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BUCKLING AND WRINKLING OF COMPRESSED SANDWICH BEAMS

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Summary: *Sandwich beams became a common engineering material. The main problem in design of these sandwich structures is prediction of the critical load. For these structures, two dominant modes of failure in compression can be distiguished. The first one is classical overall buckling and the second one is wrinkling, where only faces buckle while the core remains intact. The main goal of this contribution is the determination of influence of geometry and material properties on the critical load in compression and on the post-buckling behavior (i.e. whether overall buckling or wrinkling occurs). Numerical simulations of the post-peak behavior are carried out for a variety of combinations of material and geometrical parameters. At the end, some pieces of information for the prediction of failure mode and post-peak behavior are given.*

1. Introduction

A sandwich structure, typically consisting of two strong, stiff, thin facings and a soft lightweight thicker core, is a highly efficient way of carrying structural loads. Commonly used materials for facings are composite laminates and metals, while the core is made of metallic and non-metallic honeycombs, cellular foams, balsa wood or trusses. The facings carry almost all of the bending and in-plane loads and the core helps to stabilize the facings and carries the through-the thickness shear loads.

Sandwich structures have been widely used in aeronautics and astronautics since the early 1940s. The first theoretical works dedicated to the mechanical study of these materials are contemporary of their first applications. For example see (Heath, 1960; Hoff and Mautner, 1945; Williams, Leggett, and Hopkins, 1941). In the 1960s, the sandwich behavior began to be studied in more detail such as in the book by Allen (Allen, 1969), which sets the basis of the mechanics of sandwich structures. But it dates actually from the mid 1980s that the use of sandwich structures has moved from classical applications towards structural components, and thus they spread in other fields of engineering. The key point in using sandwich structures is the possibility of significant reduction of weight while keeping the same equivalent stiffness.

Unfortunately, since structural applications undergo multiaxial loading, the design of sandwich structures must take into account various loading modes such as bending, tension, compression, etc. Usually, classical layered material theories are sufficient for describing some aspects of these mechanical sandwich behavior typically in the elastic range. Nevertheless sand-

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wich structures exhibit complex failure mechanisms including face sheet compressive failure, adhesive bond failure, indentation failure, core failure and facing wrinkling.

The wrinkling of sandwich beams subjected to compression or bending is defined as a localized short-wave length buckling of the compressed facing. The wrinkling may be viewed as the buckling of the compressed facing supported by elastic or elastoplastic continuum, the core. Together with the overall buckling it is a common failure mode of sandwich beams, leading to the loss of the beam stiffness.

This paper is divided as follows. First, two theoretical analyses for prediction of critical wrinkling stresses based on beam and plate theory are given. Third section deals with numerical model and its accuracy. Next, the relationship between the thickness of faces and the collapse mode (i.e. overall buckling or wrinkling) is studied. The fifth section focuses on the influence of the length of sandwich beam on the critical wrinkling stress. Next section reveals some interesting information about the effect of core stiffness on the collapse mode and the last section is devoted to conclusions.

2. Theoretical determination of critical stresses

Two types of wrinkling can be distinguished. The first one is a symmetrical mode, see Fig. 1(a) and the second mode is antisymmetrical, see Fig. 1(b).

(a) (b)

Figure 1: Two wrinkling modes: (a) symmetrical mode (b) antisymmetrical mode

The complete understanding and characterization of buckling in such materials requires to take into consideration geometrical instabilities at two scales of observation. Since the 1950s, numerous theoretical studies have been conducted on the buckling in sandwich beams, panels and shells (Williams, Leggett, and Hopkins, 1941; Allen, 1969; Vinson, 1989). The classical approach developed so far is to propose two distinct models associated with the study of global and local buckling in an uncoupled way. The global mode of a sandwich beam is studied using an equivalent homogeneous beam in which the transverse shear effects must be considered. For the local forms, a beam (face) resting on an elastic foundation (the core), able to model the transverse normal and shear stiffness of the core, is generally used. The classical formula for the local form first proposed by Hoff and Mautnerr (Hoff and Mautner, 1945) is given by Eq. (1), where E_{c3} is the transverse Youngs modulus of core, G_{c13} is the shear modulus of core and E_{f1} is the longitudinal Youngs modulus of the face. As you can see the critical stress for wrinkling is independent of geometrical properties and loading; only material properties of the core and the faces are taken into account.

$$\sigma_{cr} = cE_{c3}E_{f1}G_{c13}. \tag{1}$$

The value of constant c varies from 0.5 to 0.9 depending on the authors and the wrinkling case studied. The value of c in Eq. (1) for symmetrical wrinkling obtained by an energy approach is 0.91 according to Hoff and Mautner (Hoff and Mautner, 1945). However Hoff and Mautnerr suggested an empirical value of $c = 0.5$, which better fitted their experiments. Other authors proposed different values for c (for instance Gough suggested 0.63, Cox 0.6, Goodier and Neou 0.815 etc.).

Heath (for more details see Heath, 1960) suggested a different equations for the prediction of critical stresses:

$$\sigma_{cr} = \sqrt{\frac{2}{3}\frac{t_f}{t_c}\frac{E_{c3}E_{f1}}{(1-\nu_{13}\nu_{31})}}, \tag{2}$$

for wrinkling and

$$\sigma_{cr} = \frac{P_E}{2t_f}\frac{1-(\frac{1}{4}P_E/(G_{13}(t_c+t_f))(1-n))}{\left[1+(\frac{1}{4}P_E n/(G_{13}(t_c+t_f)))\right]^2} \tag{3}$$

for overall buckling, where t_f and t_c are the thicknesses of faces and the core, respectively, ν is the Poisson's ratio of the core, $P_E = 4\pi^2 D/(t_c+2t_f)^2$ is the critical loading for sandwich with a rigid core (Timoshenko, 1940), $n = \frac{G_{13}}{G_{31}}$, D is the flexural stiffness of a complete sandwich with the rigid core and $G_{ij}, i, j = 1, 3$ is the shear modulus of core in the i-th direction.

The most elaborate model, which distinguishes critical stresses for symmetrical and antisymmetrical wrinkling and overall buckling based on common assumptions was suggested by Léotoing (Léotoing, Drapier, and Vautrin, 2002). There are two main assumptions. First, faces are supposed to be Euler-Bernoulli beams and second, a higher order theory must be used for displacement of core due to good representation of a short wavelength phenomena. The resulting critical loads (P_{cr}^G for overall buckling and P_{cr}^A, P_{cr}^S for antisymmetrical and symmetrical wrinkling, respectively) are

$$P_{cr}^G = B\left[\frac{\rho_t}{\rho_l^2} + \left(\frac{6\rho_e}{(\rho_t/\rho_v)+(\rho_t/12\rho_l^2)+2\rho_e\rho_l^2)}\right)\left(\frac{1}{\rho_t}+\rho_t+2\right)\right], \tag{4}$$

$$P_{cr}^S = B\left[12\left(\sqrt{2\rho_e\left(\frac{1}{\rho_t}+2\right)}-\frac{\rho_t}{\rho_v}\right)\right] \quad \text{if} \quad \frac{2\rho_t^3}{\rho_v^2\rho_e(1+2\rho_t)} < 1, \tag{5}$$

$$P_{cr}^A = B\left[4\sqrt{6}\sqrt{\frac{\rho_e}{\rho_t}} + \frac{2\rho_e\rho_v}{\rho_t^2}\right], \tag{6}$$

where $\rho_l = L/\pi t_c, \rho_e = E_c/E_f, \rho_t = t_f/t_c$, $\rho_v = G_c/E_c$ and L is the length of the sandwich beam. In this case both materials are assumed to be isotropic and linearly elastic. Factor B can be expressed as $B = (t_c+2t_f)\rho_t E_s t_f/6$ and is always positive. The exact derivation of these equations can be found in (Léotoing, Drapier, and Vautrin, 2001).

Some other models for the computing of critical wrinkling stresses were developed. See for example (Vonach and Rammerstorfer, 2000).

3. Numerical model

In the literature, there are very few models, which are dedicated to sandwich modelling. This lack of reference models constraints to use the classical Finite Element Analysis, which is based on the displacements as unknowns. In this case the stresses on the interface are not continuous and so it is necessary to assess the quality and accuracy of such models. The first approach to modelling sandwiches is a two-dimensional model for the whole sandwich. It means the

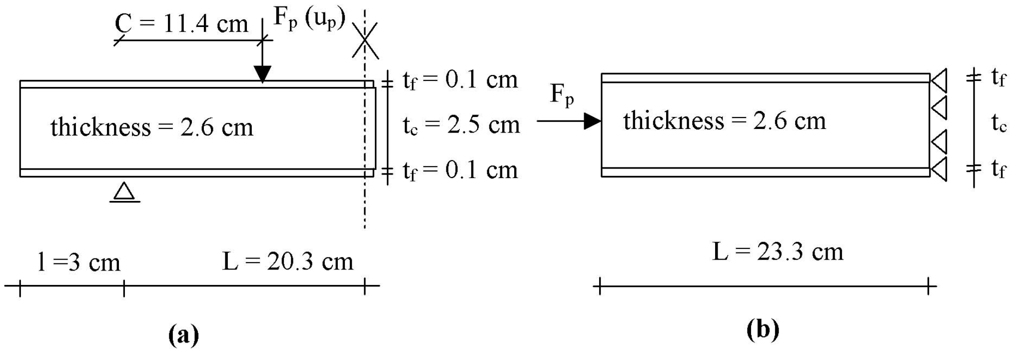

Figure 2: Two loading cases: (a) four point bending, (b) pure compression

core and the face are modelled by the standard four-noded bilinear or three-noded linear solid elements. Various tests did not show any significant discrepancy for the eigenvalue-buckling loads derived with plane stress or plane strain elements. The problem is that in order to have some reasonable amount of elements through the thickness of face, the size of these elements is very small and so the number of elements is very large (because of large ratio L/t_f). It means that the numerical analysis would be very CPU time-consuming even for the linear perturbation method (solving eigenvalue modes). Fortunately this model can be simplified by using one-dimensional beams (instead of solids) with influence of shear on the deformation (the three-noded beam is used, i.e. it has nine degrees of freedom: two translational and one rotational degree for every node). In this case the reduction of degrees of freedom (dof) is very significant. Due to the change of center of gravity of the face modelled as a solid and the face modelled as a beam, which influences the global stiffness in bending, some correction must be carried out. The easiest way is to increase the thickness of core. In this simplified model the size of the elements is not governed by the thickness of the face but the number of elements must be sufficient to capture the local instabilities i.e. wrinkling. In practice at least ten nodes (i.e. 5 elements) per one wavelength of wrinkling is used, the expression which is known analytically. Any exception of this rule will be emphasized in the following text. The comparison of both models, i.e. faces as beams or solids, is proposed in the following. Two types of loading are studied. The first one is pure bending and the second one is pure compression. The geometry including the loading and constraints for the latter case is shown in Fig. 2(b) and both meshes are depicted in Fig. 3. Only one half of the specimen due to the symmetry is modeled in the first case. This ”one half” geometry is the same as in the latter case. The loading is carried out by the rotation of the end cross section. It must be pointed out the change of t_c if the faces are modeled as beams from 2.5 cm to 2.6 cm. The comparison of first eigenvalues is done for all four cases. The face and the core are supposed to be isotropic and linearly elastic with $E_f = 147,000$ MPa, $\nu_f = 0.27$ and $E_c = 132$ MPa, $\nu_c = 0.25$, respectively. The linear perturbation method is used for the computing of first eigenvalues and eigenvalue modes for all cases. Tab. 1 shows the results. Overall buckling is the first eigenvalue mode for compression and wrinkling of the upper face is the first eigenvalue mode for pure bending. There is almost no difference between the results of both models. The number of dofs is reduced from 14,370 (faces as solids) to 5,038 (faces as beams).

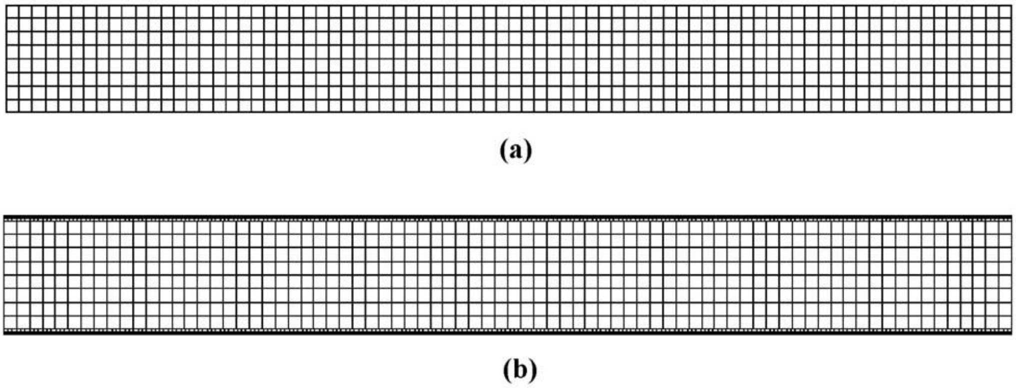

Figure 3: Two meshes: (a) faces as beams, (b) faces as solids (3 elements through thickness)

Table 1: First eigenvalues for different types of loading and models

model	**compressed solid**	**compressed beam**	**bent solid**	**bent beam**
first eigenvalue	$3.58\ 10^{-2}$ MN	$3.46\ 10^{-2}$ MN	$5.38\ 10^{-4}$ MNm	$5.42\ 10^{-4}$ MNm

Next, the accuracy of simplified model in geometrically nonlinear analysis is examined. In this example, the four point bending model is used. The geometry is shown in Fig. 2(a). Only one half of this specimen (which is depicted) is modelled due to the symmetry. It means the same meshes as in the previous analysis are used. The material parameters are the same as above. The results are shown in Fig. 4. The error between both models is only about 6%. So it can be assumed that the mechanical behavior of the sandwich beam is well presented by the model, where the faces are modeled as beams.

4. Relationship between the thickness of faces and the collapse mode

According to many studies, which were carried out in the 1960s, the main influence on the collapse mode of the sandwich beams has the length of beam, the transverse Young's modulus of core, the longitudinal Young's modulus of faces and the ratio between the thickness of core and the thickness of faces. The latter one will be objective of this study. Suppose we have a sandwich beam depicted in Fig. 5(a). Material properties of the faces and the core are $E_f = 147,000$ MPa, $\nu_f = 0.27$ and $E_c = 132$ MPa, $\nu_c = 0.25$, respectively. Both materials are supposed to be linearly elastic and isotropic. The faces are modeled as beams because of significant reduction of dofs and good accuracy as was shown above. The loading is applied at the one end of the sandwich beam while the opposite end is fixed. The thickness of faces is varied. Standard linear perturbation method is used and the eigenvalue mode and eigenvalue are computed for every thickness of face. The influence of the thickness of faces on the collapse mode is crucial. If the thickness of face is smaller than 0.75 mm then the collapse mode is

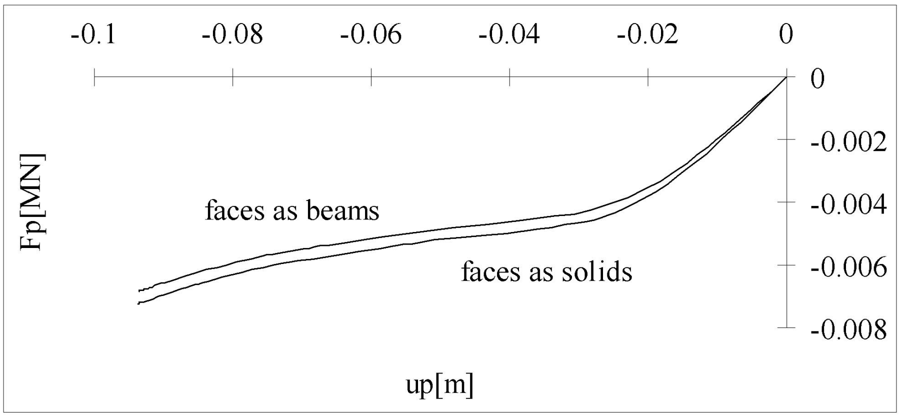

Figure 4: Deflection of loading point in four point bending

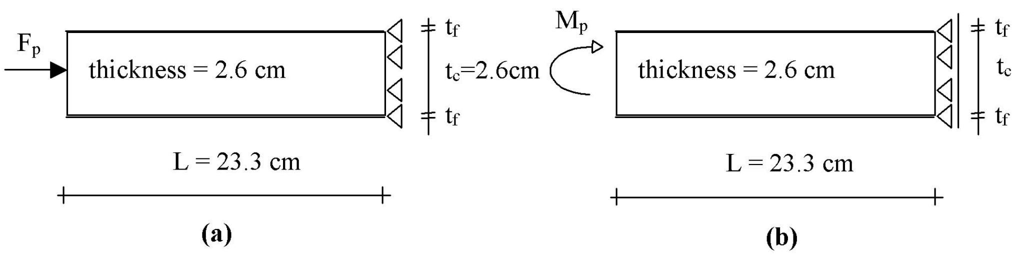

Figure 5: Geometry and loading

wrinkling and vice versa if the thickness of face is more than 0.87 mm the collapse mode is overall buckling. Four types of collapses are shown in Fig. 6.

The next analysis which is carried out is very similar to the previous one, but the loading is different. Instead of compression, only bending is involved. It means only normal forces in the faces carry the load, there is no shear force in the core. The geometry and loading is depicted in Fig. 5(b). In this case only wrinkling of the upper compressed face can occur. The deformed shape (first eigenvalue mode) of this beam for two different thicknesses of faces is depicted in Fig. 7. Fig. 8 shows all results together. The analytical solutions according to Heath's formula and according to Léotoing's formula are depicted too. The accuracy of Léotoing's formula for the sandwich beam loaded by compression is very good, but in the case of pure bending none of all three types of critical load (global overall buckling, symmetrical and antisymmetrical wrinkling) can be used. The Heath's formula is not good at all. The critical stresses for both types of loading, when wrinkling occurs, are very similar. See left side of Fig. 8. The arrow "buckling" shows, where wrinkling switches to overall buckling for the compressed sandwich beam. Higher load-carrying capacity of the nonlinear analysis than the loading capacity solved as the eigenvalue problem in the case of pure bending is very interesting. One possible reason is that the postpeak behavior is stable i.e. no softening occurs and so it is little difficult to determine the exact critical load in the latter case.

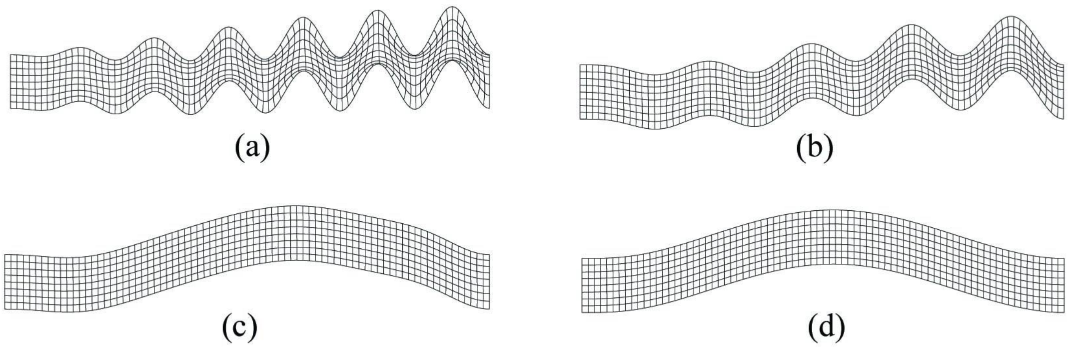

Figure 6: Eigenvalue modes for different thicknesses of faces: (a) $t_f = 0.75$ mm, (b) $t_f = 0.87$ mm, (c) $t_f = 0.94$ mm, (d) $t_f = 1.0$ mm

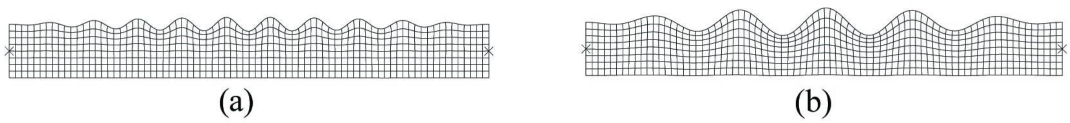

Figure 7: Eigenvalue modes for pure bending:(a) $t_f = 0.5$ mm, (b) $t_f = 1$ mm

5. Influence of the length of sandwich beam on the critical wrinkling stress

The following numerical simulations are devoted to assess the influence of the length of sandwich beam on the critical wrinkling stress in the case of pure bending and compression. The same geometry as in the previous example (see Fig. 5) is used but the whole beam is modeled (not only one half as before) and the thickness of faces is reduced to $t_f = 0.7$ mm. This thickness guarantees that only wrinkling occurs. See Fig. 8 for details. The material properties are again linearly elastic and isotropic with material constants $E_f = 147,000$ MPa, $\nu_f = 0.27$ and $E_c = 132$ MPa, $\nu_c = 0.25$. The length varies from $L = 46.6$ cm to $L = 5.8$ cm in eight steps. Results are given in Fig. 10. As you can see there is almost no influence of the length of the sandwich beam on the critical wrinkling stress in both cases. The change of critical wrinkling stresses for shorter sandwich beams is given mainly by the influence of boundary conditions. The first eigenvalue modes for the length $L = 46.6$ cm is shown in Fig. 9.

6. Effect of core stiffness on the collapse mode

The study of influence of core stiffness on the collapse mode is carried out in this section. The geometry and material parameters are the same as in the section 4 but the thickness of faces is fixed to 2 mm and 0.75 mm, respectively. The stiffness of core E_c changes from 0.001 to 400 MPa. No influence of E_c variation in the case of $f_c = 2$ mm can be observed on the collapse mode. Overall buckling always occurs. In the case of $f_c = 0.75$ mm the situation is completely different. Wrinkling as the first eigenvalue mode is valid only for models with E_c >33 MPa. In the opposite case overall buckling occurs. It is very interesting that with the softening of

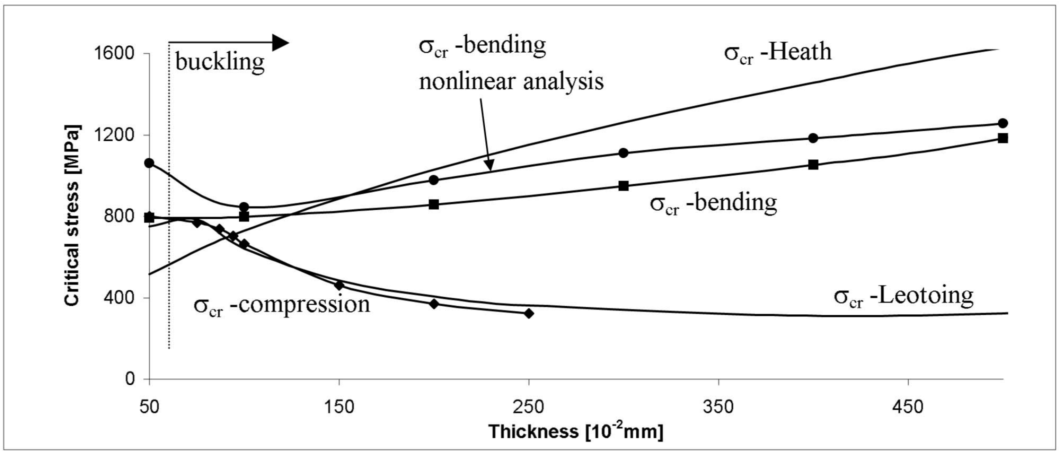

Figure 8: Results for both types of loading including analytical solutions according to Heath and Léotoing

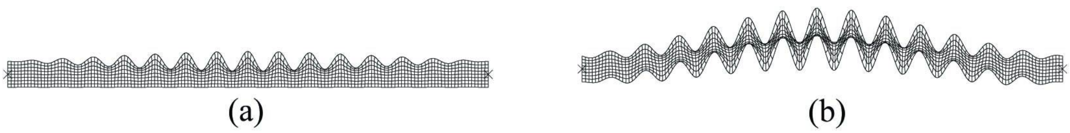

Figure 9: First eigenvalue modes for (a) bending and (b) compression ($L = 46.6$ cm)

core wrinkling switches to overall buckling. For example, according to Léotoing's theory only a reverse behavior is possible. The results of loading capacity for both specimens are depicted in Fig. 11. The x-axis is plotted in the logarithmic scale.

7. Conclusion

This study was devoted to some interesting aspects of sandwich beam collapse modes. Primarily, a low CPU time-consuming simplified numerical model was assessed. It was shown that results obtained using this model have good accuracy comparing to a full numerical model. Then it was shown the decisive influence of the thickness of faces on the collapse mode. Léotoing's formula proved to be very useful in the case of pure compression but almost useless in the case of pure bending. Next, the influence of length on the critical wrinkling stress was studied. The results confirmed independence of critical wrinkling stress on the length of sandwich beam for both types of loading. The analysis in sixth section was devoted to the effect of stiffness of core on the collapse mode. Very interesting phenomenon was simulated in this case. While Young's modulus of core was decreasing, the collapse mode changed from wrinkling to overall buckling, which is in contrary to the predictions of analytical analyses.

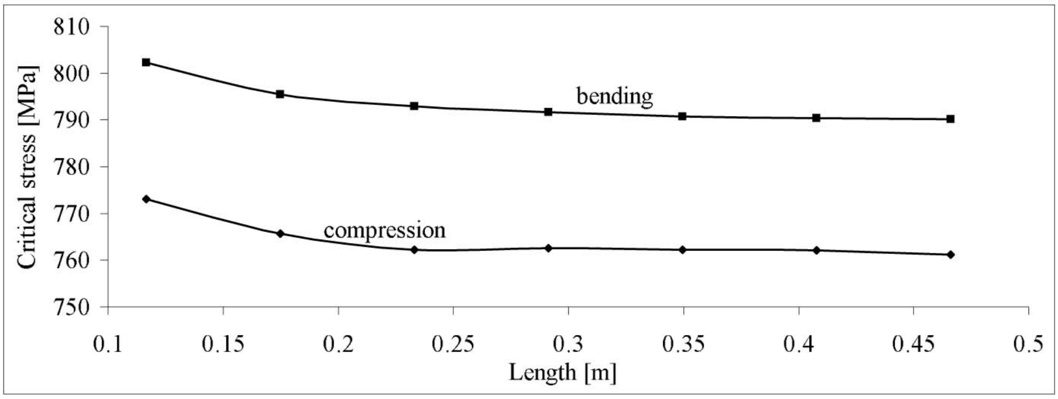

Figure 10: Influence of length on the critical wrinkling stress

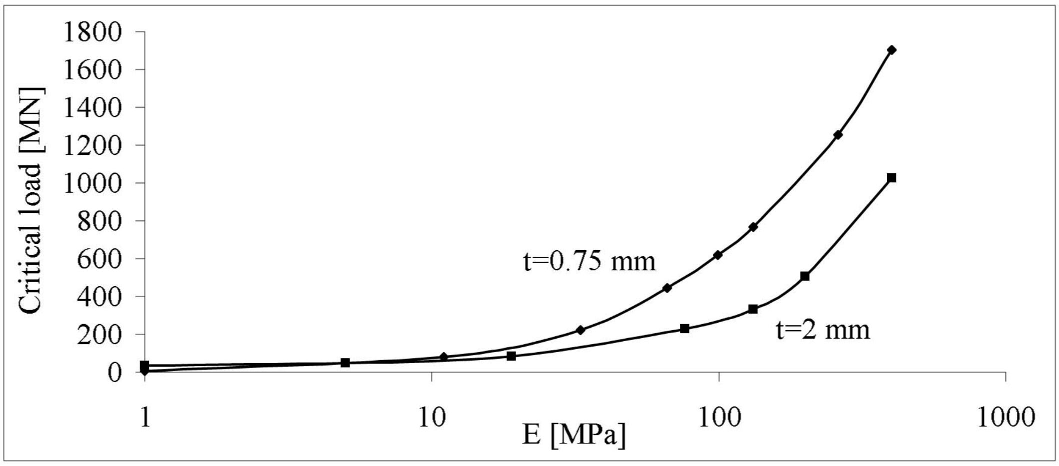

Figure 11: Influence of E_c on the critical wrinkling stress

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