ENGINEERING MECHANICS 2004
NATIONAL CONFERENCE
with international participation
Svratka, Czech Republic, 10 - 13 May 2004

ASSESSMENT OF EXPERIMENT REPRODUCIBILITY BY FINITE ELEMENT ANALYSIS

S. Pták, M. Okrouhlík, J.Trnka *)

Summary

Cylindrical metallic shell subjected to localized impact loading evoking wave response was considered. Experiments were carried out employing the Double pulse holographic interferometry (DPHI) for evaluation of displacements, the Doppler interferometry (DI) for velocities and the piezoelectric accelerometers (PA) for the registration of accelerations, while the finite element (FE) method was used for the numerical analysis. The paper compares experimental and numerical responses with the intention to discuss the accuracy limits of experimental and numerical approaches.

Introduction

The experimental investigation of high-speed phenomena accompanying the stress wave propagation in solids is a difficult task. Preparing and carrying out the experiment is expensive both in terms of required sophisticated hardware, material costs and manpower resources; furthermore the satisfactory reproducibility is hard to achieve in microsecond time scales. On the other hand employing the FE method for treatment of stress wave propagation in linear elastic continuum is a well reproducible routine, even if it is highly time and memory consuming. A lot of finite element results could be generated easily after the initial exertion is invested in proper design of the FE model. Still the detailed evaluation of finite element results requires additional both menial and mental effort. The former stems from the fact that the data sets needed for comparison with experimental data have to be mined out from extensive data files generated for every node at each time step. The latter emanates from an impossibility to distinguish precisely between the actual fast-frequency phenomena and spurious side effects due to time and space discretizations.

*) Ing. Svatopluk Pták, CSc, Doc. Ing. Miloslav Okrouhlík, CSc, Ing. Jan Trnka, CSc,
Institute of Thermomechanics, Academy of Sciences of the Czech Republic, Dolejškova 5, 142 00, Prague,
phone: +420 266 053 973, fax: + 420 286 584 695, e-mail: svata@it.cas.cz

Reaching satisfactory precision, reliability and robustness of experimental and numerical methods is a never-ending but still a worth pursuing effort. Comparing results obtained by experimental and numerical analysis allows shedding light on the accuracy of the measurement as well as to ponder about the range of applicability of both approaches since none of them has built-in self-limiting features. See [Barrow, 1999].

Vehicle for comparison – impact loaded cylindrical shell

A transient response of two cylindrical shells to impact loading was studied. Coordinate system, geometry, dimensions, loading and measuring points are indicated in Fig. 1 and in Table 1. Material constants for individual shells were precisely set by measurements and used in computation.

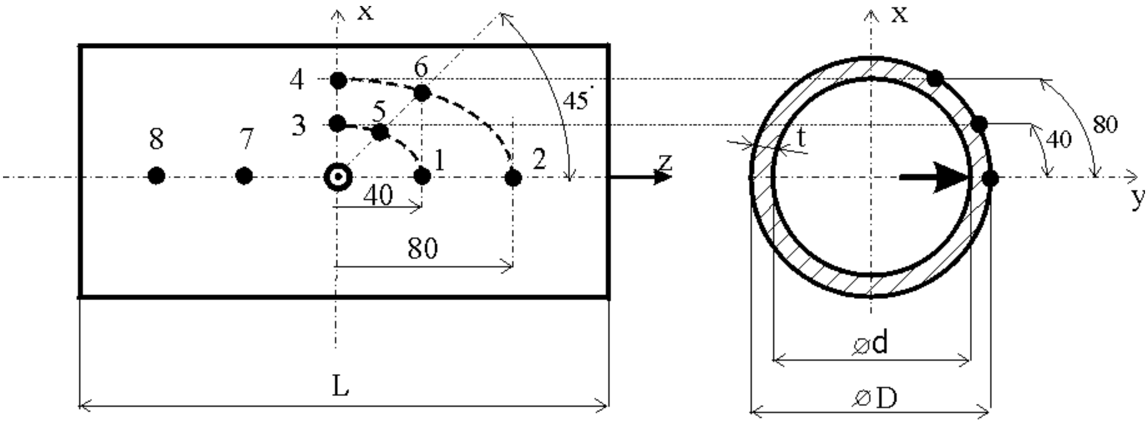

Figure 1 Shell geometry and loading

Case	ϕD	ϕd	t	L	d/t
A	321.2	310.8	5.20	280	61.77
D	106.6	96.3	5.15	302	20.70

Table 1 Dimensions of considered shells in [mm]

Experimental method and tools

The experimental results were carried out and kindly provided by the third author. They were obtained by three methods, i.e.

(i) Double Pulse Holographic Interferometry (DPHI) with double pulse ruby laser Lumonics as a source of light for visualisation of the displacement field was used. A qualitative interpretation of the recorded field (frozen fringe pattern) gives the deformed shape of the shell surface along a chosen surface line at a given time.

(ii) Contact-less Doppler interferometry (DI), Polytec OFV-3000, which produces normal velocity as a function of time at a given location.

(iii) Measurement by piezoelectric accelerometer (PA), Bruel & Kjaer 4374, giving normal acceleration as a function of time at a given location.

The methods and their implementations are described in detail in [Trnka, 2002].

Numerical method and tools

Different types of finite elements were used for model verification. Thin shell (semi-Loof, [Irons, 1976]) and solid (triquadratic, [Bathe, 1996]) elements were employed for the task. Coarse and fine spatially uniform meshes are characterized by data in Tab. 2.

Geometry		Shell elements		Solid elements	
		coarse	fine	coarse	fine
A	No. of elements	4 836	19 344	9 672	-
	No of dof's	64 031	253 795	163 083	-
	Max. frequency[MHz]	0.987	1.974	0.997	-
	Min. wavelength[mm]	5.36	2.68	5.31	-
	Mesh_size [mm]	2.68	1.34	2.68	-
D	No. of elements	6 840	27 360	13 680	44 100
	No of dof's	90 315	358 467	229 911	637 272
	Max. frequency[MHz]	0.97	1.94	0.98	1.96
	Min. wavelength[mm]	5.31	2.65	5.25	2.63
	Mesh_size [mm]	2.65	1.33	2.63	1.31

Table 2 Finite element mesh characteristics

Meshsize and time step were designed from geometry considerations and from the Fourier spectrum of the loading pulse as described below. The Newmark time-step operator with no algorithmic damping solved the system of ordinary differential equations describing the transient task. Time step was set up in such a way that it equals the time interval needed by the longitudinal 1D stress wave to pass through a quarter the length of the smallest element. For details of methodology see [Okrouhlík, Pták, 2003].

The PMD software – a general-purpose finite element package designed and maintained in the authors' institute – carried out the FE analysis. For details see www.it.cas.cz/manual/pmd/index.htm. The four-processor Compaq Alpha Server ES 40 was employed.

Loading – experimental and FE pulses

The impact pulse is produced by a loading element with exploding wire. The element has a shape of a tiny Plexiglas cylinder (diameter of 6 mm with 30 mm length) with a radial hole in which a thin (0.1 mm) bronze wire is embedded. The loading element is glued to the inner surface of the cylindrical shell. Gluing the element to the inner shell surface in perpendicular or oblique (a small adjustment of gluing surface is required) directions makes possible to evoke perpendicular or oblique loadings, respectively.

Applying a high voltage pulse to the wire causes its explosion, inducing a mechanical pressure pulse whose shape and magnitude were calibrated by the classical Hopkinson's bar technique. [Kolsky, 1963]. Since there is up to 30% uncertainty in the amplitude and shape of experimental pulses (see [Trnka, 2002, 2004]) an average pulse was elaborated out of the series of calibrated ones (see Fig. 2). Then its representative part (within the $50 m\,s$ range) was cut out of the average pulse and used as the 'reference' pulse.

The quantities resulting from experiment are interpreted as if the actual loading is identical with a reference loading mentioned above. This might not be true.

First – the explosion of the wire, the quality of gluing and soldering, the process of breaking the glue layer after the explosion, if any, and other effects are not perfectly reproducible.

Second – even for a perfectly reproducible pulse, being applied to different shell geometries considered, one could expect different results due to the fact that the loading is of contact nature.

For the numerical treatment it was the reference pulse, which was used as the input loading. One can observe (Fig. 2) that the approximation of the reference pulse by means of Fourier's series using 20 sine and cosine harmonics is adequate. Furthermore, the frequency analysis of the pulse proves that such an approximation is satisfactory since the frequency of the 20^{th} harmonics, i.e. 0.4 MHz, is still substantially smaller than the minimum of maximum eigenfrequencies appearing in Tab. 2.

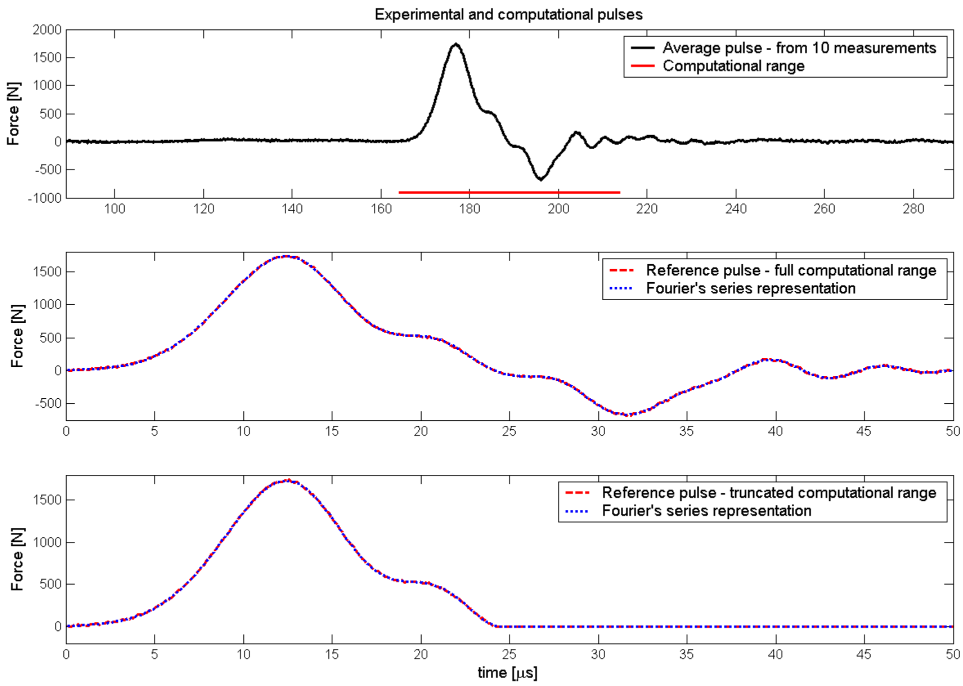

Figure 2 Loading pulse considerations

Full and truncated pulses, approximated by 20 sine and cosine harmonics and depicted in Fig. 2, were considered in numerical analysis with intention to ascertain the influence uncertainties in the glue-breaking process occurring after the wire explosion. The full pulse contains the rise and fall of the force distribution within the 50ms interval. The truncated pulse ends when the tensional force appears for the first time. This might represent the glue breaking process between the loading element and the shell.

Comparison of results

The number of carried-out experiments limits the number of experimental results available. Actually, each set of results is obtained using new and slightly different loading conditions.

The obtained experimental results are of 'field' and/or 'point' nature. The former are confined to the outer (visible) surface of the shell and consist of displacements at the shell surface at 3 time instants and are produced by the DPHI method.

The latter are available at selected locations at the outer surface. The DI method gives normal velocities as functions of time while the PA measurements provide normal accelerations as functions of time

In comparison with experiment there is an abundance of FE results. The field quantities (all components of displacements, velocities, accelerations, strains, stresses, values of energies, etc.) can be presented either as the spatial distributions for a given time or as the time sequence of spatial distributions (i.e. movie). The point quantities (sorted out of the field ones) can be visualized for a given space coordinates as a function of time or for a given time as a function of space coordinates. Furthermore the results are fully reproducible.

Comparison of displacements

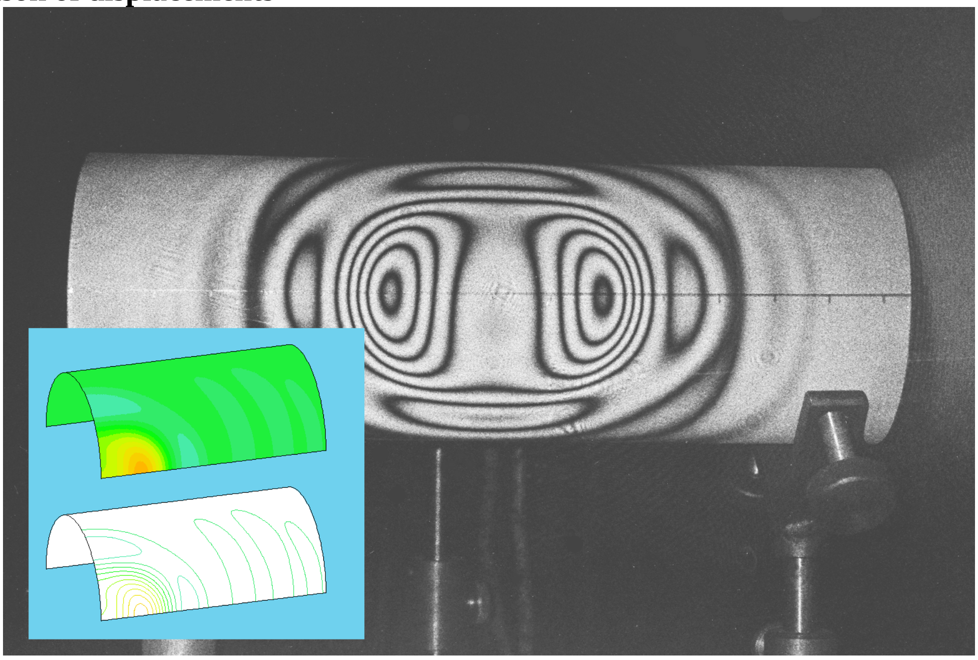

Figure 3 Experimental and FE displacement pattern

Fig. 3 shows comparison of displacements for D geometry obtained by the experimental and FE analysis corresponding to time $44.2\,ms$ after the beginning of impact. Experimental displacements are actually 'almost normal' components of displacements as seen by a camera in central projection. They are represented by black and white fringe pattern. The distance between the contour lines (both in experimental and FE results) corresponds to the quarter-wave length of the ruby light used. The FE displacements, plotted as contour lines, are those in *y*-direction (see Fig. 1) obtained by coarse-mesh shell elements. On the 'upper' surface of the shell both directions almost coincide. There is a qualitative agreement. A closer look reveals, however, that the fringe count does not precisely check and that there is a poor agreement in experimental and numerical results.

Using the DPHI method, it is the quarter-wave length of the ruby laser light, i.e. 1.735e-7 m, which defines the absolute threshold of measurement. Bellow this threshold the method is blind. For this type of loading the relative threshold is of the order of 2 %.

Comparison of velocities

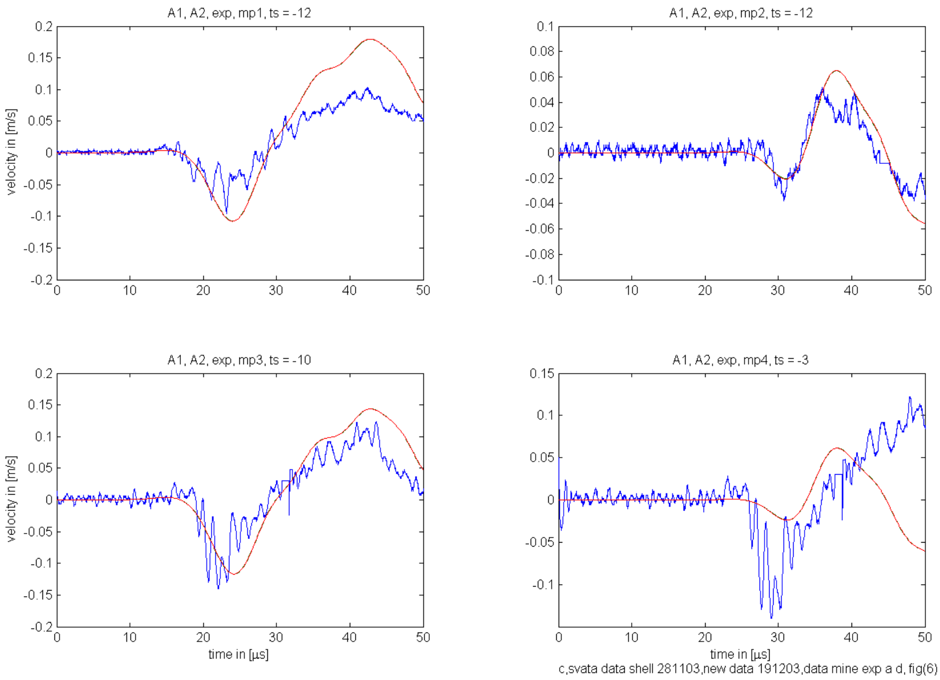

Figure 4 Comparison of velocities; FE (A1 and A2) – red, experiment - blue

The Fig. 4 shows the time distribution of experimentally obtained normal velocities for the A geometry at locations 1, 2, 3 and 4 (denoted by `mp` identifier) obtained by DI method compared with that of normal velocities by FE analysis. Four experiments were carried out to get velocity distributions at the above-mentioned four locations. The high-frequency contents of the experimental velocity distributions are attributed to speckle properties of coherent laser light and to thermal and electronic noise [Landa, Oral communication]. Using DI method the time origin of the experimental pulse is difficult to ascertain precisely [Trnka, 2002]. When presenting the comparison with FE data one has to shift the experimental time history for velocity forward or backward to get a sort of 'best' fit. The actual time shift is denoted by `ts` identifier in figures and is expressed in microseconds.

The FE shell results for the coarse (A1) and fine (A2) meshes are not distinguishable in the scale of the figure. Since there is no visible difference of results with mesh refinement one can conclude that time step and mesh size are set correctly. From this fact, however, we cannot deduce that the results are 'physically correct'. They are 'correct' within the validity range applicable to this particular FE model – the shell element a priori assumes what is happening within the shell thickness and does not take into account the actual wave processes occurring there.

The distance of locations 1 and 3 from the loading point is 40 mm, while that of locations 2 and 4 is 80 mm (see Fig. 1). Nevertheless, the observed results in mentioned couples of measuring points couldn't be identical, since they are living at non-symmetrical positions of the cylindrical shell.

A sort of 'similarity pattern' in results for locations 1,3 and 2,4 is a necessary (but not sufficient) condition for consistency of results. The FE data confirm this observation. The experimental data follow the similarity pattern and resemble FE results only at locations 2 and 3. The experimental data at location 1 do not follow the similarity pattern with those at location 3, furthermore they do not agree at all with FE data. A similar statement could be made for experimental data at location 3. One could conclude that the experimental data obtained at locations 1 and 3 are corrupted. Still, there is a good agreement in the occurrences of maxima and minima of the velocity response seen in Figure 4.

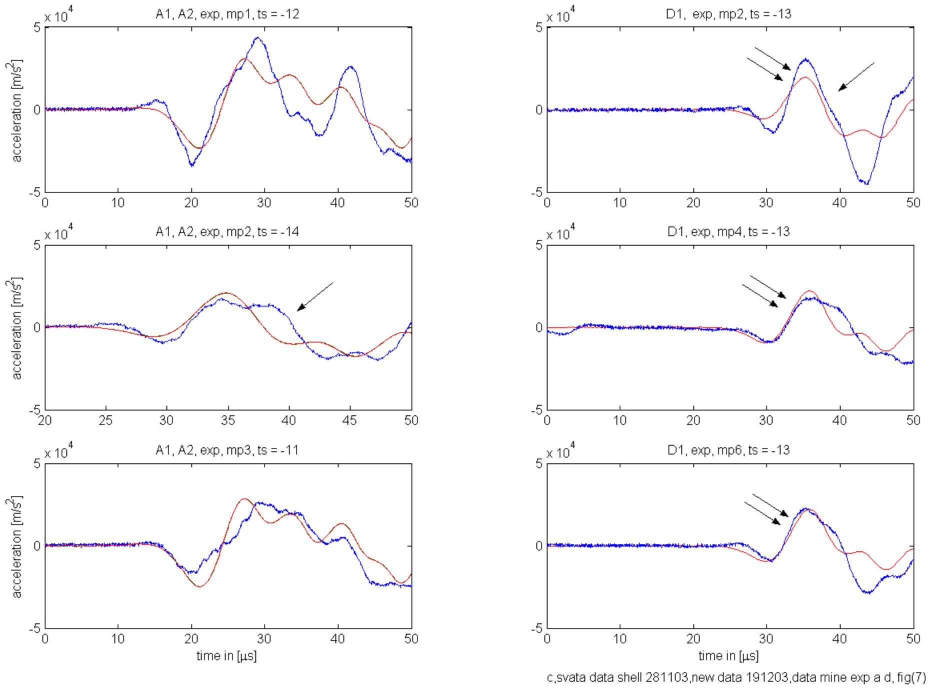

Figure 5 Comparison of accelerations; FE – red, experiment - blue

Comparison of accelerations

The left column of subfigures (Fig. 5) shows the comparison of the time distributions of normal accelerations for the A geometry at locations 1, 2 and 3. The right column of subfigures presents that of the time distribution of normal accelerations for the D geometry at locations 2, 4 and 6. Experimental data were obtained by the PA method. The FE data are due to the computation with coarse and fine meshes of shell elements applied to A geometry. For D geometry only the results for coarse shell elements are plotted. The overall agreement is questionable. As before a different time shift of experimental data is needed to get a sort of agreement with FE data. Differences in experimental amplitudes, probably due to differences in the brisance of explosion and the variations of un-gluing process, are observed.

Let's concentrate in more detail on the time responses of the same location `mp2` at different (i.e. A and D) shells. The responses (smooth lines) cannot be identical but evidently should have a similar pattern. The presented FE data behave this way. The experimental data are not similar in this respect (the differences are indicated by a single arrow in each figure – the hump in A shell data does not have its equivalent neither in D shell data for the same location nor in A data for different locations, i.e. in `mp1` and `mp3`), furthermore the rise and decline pulse behaviours are completely different.

Comparing the D shell data (Fig. 5) we observe the agreement in rise part of the pulse at `mp3` and `mp6` locations and disagreement at `mp2` location. Double arrows indicate this.

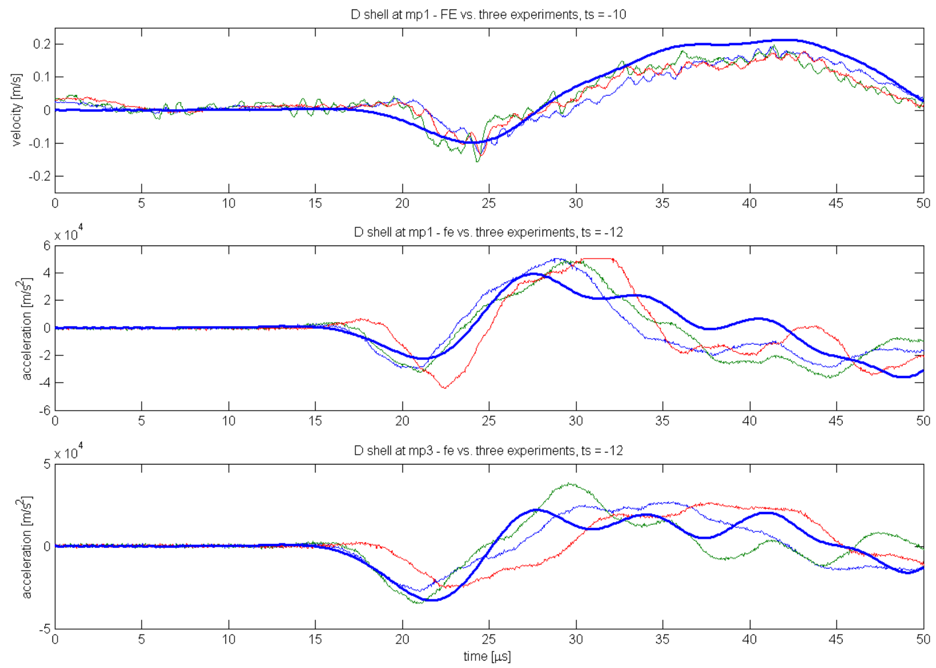

Figure 6 Results of multiple experiments (thin) compared with FE solutions (thick)

Agreement between results obtained experimentally and numerically indicates that the acceptable reproducibility of the loading process by means of exploding-wire elements is hardly attainable. Furthermore, only one displacement pattern, two acceleration distributions and one velocity distribution can be obtained as a result of a single measurement.

Fig. 6 depicts comparison of computed histories of velocities and accelerations for two measuring points with those obtained by repeated experiments. One can observe that there is a significantly smaller deviation between experimental velocity results than between those for acceleration. This might be attributed to the fact that the velocity measurement is secured by the Doppler interferometry, which is of contactless nature. The influence of added mass of accelerometer device was assessed computationally – only insignificant influence was observed, so the relatively large deviation in acceleration histories cannot be attributed to it.

The shown comparison indicates presently achievable measure of agreements between different single-shot experiments and between experiments and FE analysis. The quality and adjustment of individual components of the measurement process is important but in this case it is the reproducibility of the loading process, which plays the crucial role in the sought-after robustness of the experimental method. From this point of view the concept of creating the above-mentioned reference pulse by averaging is rather questionable.

A better consistency of the present experimental procedure might be achieved by simultaneous measurements of as large as possible number of quantities during a single explosion.

FE self-check considerations

Using FE modelling we are dealing with a discretized ‘continuum’. There is a countable number of dof’s, eigenfrequencies and a limited frequency spectrum. This means that the numerical model, for a given meshsize, is not able to distinguish frequencies above a certain limit.

Internal forces acting between elements are limited to adjacent neighbours only. That’s why the stiffness and mass matrices are sparse in FE analysis. They could also be relatively narrow-banded if an efficient node or element numbering is secured.

To find an evolution of the body response in time due to impact loading requires solving the system of differential equations in which the inversion of stiffness or mass matrices (depending on the mass matrix formulation and on the time integration method being employed) is needed. This way, the sparse structure of relevant matrices vanishes, and a sort of *unlimited range of influence* prevails. Theoretically – working with infinite number of significant digits – it means that we would get a nonzero response over the whole domain for any time greater than zero. Working with a standard double precision arithmetics, however, we would not observe any response which, in absolute value, is less than the *smallest positive floating point number* (around 2.2251e-308 in Matlab).

All this fuzz is about margins of our ability to distinguish something against nothing. This is, however, crucial for any meaningful human activity.

Practically we would have a limited range of influence given by energy transfer since the substantial part of the energy is always carried by low frequency components.

Seemingly unproblematic model of linear elastic continuum has embedded singularities in it – they are for example manifested by the fact that the displacement under the application of a point force tends towards infinity. To a certain extent this property is retained when the continuum is treated by means of a FE model. Actually it is smeared out by the existence of shape functions but it is still well manifested by the observable increase of the displacement under the application of a nodal force with diminishing meshsize. The FE mesh made of ‘null-sized’ elements would provide the infinite displacement under the application of a nodal force as the continuum model. So making a finer and finer mesh we are representing better and better those continuum properties that are mathematically correct but physically unacceptable. This is a sort of paradigm we are used to live with. Singularity in displacement response to a point loading is a well-known feature both in continuum and its FE representation as well and was treated elsewhere [Okrouhlík, Pták, 2003].

Modal properties of the continuum model pose another example of the continuum singularity. The continuum model has an unlimited spectrum of eigenfrequencies and infinite number of modes of vibrations. The FE model has finite number of eigenfrequencies and eigenmodes, furthermore the higher ones are distorted by dispersion. The highest eigenfrequency of the FE model is related to dimensions of the smallest element appearing in the mesh while that of the continuum model tends to infinity. The period of the FE eigenmode corresponding to the highest eigenfrequency is the shortest one, while that of the continuum model tends to zero. This is not, however, troubling us too much since the frequency corresponding to an element of continuum of the size of inter-atomic distance (10^{-10} m for metals) is of the order of GHz.

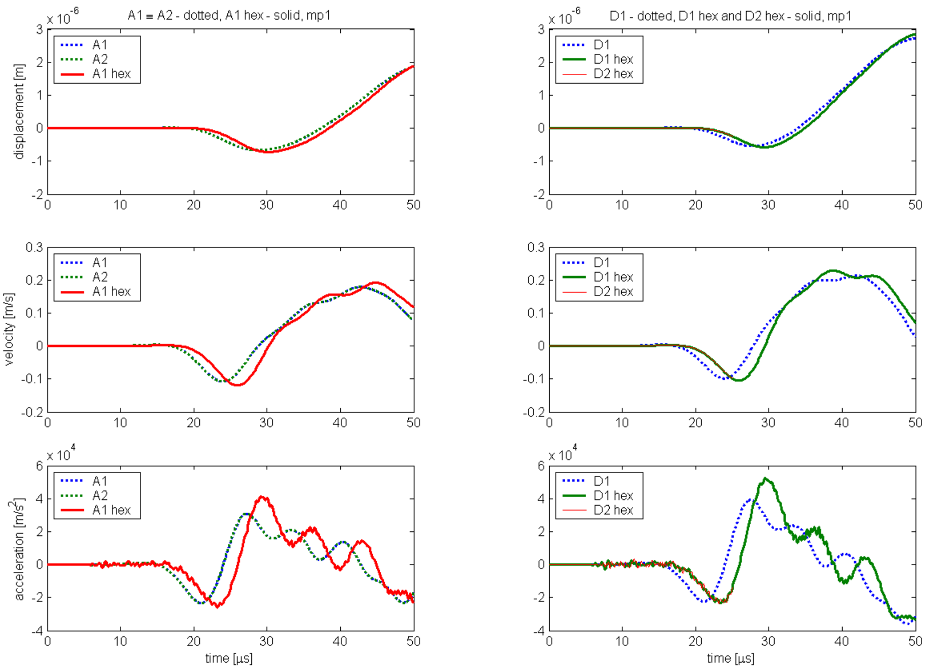

Figure 7 Comparison of results obtained by shell and solid elements

Fortunately, the nature is kind to us, since the energy is predominantly carried out by the lower modes of the spectrum.

Fast transient problems, however, contain high frequency harmonics, and in FE modelling they require small elements and a lot of eigenmodes to be taken into account. If step-by-step approach is used instead of modal superposition a very short time step of integration should be employed.

The Fig. 7 shows comparison of displacements, velocities and accelerations as functions of time at the location `mp1` as obtained by different elements (shell and solid) applied to two different shell geometries (A and D). The left column shows results obtained for the A shell computed by coarse and fine shell (A1 and A2) and coarse solid (A1 hex) elements. The right column shows results obtained for the D shell computed by coarse shell (D1) and coarse and fine solid (D1 hex and D2 hex) elements. The results for both geometries are similar, since the location `mp1` is close to the loading point and the influence of different shell shapes is small. The results in this location are not yet influenced by wave reflections from the boundaries of the finite element model. There is not a visible difference between A1 and A2 shell results – the finer mesh does not 'improve' the solution. It only means that for a given geometry and loading spectrum the mesh size is set correctly.

In Fig. 7 there is a visible time shift in results obtained by shell and solid elements. This is attributed to the fact that solid and shell elements represent different models of continuum. The shell element knows nothing about wave processes within the shell thickness since the thickness is just a computational constant and the mechanical phenomena occurring there are a priori assumed. The solid element has, however, the 'active' dimension in the normal direction of the thickness and is able to capture the wave and energy processes in this direction.

One should realize that a too small number of elements in this direction disqualifies, to a certain extent, the results (notice the high frequency contents) due to false dispersion effects and thus devaluates the extended effort required for the fine mesh processing.

Having, however, a really thin structure (thinness related to the frequency spectrum of the loading) and realizing that the energy contents of wave processes occurring within the thickness is negligible, we have to conclude that simple shell elements offer a more 'reasonable' solution devoid of false high frequency components.

On the other hand the finer and coarse mesh results for solid elements agree remarkably well (Fig. 7) confirming thus the consistency of the approach.

The visible time shift due to different element modelling, which is about 2 ms as shown in Fig. 7, could be attributed to the fact that the loading effects, applied from inside of the shell, need of about 1 ms to pass through the shell thickness.

The above observations lead us to the opinion that the solid element model of the body being studied (Fig. 1 and Tab. 1) is actually more reliable, since it better represents the more flexible and more complex nature of shell geometry than that of a thin shell element model. The experiment, however, gives us no hint in this respect.

Conclusions

When trying to ascertain the reliability of modelling approaches and the extent of their validity one has to realize that the models as a rule do not have self-correction features. That's why we have to let the models to check themselves, be checked by independent models and let the systematic doubt be our everyday companion.

It was shown how numerical methods could help ascertaining the reproducibility and precision of experiment and how the experiment could provide data for making computational procedures reliable, efficient and robust.

We believe that the experimental and numerical synergy that has been pursued is needed for a proper establishment of precision of applicability limits of methods used for modelling nature.

Acknowledgement: The support of the grant A2076904, the project K1019101 of the Czech Academy of Sciences and the grant 101/02/0072 of GACR is greatly appreciated.

References

Barrow, J.D., 1999, Impossibility. The limits of Science and the Science of Limits, Oxford University Press

Bathe, K.-J., 1996, Finite element procedures, Prentice-Hall, Inc., Englewood Cliffs, N.J.

Irons, B.M., 1976, The semiLoof shell element. In: Finite Elements for Thin Shells and Curved Members. Edited by Gallagher, R.H. and Ahswell, D.G. John Wiley, New York.

Kolsky, H., 1963, Stress waves in solids, Dover Publications, N.Y.

Landa, M., 2003, Institute of Thermomechanics, Oral communication.

Okrouhlík, M., Pták, S., 2003, Numerical Modelling of Axially Impacted Rod with a Spiral Groove, Engineering Mechanics, Vol. 10, No. 5, p. 1 – 16.

Okrouhlík, M., Pták, S., 2003, Pollution-free energy production by means of a proper misuse of finite element Analysis, Engineering Computations, Vol. 20, No. 5/6, pp.601-610, 2003

Trnka, J., Landa, M., 2002, Double pulse holointerferometry and PZT transducers use in the study of ultrasonic guided waves propagations in thin cylindrical shell. Acta Technica, CSAV. 47(1), p. 89 – 97.

Trnka, J., Dvoráková, P., 2004, Contribution to quantitative interpretation of double pulse holointerferograms of thin-walled cylindrical shells. (In Czech), Colloquium 'Dynamics of Machines 2004', Prague, February 10-11, 2004, p. 149-154.