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DEVELOPING DAMAGE MODELS FOR CONCRETE

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Summary: *The aim of this paper is to characterize the recent methods grounded on damage and fracture mechanics, for the structural analysis of plain concrete and reinforced one. Both smeared and also localized numerical fracture simulations are contemplated to model the damage regions and the discrete cracks generating in loaded concrete elements. In dependence on the dimension of these members, eligible constitutive material laws may be postulated, extending from linear elasticity or viscoelasticity for large construction, to non-linear strain-softening or plasticity for smaller components. Special loading conditions are embraced if the use of fracture mechanics is proper ; e.g., shearing, punching and anchorage pulling – out.*

1. Introduction

Stress-strain constitutive equations depicting softening result in mesh-unobjective issues except when associated with a localization criterion or localization limiter. For that reason we suppose that stress-strain relations based on continuum damage formulations are always connected with such a localization criterion, which, to be particularized, can be selected to be a band concept so that we contemplate (x, x, b) models in compliance with their classification scheme. We analyze the manner in which one may model the bulk characteristics and the behaviour of the material in the fracture zone by continuum mechanics conceptions.

It is worthy of mention that the restriction we foist on ourselves of considering band models entirely is mostly random for the essential interpretations of damage theory may be applied to create stress-crack displacement expressions for the crack approaches. Besides, the possibility is still open of finding localization criteria more general than those of crack or band simulations.

2. CDM model parts

General continuum damage model has three fundamental segments:

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a. A set of independent internal variables, p_k, which in company with the infinitesimal strain tensor ε (or the stress tensor σ) are supposed to specify uniquely – the momentary state of the solid at a given point.

The internal variables may stand for a physical quantity or be abstract in nature. They can be related to kinematic properties or to structural characteristics. E.g., the vector e^c in the smeared crack model is meant to represent the internal kinematics of cracks and n, the crack direction, is a structural internal variable. It has to be noted that when a set of internal variables is selected, any other set, biunivocally related to the first, is rigorously equivalent to it and, that is why, can be employed instead of the first. This makes the physical interpretation of a given set of internal variables marginally equivocal.

b. An equation set expressing the stress to the strain and to the internal variables:

$$\sigma = \mathbf{S}(\varepsilon, p_k) \tag{1}$$

In up-to-date thermodynamic formulations, Eq. (1) is deduced from a free energy function that represents a scalar function to be determined instead of (1). As a rule, Eq. (1) is assumed to be linear in the infinitesimal strain tensor

$$\sigma = \Lambda(p_k)\varepsilon - \mathbf{T}(p_k) \tag{2}$$

where Λ (p_k) is a fourth-order tensorial function being subject solely to internal variables, and $T(p_k)$ is a second-order tensorial function of the internal variables. If **T** is **0**, the zero second order tensor, and $\boldsymbol{\Lambda}(p_k) = \boldsymbol{\Lambda}_0$ is constant, we get a classical elastic characteristics. When **T** changes, and $\boldsymbol{\Lambda}(p_k) = \boldsymbol{\Lambda}_0$ is constant, a model depicting flow-stress degradation but for stiffness degradation is won. If **T** is 0, the zero second order tensor, and $\boldsymbol{\Lambda}(p_k)$ is variable, a model displaying stiffness degradation is obtained that all the time unloads to the origin (σ = **0** for ε = **0**, and reciprocally). If both $\boldsymbol{\Lambda}(p_k)$ and $\mathbf{T}(p_k)$ are variable, we win a general damage model. This permits us to classify material characteristics (see the following table).

Table 1 Analogy between ranking proposed and features of functions $\boldsymbol{\Lambda}(p_k)$ and $\mathbf{T}(p_k)$

BEHAVIOUR	KIND OF MODEL	$\Lambda(p_k)$	$T(p_k)$
BULK	a	Variable	Variable
	b	Variable	0
	c	Constant up to peak load	0
FRACTURE ZONE	a	Variable	Variable
	b	Variable	0
	c	Constant	Variable

c. A set of „flow rules“ stipulating the manner in that the internal variables enhance if loading progresses. This is a sensitive and fundamental point for prescription of diverse flow rules to simulations possessing the same set of internal variables and the same composition for the stress strein relation, Eq. (2), will result in very different characteristics. Besides, the flow rules have to be consistent with the irreversibility condition put by the Second Principle of Thermodynamics.

The flow rules may be mentioned at many grades of generality: A rather universal approach for time-independent behaviour is to apply one or more loading functions gained by direct generalization of the classical plasticity theory. In consequence, the internal forces q_k linked to the internal variables must be selected, and a loading function $F(q_k)$ stipulated and so the region in which the characteristics are elastic (ie: $dp_k = 0$ for any k) is related in the way:

$$F\,(q_k) \leq 0 \tag{3}$$

and the associated flow rules have a form:

$$dp_k = (\partial F / \partial q_k)\, d\mu\,, \qquad d\mu \geq 0 \tag{4}$$

It is evident that a hierarchical arrangement of internal variables is possible so that the primary set p_k occurring in Eq. (2) is completed by a secondary set of hardening-softening parameters going in the loading function (3). Even though this is a rather general statement it is not the nothing but one possibility. Multi-yield surface type formulations are also possible. We also can create restricted flow rules for especial loading occurrences among them the monotonic loading case is the simplest and most of use.

3. Overview of models

Some lately developed simulations come after.

a. Damage model with permanent strains and induced anisotropy

Realistic and according to the watching of the growth and direction of microcracks, this model embraces the anisotropy influence being the damage parameters, under consideration the nine independent constituents of an orthotropic fourth-order tensor $\mathbf{\Gamma}_D$. Further, a symmetric second order tensor standing for plastic strains ε^p is defined as an internal variable to simulate strain irreversibility.

The strain reads

$$\varepsilon = \mathbf{\Gamma}_D\, \sigma + \varepsilon^p \tag{5}$$

In compliance with Table 1, either a (*c, a, b*) or a (*a, a, b*) model is the matter.

F. Collombet, in conformity with Mazars (1985), restricted the identification of the damage variables to the axisymmetric eventuality when solely 4 variables are desired. The model supposes that the compressibility is constant whatever the damage grade, and damage is formed only in tensile strain states. The flow rules for Γ_D and ε^p are independent.

b. Microplane model

Bazant´s microplane simulation, according to Bazant, Lin & Pijandier (1987), employs a continuous distribution of internal variables that are kinematically conjugated. The nature of the model is the contemplation that at a microstandard, cracking arises at random directions rather than in a parallel array. In the case of normal concrete, the cracks occur through the mortar gathering around the aggregates. According to Fig. 1, a macrocrack has to run through microplanes oriented accidentally.

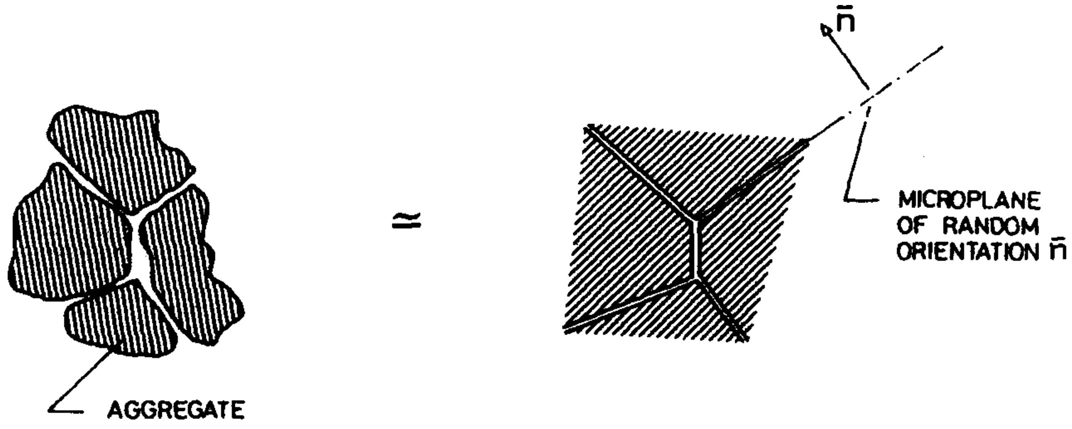

Fig. 1 Idealization of concrete structures by means of a microplane distribution

The fundamental issue is that if the microplane strains are kinematically united to the total strain, the knowledge of the fracture characteristics of a single microplane will do to piece together by integration the behaviour of all the medium.

In Bazant´s model, the role of the fundamental internal variables play the normal strains of the microplanes. These quantities, with arbitrary unit normal n, are denoted being e_n and are associated to a global microplane strain tensor ε^* by dint of the equation:

$$e_n = \varepsilon^* n \cdot n \qquad (6)$$

Next, it is supposed that the stress operating on a microplane is normal to that microplane and depends wholly on its normal strain. For monotonic microplane extension the pertinent equation may be written down:

$$t_n = f(e_n) = f(\varepsilon^* n \cdot n) \qquad (7)$$

The global stress is won by integration over the unit sphere, which, for isotropic distribution of microplanes, yields:

$$\sigma = (3/4\pi) \int t_n n \otimes n \, d\Omega_n \qquad (8)$$

where $d\Omega_n$ represents the element of solid angle round a unit normal n.

For the global strain we will get:

$$\varepsilon = \Gamma\sigma + \varepsilon^* \qquad (9)$$

where Γ is a constant fourth-order isotropic compliance tensor (not equal to the global initial elastic tensor inasmuch as the initial characteristics of the microplanes may be elastic).

The equation set describing the simulation appear rather portentous and, actually, the computations demanded for the implementation of the model are fairly troublesome, due to the need of performing the surface integral (8). However, the simulation is conceptually simple, and, over and above, it is more important, is easy to match into experimental data. This is so because the model is being grounded solely on a scalar microplane stress-strain relation and on two scalar constants. The $t_n - e_n$ relation can be chosen to demonstrate cracking, crushing, degradation of both stiffness and flow stress, and secondary internal variables may be used to set up these equations. In this sense, the microplane model involves a whole family of simulations displaying anisotropic damage in a reasonably natural manner.

c. Mixture model

The most striking characteristic of Ortiz´s model Ortiz (1987) is the dealing of concrete being a two-component mixture, the first constituent as mortar (index 1) and the second aggregate (index 2). Thereafter, the macroscopic stress is expressed as the volume average of the partial stresses of the two components and, in so much as diffusion is supposed to be prevented, the strains of the two components are the same and equal to the macroscopic strain in the following manner

$$\sigma = \alpha_1\sigma_1 + \alpha_2\sigma_2 \tag{10}$$

$$\varepsilon_1 = \varepsilon_2 = \varepsilon_3 \tag{11}$$

where σ_1 and σ_2 represent the partial stresses of the mortar and aggregates, and α_1 and α_2 are their volume fractions. The simulation is supplemented with the models for mortar and aggregate characteristics. In the simplest version, two primary internal variables were introduced. One is a fourth order tensor Γ^d, rendering the compliance change and possessing the sense of a fourth order tensorial damage, and the other stands for a plastic second order strain tensor ε_1^p. The partial stress-partial strain relationship may be expressed in the form:

$$\varepsilon_1 = (\Gamma^0 + \Gamma^d)\sigma_1 + \varepsilon_1^p \tag{12}$$

Interdependent flow rules are determined for ε_1^p and Γ^d:

$$d\varepsilon_1^p = \alpha\left(\langle\sigma_1\rangle_+ + c\langle\sigma_1\rangle_-\right)d\mu \tag{13}$$

$$d\Gamma^d = (1-\alpha)\left[\left(\langle\sigma_1\rangle_+ \bullet c\langle\sigma_1\rangle_-\right)^{-1}\langle\sigma_1\rangle_+ \otimes \langle\sigma_1\rangle_+ + c\left(\langle\sigma_1\rangle_- \bullet \langle\sigma_1\rangle_-\right)^{-1}\langle\sigma_1\rangle_- \otimes \langle\sigma_1\rangle_-\right]d\mu \tag{14}$$

where $<\sigma_1>_+$ and $<\sigma_1>_-$ mean again the positive and negative components of the partial stress tensor, ● denotes inner product of tensors and ⊗ tensorial product, α and c are constants and the multiplier μ develops in compliance with the expressions

$$\begin{aligned} d\mu > 0 \quad & \text{if } \left(\langle\sigma_1\rangle_+ \bullet \langle\sigma_1\rangle_+ + c\langle\sigma_1\rangle_- \bullet \langle\sigma_1\rangle_-\right)^{1/2 - t(\mu) = 0} \\ & \text{and } \left(\langle\sigma_1\rangle_+ + c\langle\sigma_1\rangle_-\right) \bullet \left(\Gamma^0 + \Gamma^d\right)^{-1} d\varepsilon_1 > 0 \\ d\mu = 0 \quad & \text{otherwise.} \end{aligned} \tag{15}$$

Function t (μ) represents the hardening-softening dependence that may be specified from the uniaxial tension test. The aggregate is simulated as an elastoplastic material for that the Drucker-Prager loading criterion and a non-associated plastic flow rule hold which form the simplest model being close at hand for a granular material. The final simulation after using the mixture rules is of type (*a, a, b*). And what is more, the model shows hysteretic behaviour in unloading-reloading cycles due to the coupling of mortar and aggregate characteristics.

Two following models are included in the publication "Computational Fracture Mechanics in Concrete Technology" (1999)

d. Gradient damage simulation

It is indicated that damage mechanics offers a suitable framework for smeared crack models. On top of that, the standard damage is enhanced by spatial and/or temporal rate dependent gradients. Physical reasons, eg the inherent rate dependence occurring in course of an impact, motivate the foregoing improvement.

The performance of the model is demonstrated by numerical simulations on notched specimens with curved crack paths.

Results of the stated analysis are given in Figs 2 - 4 for a single-edge notched beam subject to an antisymmetric four-point loading.

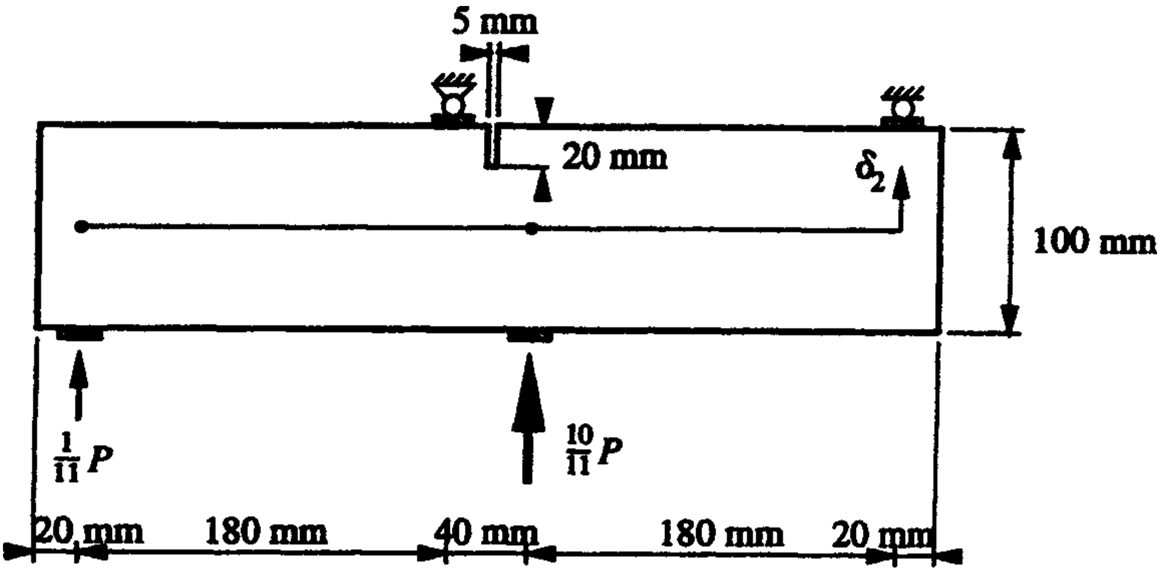

Fig. 2 Test set up of single-edge notched

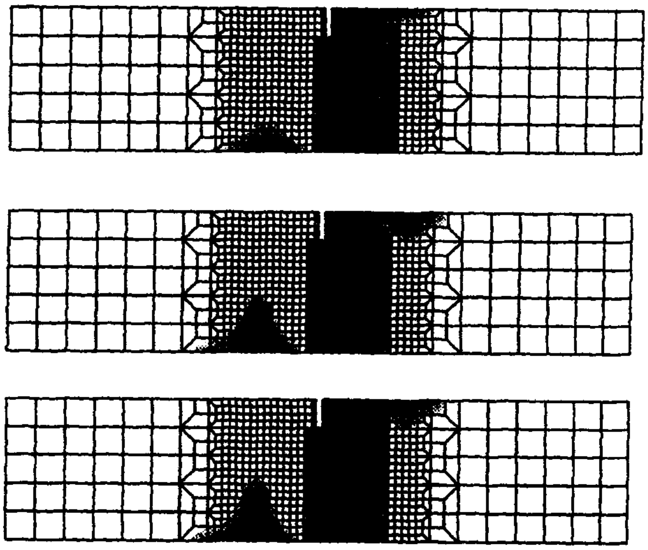

Fig. 3 Damage evolution in single-edge notched beam

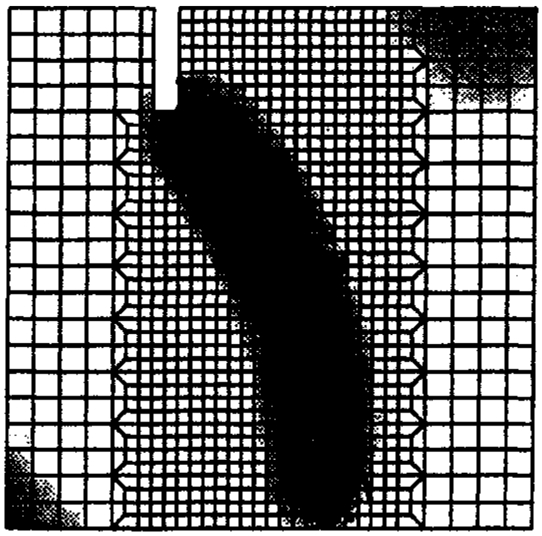

Fig. 4 Final damage distribution for modified von Mises equivalent strain definition and experimental crack pattern

e. Modelling geometrically oriented damage

Microstructural degradation and continuum damage mechanics for describing the elastic property alterations are the matter. Concurrently, an internal length is usually incorporated in the model to avoid the difficulties connected with strain-softening. To generalize a problem formulation, elastoplatic damage and crack closure effects are used. As in the majority of cases damage is not isotropic though rather geometrically directed, to compare the scalar damage model with a tensorial damage one, it is purposeful to take the induced anisotropy into consideration. In so doing, the analyses on bending and shearing geometries are performed, and these instances foreshadow circumstances when an isotropic attitude to the characterization of damage is adequate.

Outcomes of the investigation aforementioned are shown in Fig. 5.

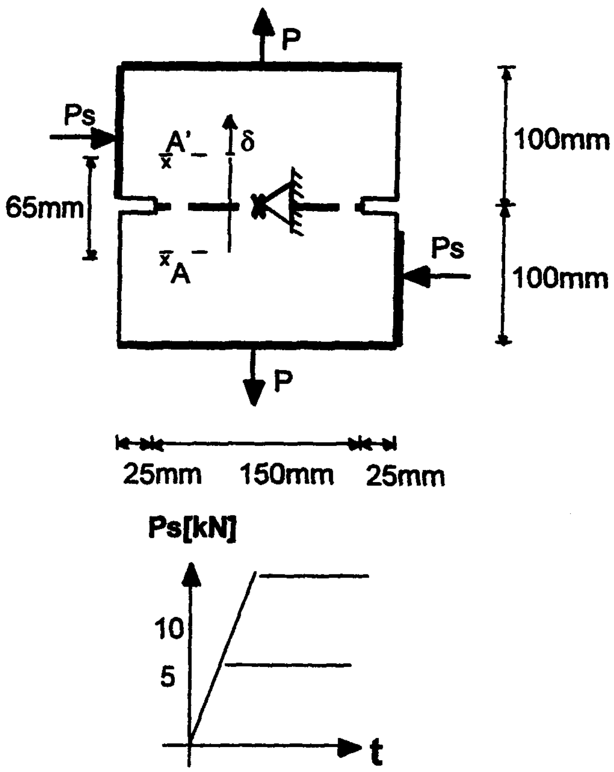

Fig. 5 Double-edge notched specimen: geometry and loads

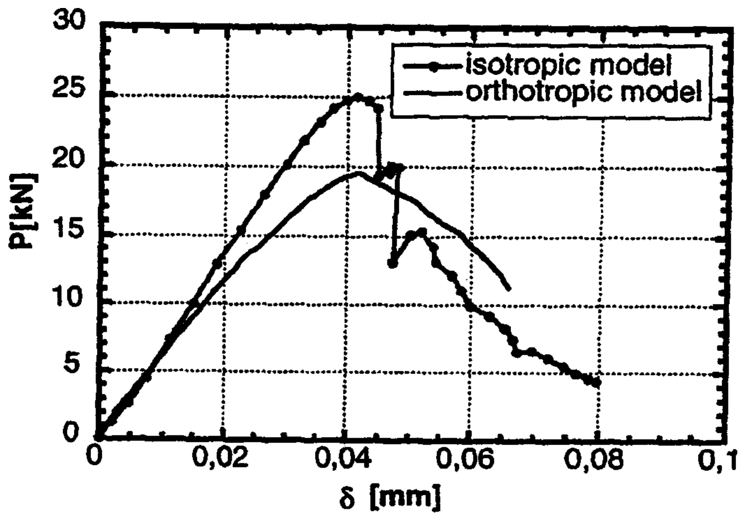

Fig. 6 Double-edge notched specimen: tensile load vs. vertical displacement for P_s = 10 kN

4. Conclusion

Continuum damage simulations provide a framework for characterizing the degradation of the elastic qualities of concrete owing to microcracking and crushing. Concurrently, this theory offers a description of macrocracking that issues from the localization of strain and damage. Though the theory is markedly phenomenological, a method is presented which yields rational bases for selecting the type of damage variable to be applied in the constitutive response. Two underlying models have been used: the classical scalar damage simulation and a model that embraces damage induced anisotropy.

5. Acknowledgements

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