits of the rotor journal centres in bearings B1 and B2 are drawn in Fig. 4 and 5. It is evident that the trajectories are closed curves and it implies that the rotor vibration is periodic. Images of the Fourier transformation of time histories of z-displacements of the rotor journal centres in bearings B1 and B2 are in Fig.6 and 7. The results show that the steady state response of the rotor consists of the principal and of several lowest ultraharmonical components. The pressure profile in the axial direction at a specified location of the bearing gap for 4 instants of time during one period is drawn in Fig. 8 - 11. It is evident that at the moments of time 1/4 and 4/4 of the period a cavitation takes place. A pressure distribution in the oil film in bearings B1 and B2 corresponding to the state of equilibrium of the rotor system can be seen in Fig.12 and 13. Time history of the pressure at specified location in the gap of bearing B1 is edvident from Fig.14. The history is characterized by short and high pressure peaks.

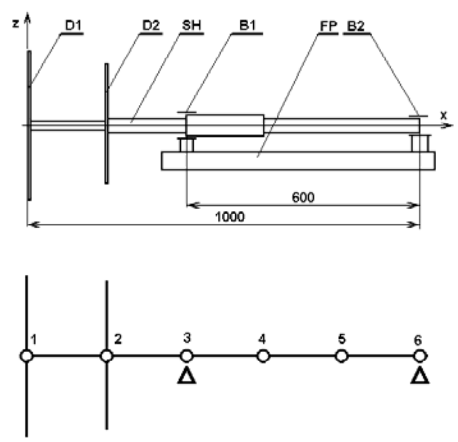

Fig.2 Scheme of the rotor system ROT7

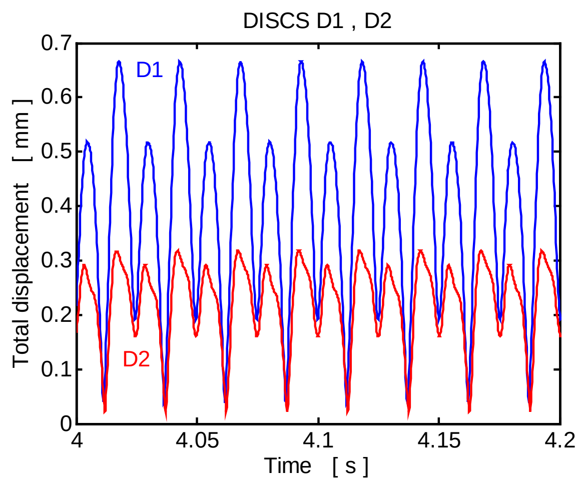

Fig.3 Displacements of the discs centres

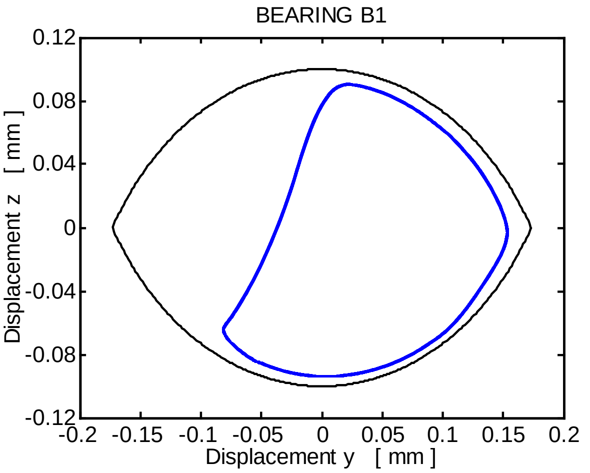

Fig.4 Orbit of the journal in bearing B1

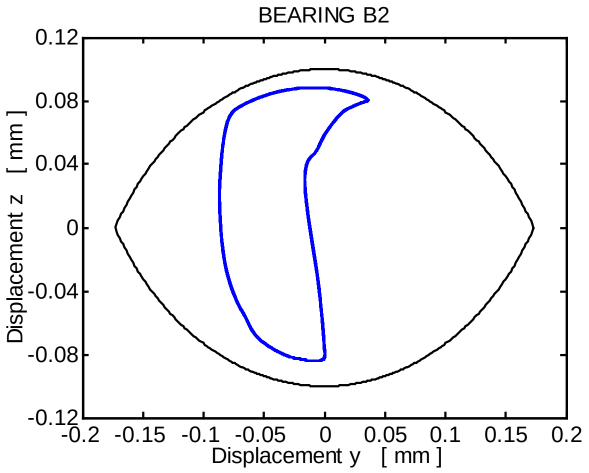

Fig.5 Orbit of the journal in bearing B2

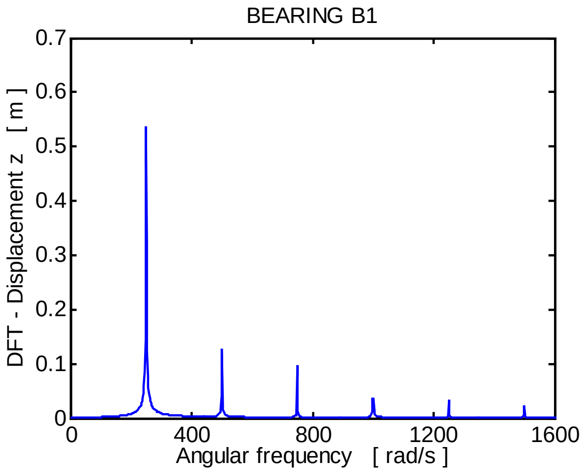

Fig.6 DFT of z-displacement in bearing B1

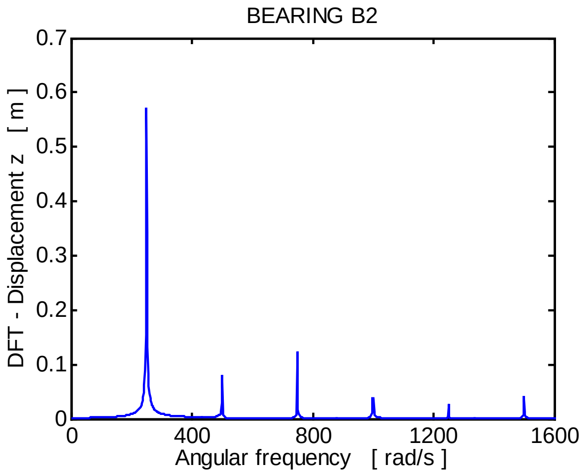

Fig.7 DFT of z-displacement in bearing B2

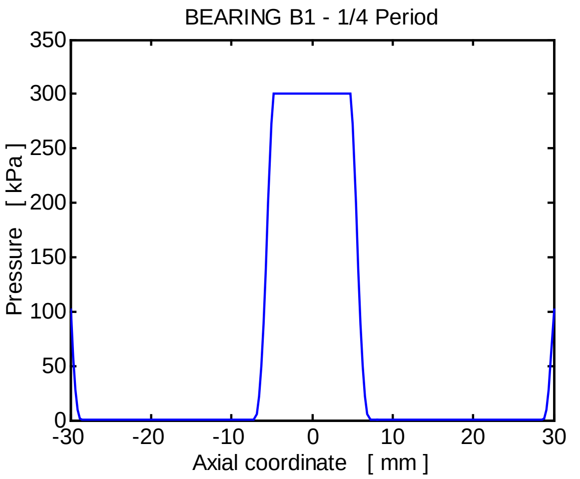

Fig.8 Pressure profile in the axial direction

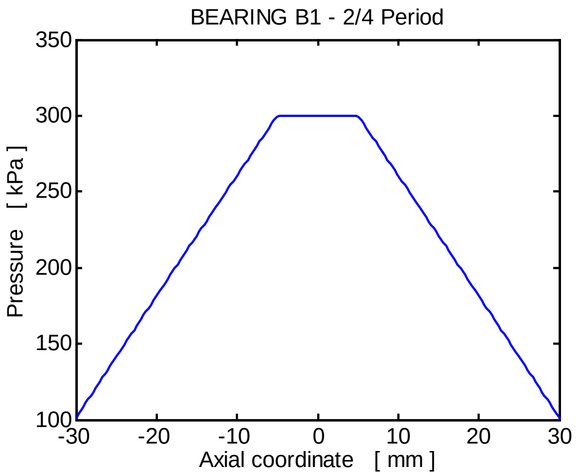

Fig.9 Pressure profile in the axial direction

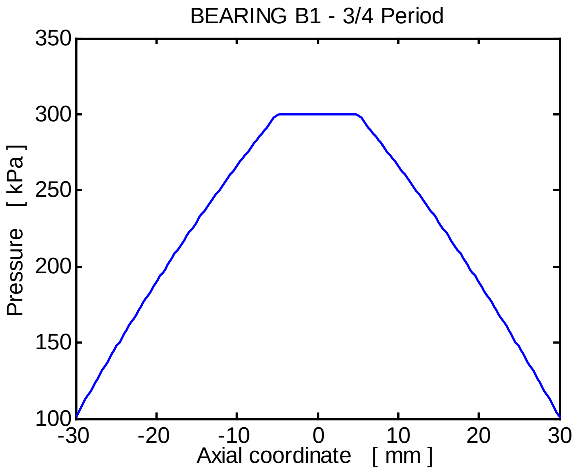

Fig.10 Pressure profile in the axial direction

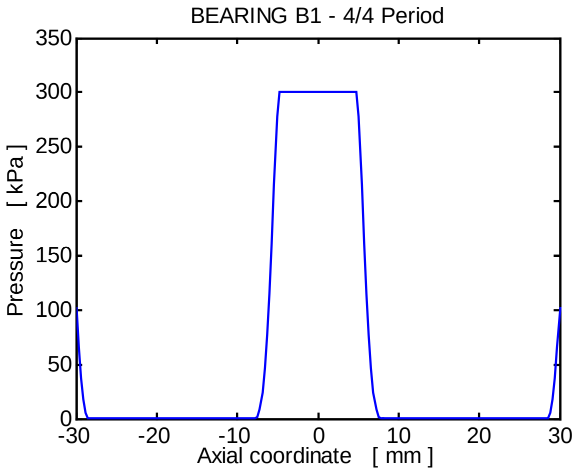

Fig.11 Pressure profile in the axial direction

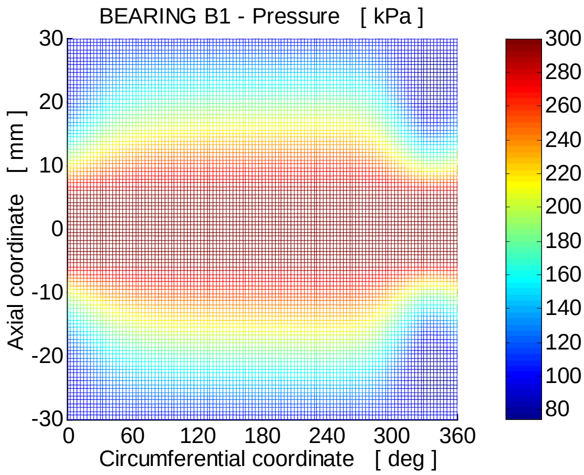

Fig.12 Pressure distribution in bearing B1

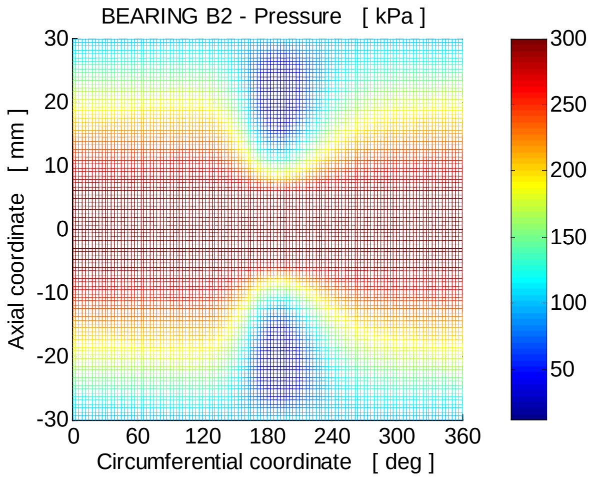

Fig.13 Pressure distribution in bearing B2

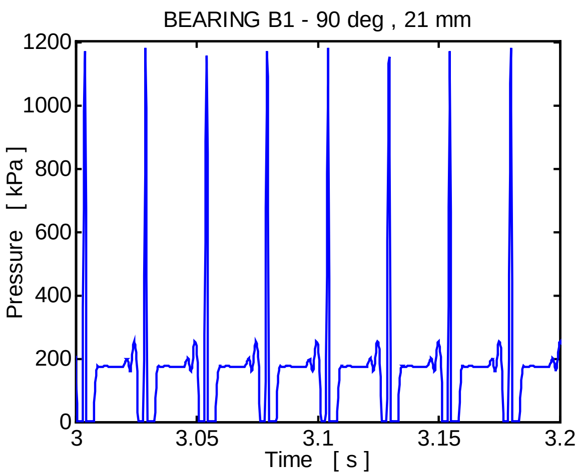

Fig.14 Time history of pressure at specified location of bearing B1

3. Conclusions

The described numerical approach is intended for investigation of imbalance response of rotors supported by lemon fluid film bearings with a deep central groove in which during their operation a vapour cavitation takes place and whose geometry and design parameters make possible to consider them as short.

Advantage of the described approach consists in making possible to express the pressure distribution in the oil film in a closed form and the pressure as a continuous function of both the axial coordinate and the time. This speeds up the calculation and makes easier to set up the matrices of partial derivatives.

Experience from the carried out computer simulations shows that the developed procedure is numerically stable which enables to apply a reasonable long integration step to solve the equation of motion.

Acknowledgment

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