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ANALYSIS OF THE EFFECT OF WINDING ANGLE ON THE MECHANICAL BEHAVIOR OF FILAMENT WOUND COMPOSITE STRUCTURES

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Abstract: *Filament winding is an important manufacturing technology of composite. In general, filament wound composite elements can be taken as laminated composite, but due to the interweaving of filaments, the filament wound composite has its own characteristics. In this paper, the stresses of a thin-walled beam with circular cross-section are calculated according to laminated theory and theory for filament wound composite in order to find out the difference and relationship between them.*

Keywords: laminated theory, filament wound composite, lay-up angle

1. Introduction

Filament wound composite elements are finding increasing applications for thick or thin structures, owing to their innovative and cost effective manufacturing technology which can provide the widest possibilities in the selection of types of layers used during fabrication. In particular, the axial, circumferential and cross-winding layers can be wound. Thus, in contrast to the traditional materials of construction, the properties of these systems can be tailored more effectively to match the stiffness and strength requirements. Many primary or secondary elements of composite structures such as aircraft wings, helicopter rotor blades, robots arms, bridges and structural elements in civil engineering constructions can be idealized as thin- or thick-walled beams. It follows that the design and analysis of the mechanical behavior of filament wound composite [1,2] beams are necessary. Up to now, there is no lack of composite beam theories based on laminated theory, but for filament wound beam, theory is lacking. In general, Filament wound composites can be taken as laminated composites, but some considerations should be given about the influence of the manufacturing process on the mechanical performances of the filament wound products. This manufacturing process often produces fiber crossovers in the architecture of the parts, although these structures are often modeled as laminated $[+\theta, -\theta]_n$ lay-ups. Therefore,

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the stresses induced in laminated composite and the stresses induced in filament wound composite under same loading should have some relationship and difference. Can laminated composite theory be utilized directly for filament wound composite, or be utilized after some modification?

In this paper, first, we will calculate stresses of a thin-walled beam with circular cross-section under free torsion based on two theories, one is Wu's theory [3] based on laminated composite theory which is simple, straightforward, suitable for engineering practice, the other is Yuan's theory [1] based on filament wound composite and then compare the results from two theories.

2.**Stress/strain relationship in thin-walled laminated beams**

Figure 1 shows the sketch of a thin-walled composite beam with circular cross-section subjected to a torsional load. We define a coordinate system (*oxyz*) with the *x*-axis (longitudinal direction of the beam) perpendicular to the cross-section of the beam. When, the plies of the beam can be assumed to be in the state of plane stress. According to [3], the stress-strain relationship of the *i*th lamina of the beam can be expressed as

$$\begin{Bmatrix} \sigma_x \\ \sigma_s \\ \tau_{xs} \end{Bmatrix} = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{21} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_s \\ \gamma_{xs} \end{Bmatrix} \tag{1}$$

Where $\overline{Q}_{jk}$ (j and k=1,2,6) denotes the off-axis stiffness of the *i*th lamina and can be determined by elastic constants E_1, E_2, G_{12}, v_{12}; $\varepsilon_x, \varepsilon_s$ and γ_{xs} denote normal strain, circular strain, and shear strain, respectively.

Owing to the assumption of no configuration distortion on the cross section, $\varepsilon_s = 0$ and assume that ε_x and γ_{xs} remain constant along the thickness, so Eq. (1) simplifies to

$$\begin{Bmatrix} \sigma_x \\ \sigma_s \\ \tau_{xs} \end{Bmatrix}_i = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{16} \\ \overline{Q}_{21} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{66} \end{bmatrix}_i \begin{Bmatrix} \varepsilon_x \\ \gamma_{xs} \end{Bmatrix} \tag{2}$$

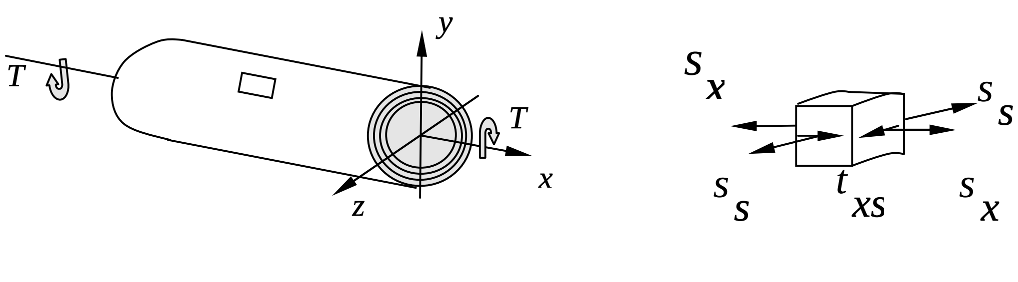

Fig.1. A composite beam and its stress state

If *t* and *p* denote the thickness and ply number, respectively, the average normal stress, the average circular stress and the average shear stress performing on the cross-section can be expressed as follows:

$$\begin{Bmatrix} s_x^* \\ s_s^* \\ t_{xs}^* \end{Bmatrix} = \frac{1}{t}\sum_{i=1}^{p}\begin{Bmatrix} s_x \\ s_x \\ s_x \end{Bmatrix}_i t_i \tag{3}$$

Where $t = \sum_{i=1}^{p} t_i$.

Substituting Eq.(2) into Eq. (3) gives

$$\begin{Bmatrix} s_x^* \\ s_x^* \\ t_x^* \end{Bmatrix} = \begin{bmatrix} \overline{Q}_{11}^* & \overline{Q}_{16}^* \\ \overline{Q}_{21}^* & \overline{Q}_{26}^* \\ \overline{Q}_{16}^* & \overline{Q}_{66}^* \end{bmatrix}\begin{Bmatrix} e_x \\ g_{xs} \end{Bmatrix} \tag{4}$$

Where $\overline{Q}_{jk}^*$ denotes the average off-axis stiffness and can be given by

$$\overline{Q}_{jk}^* = \frac{\sum_{i=1}^{p}\left(\overline{Q}_{jk}\right)_i t_i}{t} \quad (j \text{ and } k = 1,2,6) \tag{5}$$

When the beam is subjected to a torsional load without external restraint, the equilibrium conditions of the cross-section must satisfy $\int_A s_x^* dA = 0, \int_A s_x^* zdA = 0$ and $\int_A s_x^* ydA = 0,$ which means $s_x^* = 0$. Considering Eq. (4) and Eq. (2), we have

$$\begin{bmatrix} g_{xs} & e_{\mathrm{x}} & s_{\mathrm{s}}^* \end{bmatrix}^T = \begin{bmatrix} 1 & -a_{16} & B_{26}^* \end{bmatrix}^T \left[\frac{t_{xs}^*}{G}\right] \tag{6}$$

$$\begin{bmatrix} s_x & s_x & t_{xs} \end{bmatrix}_i^T = \begin{bmatrix} B_{16} & B_{26} & B_{66} \end{bmatrix}_i^T \left[\frac{t_{xs}^*}{G}\right] \tag{7}$$

Where $a_{16} = \overline{Q}_{16}^* / \overline{Q}_{11}^*$ denotes the shear-extension coupling coefficient, and elasticity constants considering shear-extension coupling are given by

$$G = \overline{Q}_{66}^* - a_{16}\overline{Q}_{16}^* \,;\, B_{26}^* = \overline{Q}_{26}^* - a_{16}\overline{Q}_{21}^* \,;\, (B_{16})_i = (\overline{Q}_{16})_i - a_{16}(\overline{Q}_{11})_i \,;$$
$$(B_{26})_i = (\overline{Q}_{26})_i - a_{16}(\overline{Q}_{21})_i \,;\, (B_{66})_i = (\overline{Q}_{66})_i - a_{16}(\overline{Q}_{16})_i \tag{8}$$

Assume that f denotes the torsional angle of the cross-section, u denotes the axial displacement (in the x-direction) of fibers. According to the free torsion theory of thin-walled beams, we have

$$g_{xs} = \frac{\partial u}{\partial x} + r\frac{df}{dx} \tag{9}$$

Where r=distance between the mid-surface of the beam wall and the torsional center of the beam.

Because the torque $T = \oint t_{xs}^{*} t r ds$,so

$$t_{xs}^{*} = \frac{T}{\Omega t} \tag{10}$$

Where $\Omega = \oint r ds$.

Combining Eq.(6), (9) and (10) gives

$$\frac{\partial u}{\partial s} = \frac{T}{Gt\Omega} - r\frac{df}{dx} \tag{11}$$

Integrating Eq. (11), a circle along the s-direction, we have

$$0 = \frac{T}{\Omega Gt}\oint ds - \frac{df}{dx}\Omega \tag{12}$$

Integrating Eq. (12) along the x-direction gives

$$f = \frac{Tl}{GK} \tag{13}$$

In which, l=length of the beam, Gk=torsional stiffness of the cross-section of the thin-walled composite beam, it is given by

$$Gk = \frac{\Omega^2}{\oint \frac{ds}{Gt}} \tag{14}$$

Eqs. (7), (8), (10), (13) and (14) can be used to calculate the stresses, strains and torsional angle of composite thin-walled beam under torsion loads without external restraint.

2. Numerical results

For a carbon-epoxy beam with circular cross-section under torsional load, with radius=40mm, length=356mm, layer thickness=0.127, [-q, +q]$_n$ lay-up, E_1=146850Mpa, E_2=11030Mpa,G_{12}=6210Mpa,and v_{12}=0.28,comparing the stress results using relationship mentioned above with the results according to theory in [1] which is based on the theory of filament wound composite, we found that data from two sources really have some relationships.

1. s_x and s_s according to two theories show the same change rule when lay-up angle changes (Fig1).
2. Shearing stress t_{xs} according to Wu’s theory keeps constant due to a_{16}=0 when lay-up angle changes, but the shear stresst_{xs} according to Yua’s theory changes when lay-up angle changes, see Fig.2. Shearing stresses according to two theories all change when the numbers of layer changes, see stress t_{xs} (tau-xs(Wu)) and t_{xs} (tau-xs(Yuan)) in Fig.3.
3. The difference of s_x and s_s comparing two sources of data at different lay-up angles and different numbers of layer is mainly caused by the difference of shearing stress t_{xs}. These can be seen from Tab.1 and Tab.2. At the same angle

and the same numbers of layer, the ratio of shearing stress t_{xs}, s_x and s_s according to two theories is almost the same. We can also see these from Fig3.

From point 3, we can see two sources of data have good relationship and the keypoint is shearing stress. If shear stresses from two sources are same at the same lay-up angle and with the same numbers of layer, the other two stresses s_x and s_s will be the same. From Eq. (7), we can see stresses of laminated composite depend on the average shearing stress t_{xs}^* and t_{xs}^* depends on the geometry of the cross-section, but the shearing stress t_{xs} in filament wound composite change with the change of lay-up angle and numbers of layer. Since the specialty of the filament wound composite in which filaments crossover each other, there are some constraints of adjacent lamina. Thus, if we do some modification to the average shearing stress t_{xs}^* from laminated theory, modified shearing stress will cause the other two stresses change and then they can be utilized for filament wound composites.

To validate results obtained from Wu's theory, we also use Ansys (shell99) to do similar calculation. The results are presented on Fig.1,2 and 3. The results from Ansys are quite correlated to the results from Wu's theory.

4.Conclusion

Based above analysis, these conclusions may be drawn

1. Analysis and calculation based on laminated theory can be used for filament wound composite but need some modifications.
2. Angle of lay-up and numbers of layer have a great influence on the stresses induced by loading.

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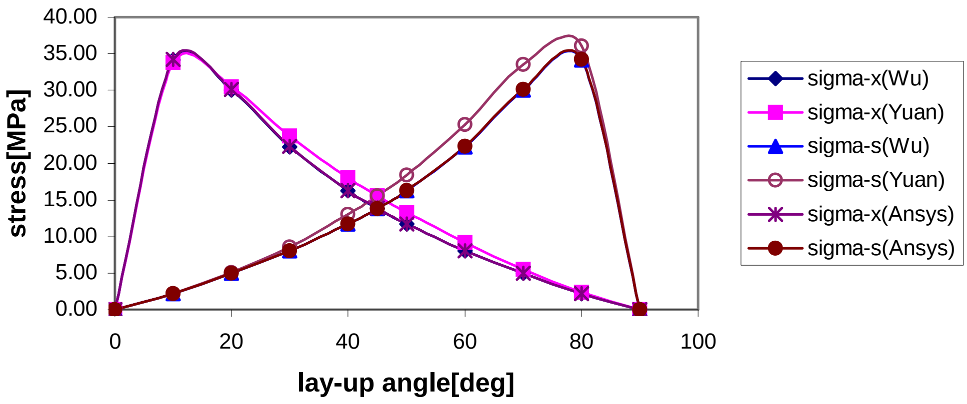

Figure 1 stresses vs lay-up angle when numbers of layer are 4.

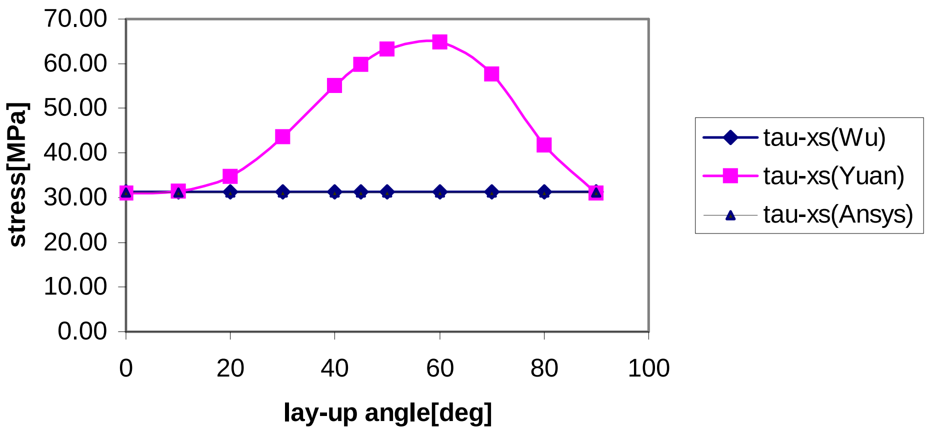

Figure 2 shearing stresses vs lay-up angle when numbers of layers are 4

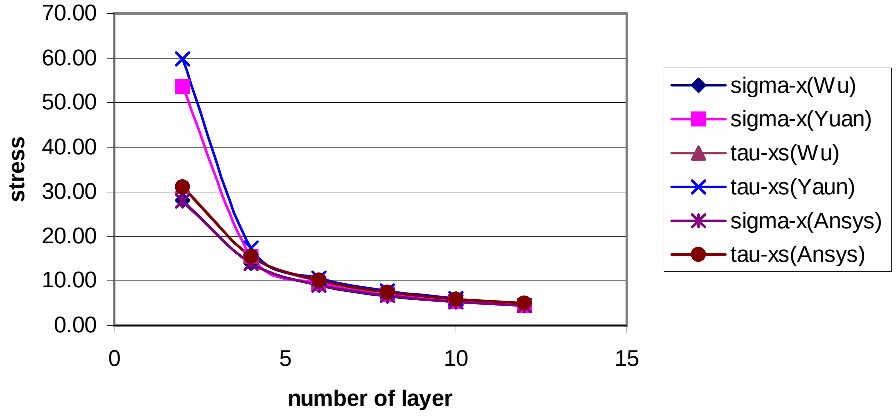

Figure 3 stresses vs numbers of layer when lay-up angle is 45 degree

Table 1 the ratio of stresses t_{xs}, s_x and s_s obtained from Wu's theory to those obtained from Yuan's theory (4 layers)

Lay-up angle	$s_x / s_x[1]$	$s_s / s_s[1]$	$t_{xs} / t_{xs}[1]$
0			1.026
10	1.022	1.022	1.022
20	1.129	0.998	0.998
30	0.951	0.951	0.951
40	0.913	0.913	0.913
45	0.902	0.902	0.902
50	0.894	0.895	0.895
60	0.892	0.891	0.892
70	0.907	1.026	0.907
80	0.958	0.959	0.959
90			1.026

Table 2 the ratio of stresses t_{xs}, s_x and s_s obtained from Wu's theory to those from Yuan's theory (12 layers)

Lay-up angle	$s_x / s_x[1]$	$s_s / s_s[1]$	$t_{xs} / t_{xs}[1]$
0			1.078
10	1.077	1.078	1.077
20	1.074	1.074	1.074
30	1.069	1.069	1.069
40	1.065	1.065	1.065
45	1.063	1.063	1.063
50	1.062	1.062	1.062
60	1.062	1.062	1.062
70	1.064	1.064	1.064
80	1.070	1.070	1.070
90			1.078