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RESPONSE OF A HARMONICALLY EXCITED COMPONENT FIXED TO A RUBBER MOUNT - INFLUENCE OF THE RUBBER MOUNT PROPERTIES RANDOM VARIABILITY

F. Fridrich[1], J. Menčík[2]

Summary: *The paper deals with sensitivity analysis of the engine injection unit electronics mounting. The presented analysis employs the Monte Carlo method and the Response Surface method to calculate variability of transmission of the vibration from engine to the injection unit electronics.*

1. Introduction

Response of elastically mounted bodies depends on parameters of the mounted body, the excitation frequency and on stiffness and damping of mounts. Stiffness and damping of rubber products vary through production reasons; furthermore, both the dynamic stiffness and damping depend on frequency. In the design phase of a dynamic system it is important to determine effects of variation of the design parameters. In this paper, an analysis of variation of the vibration transmission caused by variation of the rubber mount stiffness and rubber damping is shown on an example of the injection unit electronics mounting. The sensitivity analysis is performed by means of the Monte Carlo method and the Repose Surface Method. A relationship between hardness and stiffness of a rubber mount is calculated by means of finite element method (FEM). For generality, the vibration transmission is expressed in a dimensionless form. In this paper, the injection unit electronics mounting is investigated as a single degree of freedom system.

2. Variability and quality

Variability is a key measure of quality. That common knowledge is quoted by many authors. Deming and Taguchi relate variability to losses by function

$$L = k \cdot (x - m)^2 , \qquad (1)$$

where x is an independent variable, m is the target value and k is a constant. Function (1) says that any deviation from the required value results in loss. According to Deming, quality

[1] Ing. František Fridrich, Rubena a.s., Akademika Bedrny, 502 00 Hradec Králové, tel.: +420 495 753 528, fax: +420 495 753 210, e-mail: frantisek.fridrich@rubena.cgs.cz , + Univerzita Pardubice, DFJP, Studentská 95, 532 10 Pardubice, Czech Republic
[2] Prof. Ing. Jaroslav Menčík, CSc., Univerzita Pardubice, DFJP-KPSDM, Studentská 95, CZ – 532 10 Pardubice, Czech Republic, e-mail: jarda.mencik@upce.cz.

has to be imbedded in the product during the design phase. In the product life cycle phases following the design phase, quality could only be maintained not improved.

3. Methods used in calculation of rubber product variability

Knowledge of influence of the design parameters variability on the required parameters variability brings the highest benefit when known in the product design phase. Such information can provide the sensitivity analysis carried out in the first stages of the product design.

Among methods used in the sensitivity analysis belong the Monte Carlo method and the Response Surface method. Both the Monte Carlo and the Response Surface methods are used in this paper for calculation of variability of the vibration transmission.

The Monte Carlo method is based on multiple repetition of a trial. The Monte Carlo method is used in many different fields from integration and optimization to simulation of physical phenomenons, (Reuven, 1981).

The objective of the Response Surface method is to find a function that sufficiently approximates an unknown function, (Box, 1987).

4. Sensitivity analysis of injection unit electronics mounting

The engine injection unit electronics consists of a fixing plate and electronic components attached to the plate. The injection unit electronics fixing plate is fixed by means of rubber mounts to the engine. In operation, the engine vibration causes damage of the electronic components and the injection unit electronics emits sound.

Due to variability of some parameters during rubber mount production the properties of mounts such as the static force-deflection curve, the dynamic stiffness and loss angle change as well. Consequently the vibration transmission of the injection electronics mounting varies too. Variability of the vibration transmission is examined with respect to variability of rubber hardness and rubber loss angle.

Introduction to the computational model

A sketch of the injection electronics mounting is shown in Fig. 1 and a computational model is drawn in Fig. 2. The injection electronics mass, including the fixing plate mass, is supposed to be $m = 1.8\,kg$. The rubber mount is manufactured of rubber of hardness $Hd = 55 \pm 3\ Shore\ A$. The rubber loss angle is obtained from the measurement on the testing specimens in simple shear. The relation between rubber loss angle φ and frequency f, in the range of frequencies $f \in <10; 200> Hz$, is given by the first order polynomial

$$\varphi = b_0 + b_1 \cdot f\ , \qquad (2)$$

where $b_0 = 5.79$, $b_1 = 0.0214$. The loss angle variation is supposed to be constant in the frequency range $f \in <10; 200> Hz$ and given by the variation coefficient $v_f = 0.04$.

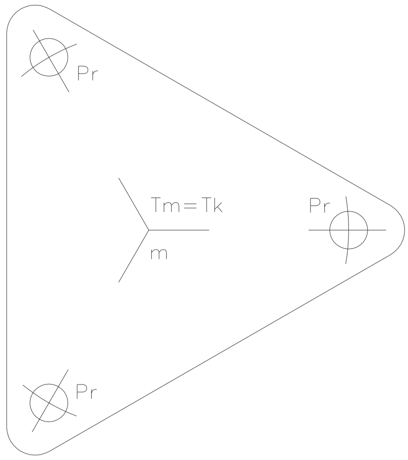

Fig. 1 Sketch of the injection electronics mounting. Pr – rubber spring, Tm – center of gravity, Tk – center of stiffness.

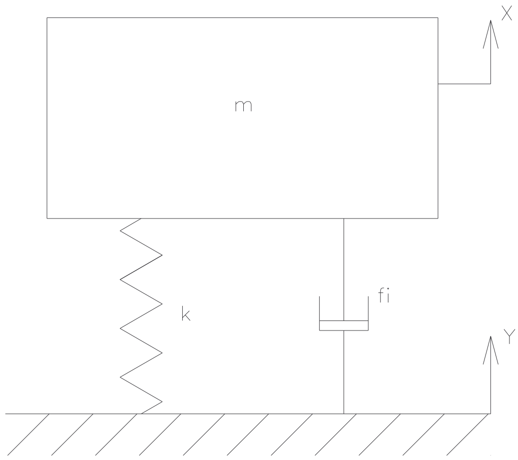

Fig. 2 Sketch of the computational model of the injection electronics mounting.

In the presented sensitivity analysis we suppose small vibration of the injection unit electronics round an equilibrium position. Variability of the vibration transmission is evaluated for harmonic excitation of a single degree of freedom system. The transmission is given by formula

$$\frac{X}{Y} = \sqrt{\frac{1+(2\cdot\xi\cdot\rho)^2}{(1-\rho^2)^2+(2\cdot\xi\cdot\rho)^2}}\,, \qquad (3)$$

where X is the amplitude of motion of the injection unit electronics, Y is the amplitude of motion of the engine, $\xi = c/c_c = tg(\varphi)/2$ is the damping ratio, $\rho = \omega/\Omega$ is the ratio of the exciting circular frequency to the undamped natural circular frequency, $\omega = 2\cdot\pi\cdot f$ is the exciting circular frequency, f is the exciting frequency, $\Omega = \sqrt{k/m}$ is the undamped natural circular frequency and $\Omega_T = \Omega\cdot(1-c/c_c)^{1/2}$ is the damped natural circular frequency. In the sensitivity analysis we investigate frequency in the range $f \in <10; 200> Hz$.

The relation between rubber hardness Hd and the stiffness of a rubber mount k is calculated by means of the finite element method and approximated by the first order polynomial

$$k = a_0 + a_1 \cdot Hd, \tag{4}$$

where $a_0 = 2813$, $a_1 = 68.3$.

The sensitivity analysis is carried out by means of the program MATLAB. Fig. 3 and 4 show the graphical representation of the transmission in coordinates $Hd - f$ and $\varphi - f$. The blue line in Fig. 4 and 5 is an intersection line of the transmission X/Y and the plane $f = const = 100\ Hz$.

<u>Presentation of the analysis results</u>

In Fig. 5 to 7, the distribution of the transmission X/Y is drawn in the frequency range $f \in <10; 200> Hz$. Fig. 8 shows a histogram of the transmission X/Y at frequency $f = 100\ Hz$ in case rubber hardness Hd and rubber loss angle φ are considered to be random quantities.

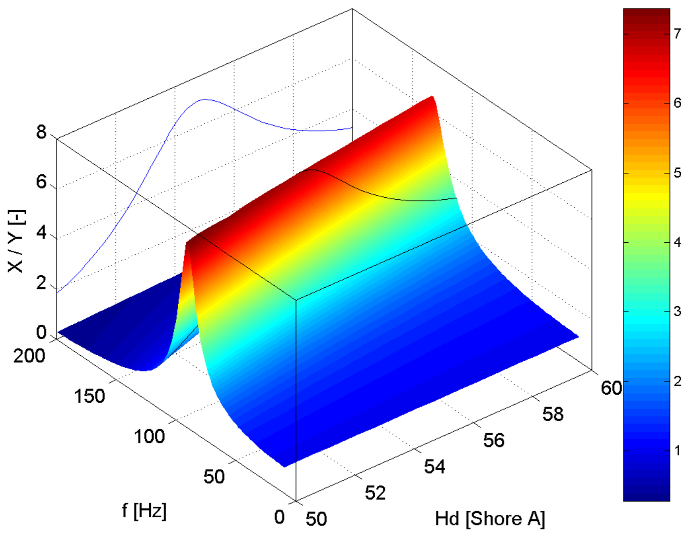

Fig. 3 Transmission X/Y in coordinates of the rubber hardness Hd and the exiting frequency f.

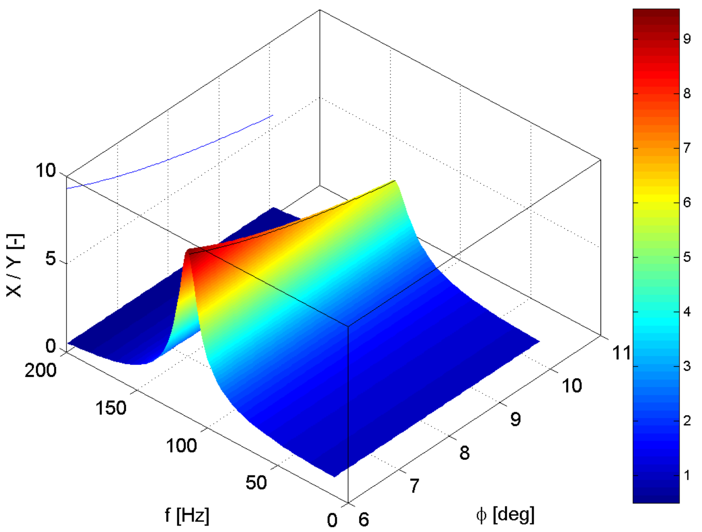

Fig. 4 Transmission X/Y in coordinates of the rubber loss angle φ and the exiting frequency f.

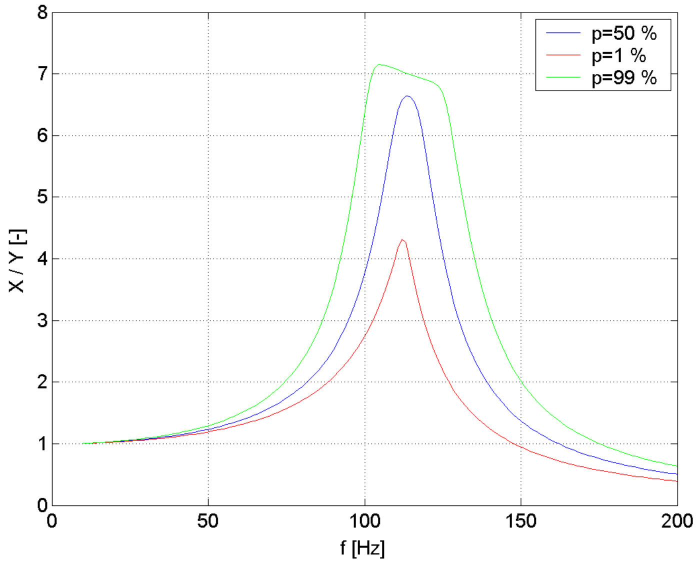

Fig. 5 Transmission X/Y. The rubber hardness Hd is considered to be a random quantity and the rubber loss angle φ is considered to be a deterministic quantity.

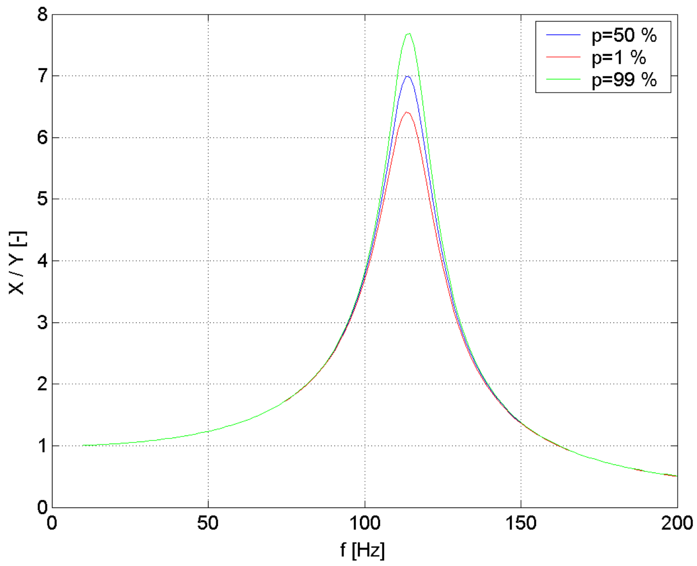

Fig. 6 Transmission X/Y. The rubber loss angle φ is considered to be a random quantity and the rubber hardness Hd is considered to be a deterministic quantity.

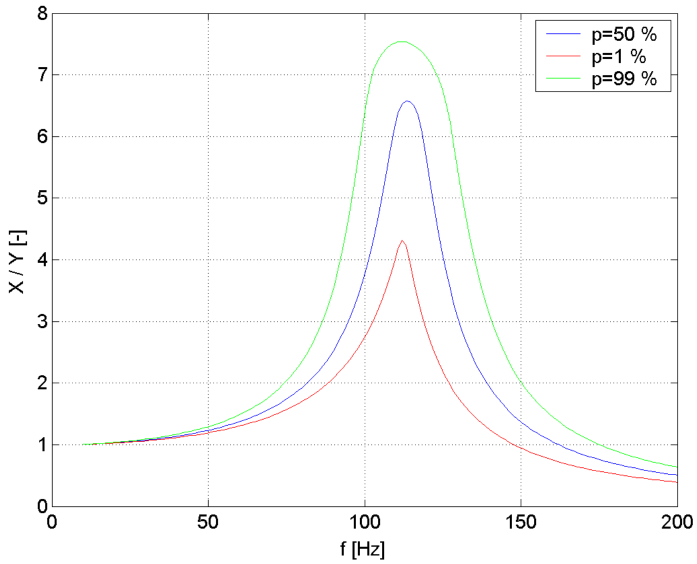

Fig. 7 Transmission X/Y. Both the rubber hardness Hd and the rubber loss angle φ are considered to be random quantities.

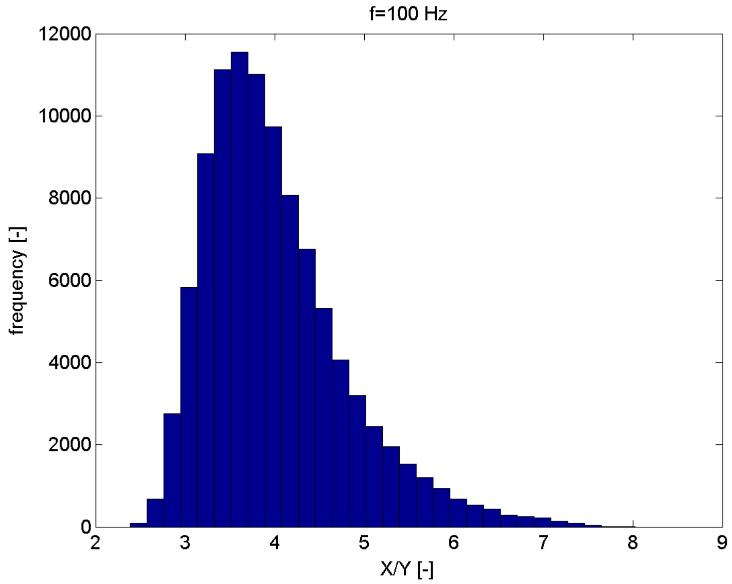

Fig. 8 Histogram of transmission X/Y at excitation frequency $f = 100\ Hz$. Both the rubber hardness Hd and the rubber loss angle φ are considered to be random quantities.

5. Conclusion

The influence of variability of rubber hardness and loss angle of rubber on variability of vibration transmission was examined on the injection unit electronics mounting. The graphs in Fig. 5 to 7 show $p = 1$ and $99\ \%$ quantiles of the transmission X/Y, with regards to random and deterministic character of the rubber hardness and loss angle.

Asymmetry of the histogram of transmission drawn in Fig. 8 is caused by shape of the transmission surface graphically represented in Fig. 4.

6. Literature

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