ENGINEERING MECHANICS 2004
NATIONAL CONFERENCE
with international participation
Svratka, Czech Republic, 10 - 13 May 2004

THERMOMECHANICAL PROCESSES IN COMPLIANT DAMPING ELEMENTS

L. Pešek, L. Půst, F. Vaněk*

Summary: *Effect of increase of heat in the rubber-like materials with high inner damping during repeated deformation is investigated. Rheological properties e.g. stiffness and damping of such materials depend considerably on temperature. Knowledge of complex modulus properties of polymeric damping materials are strongly needed for the design of systems suppressed to vibrations, for the control of vibrations and noise. Simplified mathematical model of two degrees of freedom is analysed from the point of view of lost energy, heat transfer between inner and outer part, cooling of the surface and changing stiffness and inner damping. This model is the first stage for studying of thermomechanic interaction in the 3D form of damping elements by FEM.*

1.Introduction

Basic knowledge of thermomechanic interaction, typical for behaviour of rubber-like materials and similar elastomers was presented e.g. in [1,2] in the case of 1DOF system. Such elastomer elements are used predominately for the reduction of noise and vibration in many industrial, automotive, railway and aerospace applications. The basic model was formed by a vibrating mechanical system, which is heated by the internal damping energy lost in vibrating system. This energy changes into heat, the part of which escapes into environment. The rest staying in the body increases temperature of the resilient element and changes the stiffness and damping of the whole system [3].

In the IT ASCR developed methods of measurement of temperature on the laboratory samples of resilient elements for noise and vibrations damping of tram and railway wheels enable to measure the temperature both in the resilient element and near to its surface. In the presented paper, the model of analysed system is loaded by harmonic motion $x_0 \cos \omega t$ at the static prestress x_{st}, however the general non-stationary deformation can be studied as well.

Mathematical model is very simple, with only two elements, as the temperature of the real sample was measured also only in two points. In spite of its simplicity, this model and comparison with measured dates enables to investigate the basic heat flows, the rise of temperature in the body and also the influence of this temperature on the rheological properties of the resilient element. The aim of this study is to prepare basis for the more complicated FEM model.

* Ing. Luděk Pešek, CSc., DrSc., Ing. Ladislav Půst, DrSc., Ing. František Vaněk, CSc.: Institute of Thermomechanics AS CR, Dolejškova 5, 182 00 Prague 8, phone: +420.266 053 083, +420.266 053 212, fax: +420.286 584 695; e-mail: pesek@it.cas.cz, pust@it.cas.cz, vanek@it.cas.cz.

1. Analytical framework

The symmetry of experimental sample (Fig. 1) is fully used at the creation of mathematical

The symmetry of experimental sample (Fig. 1) is fully used at the creation of mathematical model (Fig.2).

Instead of the entire sample mass $2(m_1+m_2)$, the only right half of it is investigated. Temperature θ_1 and θ_2 are measured in the point 1 inside of the sample and in the point 2 near its surface. The inner part (mass m_1) has stiffness $k_1(\theta_1, x_{st})$ and damping $b_1(\theta_1, x_{st})$, both depending on the temperature θ_1 and on the prestress x_{st}. The outer part has corresponding parameters $k_2(\theta_2, x_{st})$ and $b_2(\theta_2, x_{st})$.

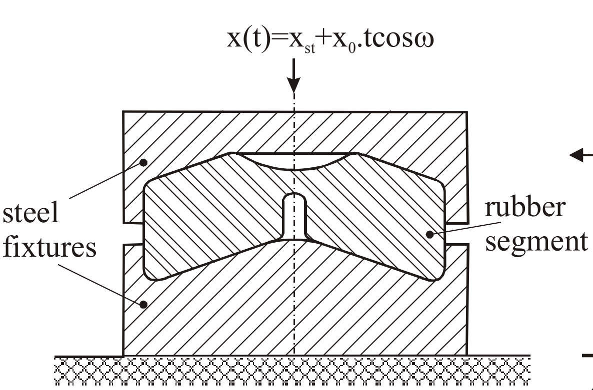

Fig.1 Mounting of the tested sample

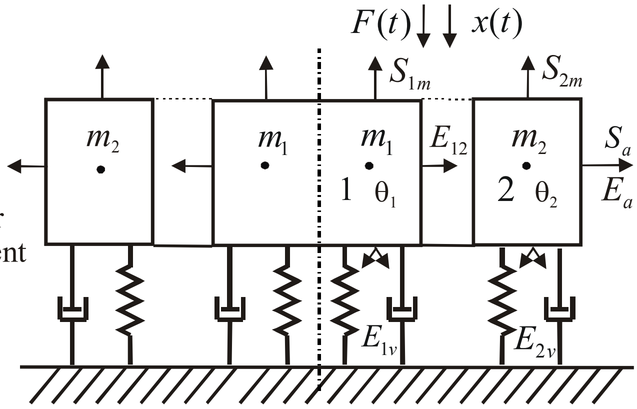

Fig.2 Scheme of mathematical model

2.1. Analysis of harmonic excitation

The element (m_1+m_2) is subjected to the common deformation $x(t)$, which we suppose to be oscillating

$$x(t) = x_{st} + x_0 \cos\omega\, t\,, \tag{1}$$

where x_{st} is the prestress, the second term gives the harmonic motion. The pressure force on the upper surface is

$$F(t) = F_{st} + F_0 \cos(\omega t - \varphi) = F_{st1} + F_{01}\cos(\omega t - \varphi_1) + F_{st2} + F_{02}\cos(\omega t - \varphi_2)\,. \tag{1a}$$

Because the body after deformation has different temperature inside and on the surface, let us split it into two parts, which can be supposed to be of the equal size and have the equal upper surfaces

$$S_1 = S_2 = S\,. \tag{2}$$

Material of both parts is the same and therefore the rheological properties have equal dependence on temperature

$$k_1(\theta, x_{st}) = k_2(\theta, x_{st}) = k(\theta, x_{st})\,,\; b_1(\theta, x_{st}) = b_2(\theta, x_{st}) = b(\theta, x_{st}) \tag{3}$$

Due to the different temperature in both parts, the instantaneous values should be different

$$k(\theta_1, x_{st}) \neq k(\theta_2, x_{st}) \qquad \text{and} \qquad b(\theta_1, x_{st}) \neq b(\theta_2, x_{st})\,. \tag{3a}$$

Rheological properties of rubber and similar synthetic materials are nonlinear and described by sophisticated terms. The functions (3) and (3a) can be considered as equivalent linear coefficients in the equivalent linearized system [2]. In the following stage of research, after gaining sufficient exact experimental data, the mode precise nonlinear models will be derived and more true nonlinear characteristics could be introduced into differential equations.

The variable x_{st} in the terms (3) and (3a) indicates that the rheological properties of investigated material vary with prestress.

The lost energy in the both parts of damping component in the time period $T = 2\pi/\omega$ is

$$E_b = E_{b1} + E_{b2} = \int_0^T \left[b(\theta_1, x_{st}) + b(\theta_2, x_{st}) \right] \dot{x}^2 dt \,. \tag{4}$$

If the motion is harmonic the lost energy in one cycle is

$$E_b = \pi \left[b(\theta_1, x_{st}) + b(\theta_2, x_{st}) \right] \omega\, x_0^2 \,. \tag{4a}$$

The energy lost in one second at frequency $f = \omega/2\pi$ is:

$$E_{b1s} = E_b . f = \tfrac{1}{2} \left[b(\theta_1, x_{st}) + b(\theta_2, x_{st}) \right] \omega^2 x_0^2 \,. \tag{5}$$

This energy equals to the increments of heat in 1*s*

$$E_{b1s} = \Delta E_{\theta 1} + \Delta E_{\theta 2} \qquad [Nm/s = kgm^2 s^{-3}] \tag{5a}$$

This heat escapes partly into environment, partly causes the increase of sample temperatureθ . The heat balance of inner (*i*=1) and outer (*i*=2) part is

$$\Delta E_{\theta 1} = \Delta E_{1v} + \Delta E_{1m} + \Delta E_{12} \tag{6}$$

$$\Delta E_{\theta 2} = \Delta E_{2v} + \Delta E_{2m} + \Delta E_a - \Delta E_{12} \,,$$

where $\Delta E_{\theta i} =$ increment of heat results from inner damping in 1*s* [kgm^2s^{-3}] i = 1,2.

$\Delta E_{iv} =$ increment of inner heat, staying in the body and increasing its temperature:

$$\Delta E_{iv} = c_v V_i \Delta\theta_i \quad i = 1, 2 \tag{7a}$$

$\Delta E_{im} =$ heat, which is drained by the metal parts of system:

$$\Delta E_{im} = S_{im} \alpha_m (\theta_i - \theta_m) \quad i = 1, 2 \tag{7b}$$

$\Delta E_{23} =$ heat, which is drained form the part 2 into air:

$$\Delta E_a = S_a \alpha_a (\theta_2 - \theta_a) \tag{7c}$$

$\Delta E_{12} =$ heat transfers from inner (1) to outer part (2):

$$\Delta E_{12} = S_{12} \alpha_{12} (\theta_1 - \theta_2) \tag{7d}$$

We suppose that temperature θ_m of metal parts and $\theta_a$of air are constants as well as the coefficients of heat transfer $\alpha_m, \alpha_a, \alpha_{12}$ and areas S_{im}, S_a, S_{12}. Temperaturesθ_1 and θ_2 of both parts vary in time. Specific heat c_v [$kg.m^{-1}.s^{-3}K^{-1}$] and volumes $V_1=V_2=V$ of both parts are equal.

From the sums (6) with respect to (5) and (7a)-(7d) we get the increments of temperature in parts 1 and 2.

$$\Delta\theta_1 = \frac{1}{Vc_v}\left[\frac{1}{2}b(\theta_1, x_{st})\omega^2 x_0^2 - S_{1m}\alpha_m(\theta_1 - \theta_m) - S_{12}\alpha_{12}(\theta_1 - \theta_2)\right] \quad (8)$$

$$\Delta\theta_2 = \frac{1}{Vc_v}\left[\frac{1}{2}b(\theta_2, x_{st})\omega^2 x_0^2 - S_{2m}\alpha_m(\theta_2 - \theta_m) - S_a\alpha_a(\theta_{21} - \theta_a) + S_{12}\alpha_{12}(\theta_1 - \theta_2)\right]$$

The relations among rheological parameters E, ν in the equation

$$\sigma = E\varepsilon + \nu d\varepsilon / dt \quad (9)$$

and stiffness k and damping b in the equation

$$F = kx + bdx/dt \quad (9a)$$

are

$$k = \frac{S_c}{l}E, \qquad b = \frac{S_c}{l}\nu\ . \quad (10)$$

Here $S_c = 2S = S_1 + S_2$ is the pressed cross-section, l is length of the sample.

Let the dependences of b and k on the temperature θ and prestress x_{st} are simple functions

$$b = b_0 + b_1\theta + b_2 x_{st} \quad (11)$$

$$k = k_0/(1 + k_1\theta) + k_2 x_{st}\ .$$

Then the periodic force $F(t)$ realizing the motion $x_{st} + x_0 \cos\omega t$ is given as a sum of forces acting on parts 1 and 2:

$$F(t) = \left[k_0/(1 + k_1\theta_1) + k_0/(1 + k_1\theta_2) + 2k_2 x_{st}\right](x_{st} + x_0\cos\omega t) + \quad (12)$$
$$+\left[2b_0 + b_1(\theta_1 + \theta_2) + 2b_2 x_{st}\right](-\omega x_0 \sin\omega t)\ .$$

This force depends both on the amplitude and frequency of excitation and on the temperatures

θ_1 and θ_2 of both parts of resilient element. These temperatures in the time $t = NT = N2\pi/\omega$ can be ascertained from equations

$$\theta_1(t) = \theta_0 + \sum_{n=1}^{N}\Delta\theta_{1n}, \qquad \theta_2(t) = \theta_0 + \sum_{n=1}^{N}\Delta\theta_{2n}, \quad (13)$$

where the increments of temperature in one period are

$$\Delta\theta_{1n} = \frac{1}{Vc_v}\left\{(b_0 + b_2 x_{st})\omega^2 x_0^2/2 + S_{1m}\alpha_m\theta_m + \left[b_1\omega^2 x_0^2/2 + S_{1m}\alpha_m - S_{12}\alpha_{12}\right]\cdot\right.$$
$$\left.\cdot\theta_{1,n-1} + S_{12}\alpha_{12}\theta_{2,n-1}\right\} \quad (13a)$$
$$\Delta\theta_{2n} = \frac{1}{Vc_v}\left\{(b_0 + b_2 x_{st})\omega^2 x_0^2/2 + S_{2m}\alpha_m\theta_m + S_a\alpha_a\theta_a + \left[b_1\omega^2 x_0^2/2 + S_{2m}\alpha_m -\right.\right.$$
$$\left.\left. - S_a\alpha_a + S_{12}\alpha_{12}\right]\theta_{2,n-1} - S_{12}\alpha_{12}\theta_{1,n-1}\right\}$$

Index n-1marks the values in previous period.

2.2. Analysis of non-stationary motion

Aforementioned equations are suitable for cases, of loading the resilient element by harmonic deformation with constants or slowly changing amplitude. At general, non-stationary loading $x(t)$, the above-mentioned procedure has to be applied on a shorter time interval than is one period.

Deformation let be supposed $x_{st}+x(t)$, where the constant prestress x_{st} is added to the time variable component. The mechanical and thermal processes are solved in a short time interval Δt, when in the increase of deformation is Δx at velocity $\dot{x}=\Delta x/\Delta t$ and the change of temperature $\theta_i(t)$ is $\Delta\theta_i$.

Resistance of resilient element is supposed in nonlinear form

$$F = f(x,\theta) + g(x,\dot{x},\theta) = S_1\sigma(x,\theta_1) + S_2\sigma(x,\theta_2) + S_1\sigma_g(x,\dot{x},\theta_1) + S_2\sigma_g(x,\dot{x},\theta_2), \quad (14)$$

where $f(x,\theta), \sigma(x,\theta_i)$ are elastic (reversible) components of force, $g(x,\dot{x},\theta), \sigma_g(x,\theta_i)$ are damping (irreversible) components of force.

Elastic components of force are potential ones and they do not produce any thermal energy during the closed cycle of deformation. This energy results from the work of damping components. Increment of the lost energy in time interval Δt is

$$\Delta E = \Delta E_1 + \Delta E_2 = \lfloor S_1\sigma_g(x,\dot{x},\theta_1) + S_2\sigma_g(x,\dot{x},\theta_2)\rfloor \Delta x\,. \quad (15)$$

Increments of energy (that are increments of heat $\Delta E_{i\theta}$) consist of particular heat described by equations (7a) and (7b), (7c), (7d), that have to be multiplied by the time interval Δt:

$$\begin{aligned} \Delta E_{im} &= S_{im}\alpha_m(\theta_i - \theta_m)\Delta t \qquad i = 1,2 \\ \Delta E_a &= S_a\alpha_a(\theta_2 - \theta_a)\Delta t \\ \Delta E_{12} &= S_{12}\alpha_{12}(\theta_1 - \theta_2)\Delta t \end{aligned} \quad (16)$$

Interments of temperature of parts 1 and 2 in the time interval Δt are

$$\begin{aligned} \Delta\theta_1 &= (\Delta E_{1\theta} - \Delta E_{1m} - \Delta E_{12})/(Vc_v) \\ \Delta\theta_2 &= (\Delta E_{2\theta} - \Delta E_{2m} - \Delta E_{2a} + \Delta E_{12})/(Vc_v) \end{aligned} \quad (17)$$

so that the temperatures in time t are

$$\theta_1(t) = \theta_0 + \sum_{n=1}^{N}\Delta\theta_{1n}\,, \theta_2(t) = \theta_0 + \sum_{n=1}^{N}\Delta\theta_{2n} \quad (18)$$

where $N = t/\Delta t$, $\Delta\theta_{in}$ $(i = 1,2)$ are temperature increments calculated for $\theta_1((n-1)\Delta t)$.

Applying formulae (7a), (15) and (16) we obtain in time $t = N\Delta t$

$$\begin{aligned} \theta_1(t) = \theta_0 + \sum_{n=1}^{N}\Big[&S_1\sigma_g\big(x(n_1\Delta t), \dot{x}(n_1\Delta t), \theta_1(n_1\Delta t)\big)\Delta x(n_1\Delta t) - \\ &- S_{1m}\alpha_m\big(\theta_1(n_1\Delta t) - \theta_m\big)\Delta t - S_{12}\alpha_{12}\big(\theta_1(n_1\Delta t) - \theta_2(n_1\Delta t)\big)\Big]/(Vc_v) \end{aligned}$$

(19)

$$\begin{aligned} \theta_2(t) = \theta_0 + \sum_{n=1}^{N}\Big[&S_2\sigma_g\big(x(n_1\Delta t), \dot{x}(n_1\Delta t), \theta_2(n_1\Delta t)\big)\Delta x(n_1\Delta t) - S_{2m}\alpha_m\big(\theta_2(n_1\Delta t) - \theta_m\big)\Delta t - \\ &- S_a\alpha_a\big(\theta_2(n_1\Delta t) - \theta_a\big)\Delta t - S_{12}\alpha_{12}\big(\theta_1(n_1\Delta t) - \theta_2(n_1\Delta t)\big)\Delta t\Big]/(Vc_v) \end{aligned}$$

where $n_1 = n - 1$, $\Delta x(n_1\Delta t) = \dot{x}(n_1\Delta t)\Delta t$.

These equations can be rearranged using values of temperature θ_1, θ_2 and deformation $x, \dot{x}$ in previous step and using increment solution

$$\begin{aligned} \theta_1(n\Delta t) - \theta_0 = \theta_{1,n-1}, + \lfloor &S_1\sigma_g(x_{n-1}, \dot{x}_{n-1}, \theta_{1,n-1})\dot{x}_{n-1}, - \\ &- S_{1m}\alpha_m(\theta_{1,(n-1)} - \theta_m) - S_{12}\alpha_{12}(\theta_{1,n-1} - \theta_{2,n-1})\rfloor \Delta t/(Vc_v) \end{aligned}$$

$$\theta_2(n\Delta t)-\theta_0=\theta_{2,n-1},+\left[S_2\sigma_g(x_{n-1},\dot{x}_{n-1},\ \theta_{2,n-1})\dot{x}_{n-1},-S_{2m}\alpha_m(\theta_{2,n-1}-\theta_m)-\right.$$
$$\left.-S_{12a}\alpha_a(\theta_{2,(n-1)}-\theta_a)-S_{12}\alpha_{12}(\theta_{1,n-1}-\theta_{2,n-1})\right]\Delta t/(Vc_v)\,. \quad (20)$$

Temperatures $\theta_{1,n}(t)=\theta_1(n\Delta t)$ and $\theta_{2,n}(t)=\theta_2(n\Delta t)$ determined from (20) can be used for ascertaining force *F*. The formulae (14) contain nonlinear functions $\sigma(x,\theta_i)$ and $\sigma_g(x,\dot{x},\theta_i)$ describing the thermo-rheological properties of rubber-like material. Comparing of calculated relations $\theta_i(t)$ with measured ones enable to identify the unknown functions $\sigma(x,\theta_i)$ and $\sigma_g(x,\dot{x},\theta_i)$. These functions will be in the further text supposed in the same form as in [1]:

$$S_1\sigma(x,\theta)=K_0x/(1+K_1\theta)$$
$$S_1\sigma_g(x,\dot{x},\theta)=(B_0+B_1\theta)\dot{x} \quad (21)$$

with free parameters K_0, K_1, B_0, B_1.

2.3. Free motion

Equations (20) can be used also for description of thermo-mechanical process without loading of external forces or deformation. At given initiate thermal condition $\theta_1(0)$ and $\theta_2(0)$ the temperatures of unloaded system $(\sigma(x,\theta)=0,\sigma_g(x,\dot{x},\theta)=0)$ vary according to the equations (20) without the first terms in square brackets. Thermal constants (or functions) $\alpha_m,\alpha_a,\alpha_{12}$ can be easily ascertained from the measured non-stationary decrease of temperature and comparison with numerical solution of abbreviated equations (20).

3. Examples

3.1 Numerical investigated effect of parameters.

The influence of different parameters of thermomechanical system on the heat distribution can be investigated by means of mathematical model (20). Temperature of absolutely isolated system, with all coefficients of heat transfer equal zero $(\alpha_m=\alpha_a=\alpha_{12}=0)$, the increase of temperature is given by the straight line. When the excitation ends, the temperature becomes constant. Increasing heat transfer into environment causes declination and bending of curve $\theta\ (t)$. Fig. 3 shows these curves for system where the heat transfer is realized only by contact with metal parts $\alpha_m(0, 200, 400)$ but no heat leaks into air, $\alpha_a=0$. System with strong heat drain can reach the balanced state, when the input and output of heat is equal, in short time as it is seen in the curve for $\alpha_m=400$ for $t\in(500, 750)$, where the temperature is rather stable $\theta\doteq 25.9\,°\mathrm{C}$. After time *t*=750s the excitation ends and the temperature decreases to the environmental temperature $\theta_s=23\,°\mathrm{C}$.

Additional leakage of heat by means of air cooling $\alpha_a=50$, at heat exchange between both parts of resilient element $(\alpha_{12}=10)$ causes the different time courses of $\theta_1(t)$ and $\theta_2(t)$ as it is seen from Fig. 4. Temperature of inner part 1 is always higher then that of outer part 2. This difference depends on the ratio α_a/α_m and increases with this ratio.

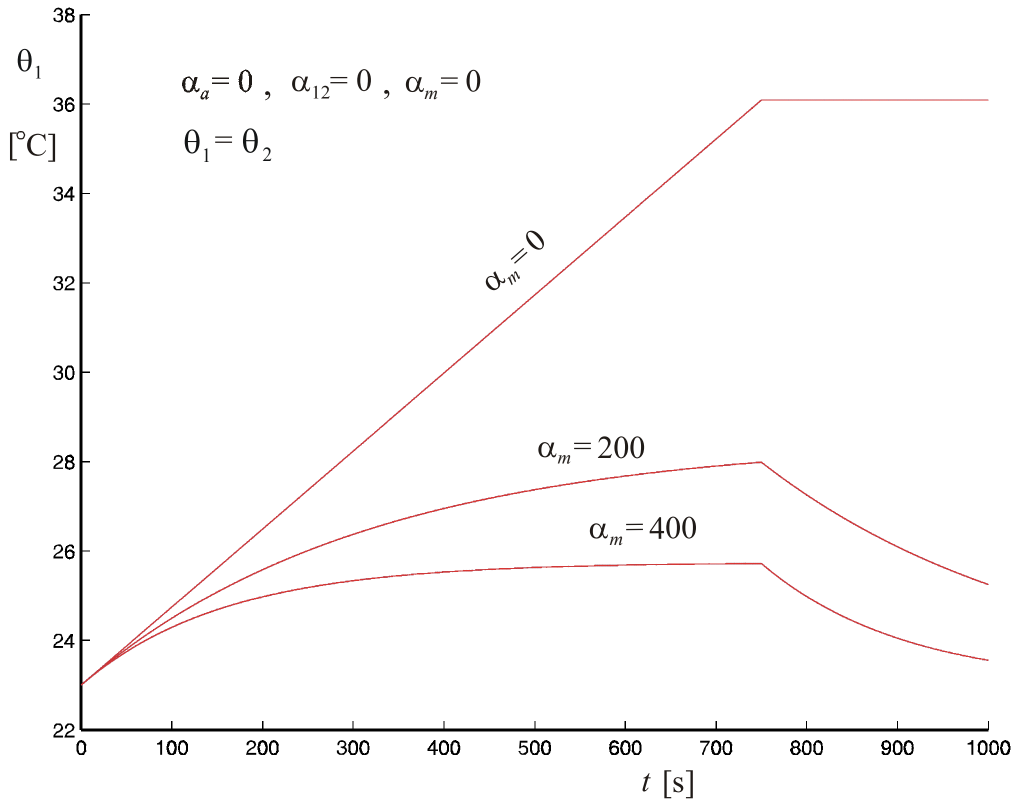

Fig. 3 Temperature versus time diagrams concerning the only heat transfer into metal

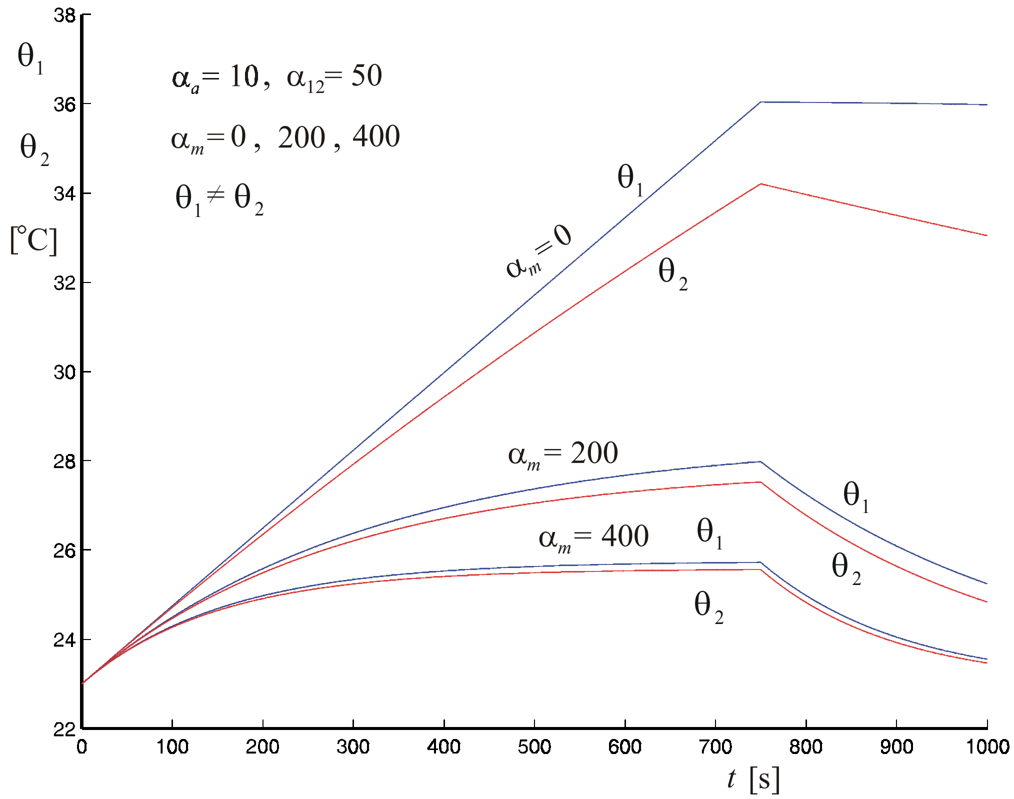

Fig. 4 Temperature versus time diagrams concerning heat transfers into metal, air and inside resilent material

3.2. Method of testing

The method, evaluation and results of tram wheel rubber segment testing were published in [4]. Therefore, the only brief description of the test will be given here. The test was realized on the test rig Hydropuls PSB 250 with servo valve type PL 250 N that is part of electro-hydraulic shaker Schenck. The loading was controlled by drift of piston rod. During the tests the force response, drift of piston rod, inner segment temperatures and strains were picked up. The time records of these signals were recorded by the digital scope YOKOGAWA at sample frequency 500Hz. The segments were statically loaded by pre-stress 2.5kN, 6kN and 10kN. For each pre-stress the dynamic tests with sinus loadings with excitation frequency 5Hz, 10Hz, 20Hz and 30Hz were performed. The amplitudes of sinus excitation were 1 and 2kN. The segments were fixed in steel fixtures (see Fig.1) that simulate their position in the tram wheel. The measurements run at the temperature of surrounding (about 23°C). For measurement of temperature two single ended bead-type thermistors with sensitivity ≈ 3.6 ^{0}C/V of type SEMI833 ET, Hydrotec Messtechnik were used. These thermistors serve for measurement in range –40° up to 120°C, however, they have non-linear character and lower sensitivity in temperature above 40°C. For accurate measurement in this range the regression curve has to be used. Thanks to very small dimensions (∅ 1,5 mm) they enable measurement of fast temperature changes with time constant $\tau < 0{,}7\,\mathrm{s}$.

As to temperature measurements of the segment two thermistors were embedded into it. For evaluation of inner heat transfer one of them, designated TU, was placed in the middle of segment and one, TK, close to the surface of segment. The loading started when these temperatures were stabilized on the same level. Then the loading time block started. When the segment became heat balanced and inner temperature changes slowed down substantially (about after 10 minutes), the loading was switched and the process of segment's free cooling started. The case of the harmonic loading 30Hz, pre-stress loading 6kN, dynamic amplitude 2kN was used for the identification presented in this paper.

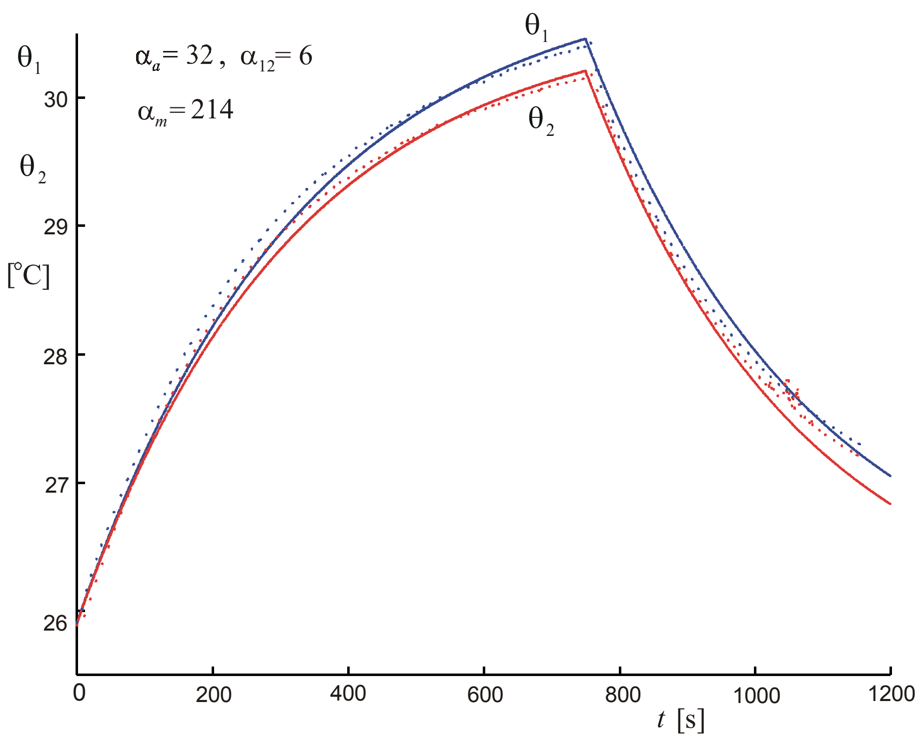

Fig. 5 Comparison of experimental (dotted) and calculated (solid lines) results

3.3 Identification of parameters

Developed mathematical model was applied for identification of some heat-transfer parameters of the resilient element. The tangent of curve $\theta(t)$ in the beginning of harmonic loading (Fig. 5, time *t*=0) enables to determine the coefficient of heat capacity of rubber-like material at room temperature $\theta_r = 23$ °C. The value of maximum temperatures θ_1, θ_2 and their difference $\theta_1 - \theta_2$ enable to estimate coefficient α_m in the main heat drain in the contact surface of rubber with metal : $\alpha_m = 214$, as well as coefficients for inner heat transfer $\alpha_{12} = 6$ and heat transfer into air $\alpha_a = 32$. These coefficients were also verified by the exponent-like curves of temperature decrease $(t = 750 - 1150\,\mathrm{s})$ at free cooling without external loading.

4. Conclusion

A new mathematical model of thermo-mechanical properties of oscillatory loaded rubber-like element was developed, analysed and verified by experimental test. This model based on 2DOF system includes the heat source caused by the inner material damping, heat flows to the metal parts, and to the surrounding air.

Numerical incremental solution enables to ascertain the temperature versus time functions at harmonic deformation as well as at free cooling without any external loading. The possibility of parametric identification of thermo-mechanical constants is presented.

Elaborated method of solution will be further extended for investigation of more complicated system and for application to damping elements for reduction of noise and vibrations of railway and tram wheels.

5. Acknowledgement: *This research has been solved in a frame of the grant GA CR no.101/02/0241.*

6. References:

[1] Pešek L., Půst L., Vaněk F.:Contribution to identification of thermo-mechanic interaction at vibrating rubber-like materials. In: IFAC Symposium on System Identification /13./ : preprints. - (Ed. Van den Hof, P. M. J.; Walhlberg, B.; Weiland, S.). - Rotterdam, Omnipress, 2003, pp. 479-484.
[2] Brepta R., Půst L., Turek F.: Mechanické kmitání, Sobotáles, Praha, 1994
[3] Nicholson, D.W.: Finite Element Analysis – Thermomechanics of Solids, CRC Press, 2003.
[4] Pešek L., Vaněk F., Veselý J., Cibulka J.: Experimental research of thermo-mechanical behavior of tram wheel rubber segment, In: Dynamic of Machines *2004*, Institute of Thermomechanics AS CR, 2004, Prague, pp. 97-104, National conference with international participation (in Czech).
[5] Schätz M.,Vondráček P.: Polymer testing, Institute of Chemical Technology, Prague, 1988, text book (in Czech) .