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Bone Tissue Modelling Based on the Orthotropic Micropolar Continuum

Josef Rosenberg[1], Robert Cimrman, Vít Nováček

Abstract: *The paper deals with the bone tissue modelling as a micropolar continuum. Isotropic and orthotropic constitutive relations were used for the description of the material. A simple bone specimen under torsional load was studied as well as a femur with hole and steel nail in order to determine stress and deformation distribution and to compare the behaviour of the isotropic and orthotropic micropolar materials.*

Keywords: bone, orthotropy, microcontinuum, micropolar continuum

1 Introduction

From the biomechanical point of view the living tissues are the most complex mechanical structures. Their hierarchical nature and the active behavior in longer (bone remodelation, aging) or shorter (e.g. muscle activation) time periods are the typical properties. Another important feature is the non-homogenity and non-uniformity. Even such highly organized tissue like striated muscles diffres significantly in various body parts. Large differences can be observed also among groups of human or animal individuals. These reasons cause that the commonly used continuum models covered by the commercial software do not give satisfactory results. In the last decade new theories appeared trying to cope with these difficulties. One of the most developed seems to be the microcontinuum theory established by Eringen and his co-workers (see e.g. [3, 4]). Although there are some new theories like unified theory of generalized continuum (Sansour, [11]), we decided to apply for the proposed bone modeling the special type of Eringen's microcontinuum theory — the micropolar continuum. The main reason for this decision is published information about the

[1]Prof. Ing. Josef Rosenberg, DrSc., New Technologies Centre, Univerzitní 8, 306 14 Plzeň, phone: +420 19 7491124, email: rosen@kme.zcu.cz

material parameters ([5, 6]). The second reason is that the micropolar continuum theory seems to be relatively simple in comparison with other theories. This contribution follows our previous papers ([9, 10]) dealing with the isotropic micropolar models of bones and enriches models by involving the orthotropy. Studying the structure of bones reveals that even the anisotropy is typical here. We limit our study to the small deformation concept which allows us to use linear formulation of the whole problem. Considering the physiological activities the small deformation assumption seems to be adequate. Since the main goal is to increase the accuracy of determination of stresses and deformations in the neighbourhood of different kinds of bone protheses we focus our attention on a femur with hole and a steel nail. The influence of the micropolarity and orthotropy on the stress distribution along the hole is studied.

2 Microcontinuum theory

Comprehensive description of the microcontinuum theory can be found e.g. in [4]. Here we will introduce only the basic concept necessary for the definition of the corresponding boundary value problems.

In the microcontinuum theory, a particle is associated to the every point of continuum, occupying the microvolume dV' and obeying both the spherical motion and deformation. The position of the point is given in the original configuration by the vector $\mathbf{x}$ and in the current configuration by $\mathbf{y}$. The position of an arbitrary point inside the particle is then given in both configurations by $\boldsymbol{\xi}$ and $\boldsymbol{\eta}$ respectively, therefore $\mathbf{x}' = \mathbf{x} + \boldsymbol{\xi}$, $\mathbf{y}' = \mathbf{y} + \boldsymbol{\eta}$. After we introduce macrovolume dV and the elements of microsurface dS' and of macrosurface dS, the momentum balance law in Eulerian approach can be written in form

$$t_{kl,k} + \rho f_l = 0, \quad m_{klm,k} + t_{ml} - s_{ml} + \rho(l_{lm} - \sigma_{lm}) = 0, \tag{1}$$

where s^{lm} denotes the micro-stress average, i.e. stress tensor of the macrovolume averaged across the volume (symmetric); t^{kl} is the stress tensor of the macrovolume averaged across the surface (non-symmetric); m^{klm} is the first stress moment, i.e. moment of the forces acting on the surface of the macrovolume with respect to its centre of gravity; l^{lm} denotes the first body moment of the volume forces with respect to the centre of gravity of the macrovolume; and finally f^l represents the averaged volume force.

Defining relations are the following

$$\begin{array}{c} t^{kl}ds_k = \int\limits_{dS} t'^{kl}ds'_k, \qquad \rho f^l dV = \int\limits_{dV} \rho' f'^l dV', \\ m^{klm}ds_k = \int\limits_{dS} \eta^m t'^{kl}ds'_k, \quad s^{lm}dV = \int\limits_{dV} t'^{lm}dV', \\ \rho l^{lm}dV = \int\limits_{dV} \rho'\eta^m f'^l dV', \end{array} \tag{2}$$

where t'^{kl} is the stress tensor in a particle, $t'^{kl} = t'^{lk}$;

Further step is to introduce the proper strain measures. According to [3] the three tensor strain measures are the Cauchy's deformation tensor, $C_{KL} = y_{l,K}y^l_{,L}$, and the micro-deformation tensors, $\Psi_{KL} = y_{l,K}\chi^l_L$, $\Gamma_{KLM} = y_{l,K}\chi^l_{l,M}$, where $\chi^l_k = \partial\eta^l/\partial\xi^k$, $\overline{\chi}^l_k = \partial\xi^l/\partial\eta^k$, $y_{l,L} = \partial y_l/\partial x_L$.

If we introduce the displacement vector $\mathbf{u}$ and the rotation matrix $[\Phi_{ij}]$, the following useful relations can be written for the linear micromorphic continuum [4]

$$\begin{array}{ll} \Psi_{KL} - \delta_{KL} = \varepsilon_{kl}\delta_{kK}\delta_{lL}, & C_{KL} - \delta_{KL} = 2\varepsilon_{kl}\delta_{kK}\delta_{lL}, \\ \Gamma_{KLM} = \gamma_{klm}\delta_{kK}\delta_{lL}\delta_{mM}, & \varepsilon_{kl} = u_{l,k} - \phi_{lk}, \\ 2e_{kl} = \phi_{kl} + \phi_{lk}, & \gamma_{klm} = \phi_{kl,m}. \end{array} \tag{3}$$

The simplest type of the microcontinuum is the micropolar continuum, where the particle undergoes only the spherical motion (without any deformation). Then

$$\lambda_{klm} = m_{klm} = -\frac{1}{2}e_{lmr}m_{kr}, \quad l_{kl} = -\frac{1}{2}e_{klr}l_r, \tag{4}$$

where m_{kr} is the couple stress tensor, and l_r is the body couple density; e_{lmr} is Levi-Civita tensor. The basic equations for the micropolar continuum have the following form:

$$t_{kl,k} + \rho f_l = 0, \quad m_{kl,k} + e_{lmn}t_{mn} + \rho l_l = 0, \tag{5}$$

where

$$\begin{array}{c} t_{kl} = \rho\frac{\partial\Psi}{\partial\overline{\Psi}_{KL}}\frac{\partial y_k}{\partial x_K}\overline{\chi}_{lL}, \quad m_{kl} = \rho_0\frac{\partial\Psi}{\partial\Gamma_{LK}}\frac{\partial y_k}{\partial x_k}\overline{\chi}_{lL}, \\ \overline{\Psi}_{KL} = y_{k,K}\overline{\chi}_{kL}, \quad \Gamma_{KL} = \frac{1}{2}e_{KMN}\overline{\chi}_{kM}\overline{\chi}_{kN}. \end{array} \tag{6}$$

The next set of equations necessary to define the boundary value problem are the boundary conditions

$$\left.\begin{array}{l} u_k = \widehat{u}_k \\ \phi_k = \widehat{\phi}_k \end{array}\right\} \text{ on } \partial\Omega_1, \qquad \left.\begin{array}{l} t_{kl}n_k = \widehat{t}_l \\ m_{kl}n_k = \widehat{m}_l \end{array}\right\} \text{ on } \partial\Omega_2. \tag{7}$$

To complete the set of equations a material constitutive law must be chosen. The basic constitutive equations of anisotropic micropolar thermoelastodynamics were derived by Eringen (see e.g. [4]) as

$$\begin{aligned} t_{kl} &= A_{klmn}\varepsilon_{mn} + C_{klmn}\gamma_{mn}, \\ m_{kl} &= C_{mnlk}\varepsilon_{mn} + B_{lkmn}\gamma_{mn}. \end{aligned} \tag{8}$$

There are restrictions following from the requirement of stability of the material thermodynamic state. The thermodynamic state of the micropolar body is said to be stable if and only if the internal energy function is nonnegative for all temperatures and strains, i.e. if the strain energy decreases (increases) with decreasing (increasing) strains and temperatures. This restriction can be generally expressed as a quadratic form

$$U \equiv \frac{1}{2}\left(\frac{\rho C_0}{T_0}T^2 + \lambda_{\alpha\beta}w_\alpha w_\beta\right) \geq 0, \quad \alpha, \beta = 1, 2, \ldots, 18, \tag{9}$$

where w_α represents the components of a vector in a 18-dimensional vector space corresponding to nine components of both ε_{kl} and γ_{kl}. The symmetric matrix $\lambda_{\alpha\beta}$ consists of linear combination of A_{klmn}, B_{klmn}, and C_{klmn}. Continuity requirements are also assumed (see Eringen, [4]), e.g. $A_{klmn} = A_{mnkl}$. The restriction (9) is fulfilled if $C_0 \geq 0$ and $\lambda_{\alpha\beta}w_\alpha w_\beta \geq 0$ is positive semidefinite.

For isotropic materials, the material symmetry group is the full group of orthogonal transformations [4]. Consequently, we take

$$\begin{aligned} A_{klmn} &= \lambda\delta_{kl}\delta_{mn} + (\mu+\kappa)\delta_{km}\delta_{ln} + \mu\delta_{kn}\delta_{lm}, \\ B_{klmn} &= \alpha\delta_{kl}\delta_{mn} + \beta\delta_{kn}\delta_{lm} + \gamma\delta_{km}\delta_{ln}, \\ C_{klmn} &= 0, \end{aligned} \tag{10}$$

and finally we obtain the constitutive relations of the form

$$\begin{aligned} t_{kl} &= \lambda\varepsilon_{mm}\delta_{kl} + (\mu+\kappa)\varepsilon_{kl} + \mu\varepsilon_{lk}, \\ m_{kl} &= \alpha\gamma_{mm}\delta_{kl} + \beta\gamma_{kl} + \gamma\gamma_{lk}. \end{aligned} \tag{11}$$

To satisfy the stability requirements we write following conditions imposed upon isotropic material moduli

$$\begin{aligned} &3\lambda + 2\mu + \kappa \geq 0, \quad 2\mu + \kappa \geq 0, \quad \kappa \geq 0, \\ &3\alpha + \beta + \gamma \geq 0, \quad \gamma + \beta \geq 0, \quad \gamma - \beta \geq 0. \end{aligned} \tag{12}$$

For orthotropic material, the material symmetry group is C_3 symmetry group (Zheng and Spencer, [13]). Since we suppose material orthotropy only in the case of macroscopic strain and isotropy on the microscopic scale, the constitutive relations can be rewritten as

$$t_{kl} = A_{klmn}\varepsilon_{kl}, \quad m_{kl} = \alpha\gamma_{mm}\delta_{kl} + \beta\gamma_{kl} + \gamma\gamma_{lk}, \tag{13}$$

where $A_{klmn}\varepsilon_{mn} = M_{\alpha\beta}w_\beta$; $k, l, m, n = 1, 2, 3$; $\alpha, \beta = 1, 2, \ldots, 9$; $\mathbf{w} = [\varepsilon_{11}, \varepsilon_{22}, \varepsilon_{33}, \varepsilon_{12}, \varepsilon_{13}, \varepsilon_{23}, \varepsilon_{21}, \varepsilon_{31}, \varepsilon_{32}]^T$; and the micropolar elasticity matrix (symmetric) has the form

$$M_{\alpha\beta} = \begin{bmatrix} M_0 & M_3 & M_4 & 0 & 0 & 0 & 0 & 0 & 0 \\ M_3 & M_1 & M_5 & 0 & 0 & 0 & 0 & 0 & 0 \\ M_4 & M_5 & M_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & M_6 & 0 & 0 & M_{12} & 0 & 0 \\ 0 & 0 & 0 & 0 & M_7 & 0 & 0 & M_{13} & 0 \\ 0 & 0 & 0 & 0 & 0 & M_8 & 0 & 0 & M_{14} \\ 0 & 0 & 0 & M_{12} & 0 & 0 & M_9 & 0 & 0 \\ 0 & 0 & 0 & 0 & M_{13} & 0 & 0 & M_{10} & 0 \\ 0 & 0 & 0 & 0 & 0 & M_{14} & 0 & 0 & M_{11} \end{bmatrix}. \tag{14}$$

The thermodynamic state stability conditions can be alternatively written as

$$|M_{\alpha\alpha}| > 0, \quad \forall\alpha = 1, \ldots, k, \quad k = 1, 2, \ldots, 9. \tag{15}$$

In [9] a variational formulation of the boundary value problem (5), (7), (8) is shown. The solution can be found as the stationary point of the potential

$$\begin{aligned} \prod(\underline{u}, \underline{\phi}) &= \tfrac{1}{2}\int_\Omega (A_{klmn}\varepsilon_{mn} + C_{klmn}\gamma_{mn})\varepsilon_{kl}\mathrm{d}x \\ +\tfrac{1}{2}\int_\Omega (C_{mnlk}\varepsilon_{mn} &+ B_{lkmn}\gamma_{mn})\gamma_{lk}\mathrm{d}x + \int_{\partial\Omega_2} (\widehat{u}_i n_j + g_{ij})\tau^{ij}\mathrm{d}s \\ + \int_{\partial\Omega_2} (\widehat{\phi}_k n_l + \gamma_{kl})m^{kl}\mathrm{d}x &- \int_\Omega \rho\widehat{f}_i u_i\mathrm{d}x - \int_{\partial\Omega_1} \widehat{\tau}_i u_i\mathrm{d}x - \int_\Omega \rho\widehat{l}^l\mathrm{d}x \end{aligned} \tag{16}$$

with the constrains

$$\begin{aligned} \varepsilon^{kl} = \tfrac{\partial u^l}{\partial x^k} + e^{lkm}\phi_m, \quad \gamma^{kl} = \tfrac{\partial\phi_k}{\partial x^l}, \\ -u_i n_j = g_{ij} \text{ on } \partial\Omega_2, \quad -\phi_i u^j = \gamma^{ij} \text{ on } \partial\Omega_2, \end{aligned} \tag{17}$$

where g_{ij} is an auxiliary variable. Then the weak solution of the problem (5), (7), (8) satisfies the conditions (we omit loading terms for brevity here)

$$\begin{aligned} \prod(\underline{u}, \underline{\phi}; \delta\underline{u}) = 0 \rightarrow \int_\Omega \tau_{kl}\delta_u\varepsilon_{kl}\mathrm{d}\Omega = 0, \\ \prod(\underline{u}, \underline{\phi}; \delta\underline{u}) = 0 \rightarrow \int_\Omega (\tau_{kl}\delta_\phi\varepsilon_{kl} + m_{kl}\delta_\phi\phi_{l,k})\mathrm{d}\Omega = 0. \end{aligned} \tag{18}$$

Following the approach presented in [9] the finite element discretization of micropolar continuum was used in a home made FEM package developed by the authors and applied to the above described boundary value problem.

3 Micropolar model of the bone tissue

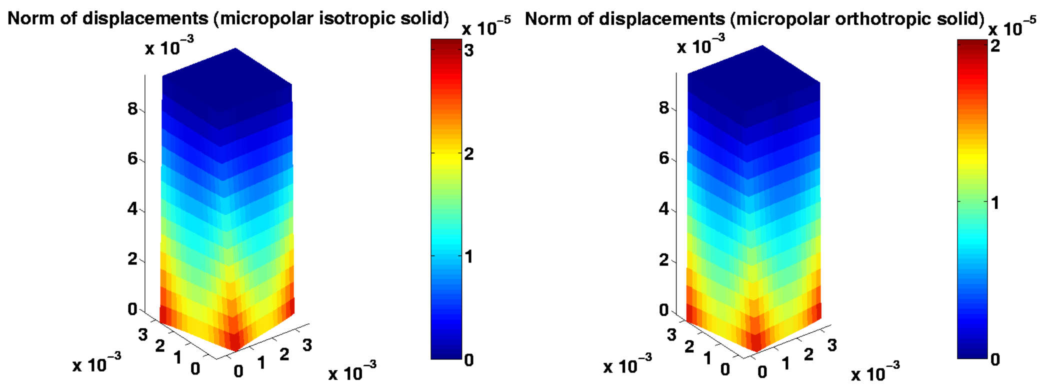

Figure 1: Displacement field (20× magnified) of the brick specimen.

In [4, 6] the experimentally obtained material parameters for the corticoidal bone are published:

Young's modulus [Pa]	$E = 2G(1+\nu)$
Poisson ratio	$\nu = \lambda/(2\lambda + 2\mu + \kappa) = 0.4$
Coupling number	$N = 0.9$
Characteristic length of microstructure [m]	$c = \sqrt{\gamma(\mu+\kappa)/\kappa(2\mu+\kappa)}$
Characteristic length for torsion [m]	$l_{torsion} = 0.2 \cdot 10^{-3}$
Characteristic length for bending [m]	$l_b = l_t/\sqrt{3} = cN$
Shear modulus [Pa]	$G = 4.5 \cdot 10^9$
Polar ratio	$\psi = 1.5$

Using the formulas $\lambda = \frac{2\nu}{1-2\nu}G$, $\mu = \frac{1-2N^2}{1-N^2}G$, $\kappa = \frac{2N^2}{1-N^2}G$, $\alpha = \frac{2(1-\psi)}{\psi}l_t^2 G$, $\beta = \frac{2}{3}l_t^2 G$, $\gamma = 2\beta = \frac{4}{3}l_t^2 G$, the material parameters used in isotropic constitutive equations (11) can be obtained: $\lambda = 1.8 \cdot 10^{10}$ [Pa], $\mu = -1.468 \cdot 10^{10}$ [Pa], $\kappa = 3.837 \cdot 10^{10}$ [Pa], $\alpha = -120$ [N], $\beta = 120$ [N], $\gamma = 240$ [N].

Equivalent linear elastic Lamé coefficients can be acquired using the formulas $\lambda_E = \lambda_M$, $\mu_E = \mu_M + \kappa/2$, where subscripts E, M denote elastic and micropolar coefficients respectively. This gives us $\lambda_E = 1.8 \cdot 10^{10}$ [Pa], $\mu_E = 4.5 \cdot 10^9$ [Pa] and hence $E = 1.26 \cdot 10^{10}$ [Pa], $\nu = 0.4$.

Material parameters used in constitutive relation of the orthotropic microcontinuum were equivalent to those of the isotropic microcontinuum except the micropolar

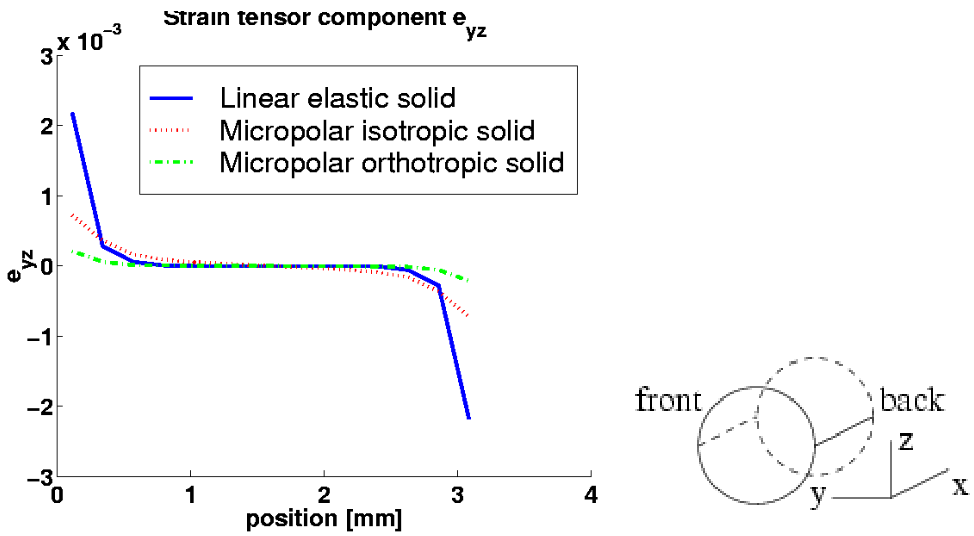

Figure 2: Left: Strain tensor component e_{yz} along the bottom element row in respect to x axis. Right: Femur example: Lines on the hole surface.

elasticity matrix components M_8 and M_{11} whose values were chosen twice greater than the previously determined isotropic parameters, that is $M_8 = M_{11} = 2(\mu + \kappa)$.

Two main examples were studied: (1) Brick-like compact bone specimen under torsional load; (2) Human femur with steel nail. The bone specimen under torsion was modelled as micropolar isotropic and orthotropic solid according to experimental and computational observations presented in [6]. Top plane nodes degrees of freedom were fixed and the torsional load was applied at the bottom plane. There are apparent differences in behaviour of isotropic and orthotropic materials studied shown in Figures 1 and 2.

Micropolar isotropic and orthotropic femur with a steel nail undergoing bending and torsion was also studied. Material parameters given above were used in the case of the isotropic model. The following material parameters (in [GPa]) were used for modelling the orthotropic bone:

$$
\begin{array}{lllll}
M_0 = 22.4 & M_1 = 25.0 & M_2 = 35.0 & M_3 = 14.0 & M_4 = 15.8 \\
M_5 = 13.6 & M_6 = 22.2 & M_7 = 22.7 & M_8 = 23.2 & M_9 = 22.2 \\
M_{10} = 22.7 & M_{11} = 23.2 & M_{12} = -16.1 & M_{13} = -15.6 & M_{14} = -15.0.
\end{array}
$$

Bending load: We investigate the influence of the microstructure on the stress computed along lines on the surface of the drilled hole in the bone (Figure 2 right).

In Figure 4 we plot the "averaged" (in the least squares sense) t_{33} stress in the 2 rows of elements on "front" and "back" curves denoted 1, 2 respectively in Figure 4 and in rows one element diameter above these rows; the abscissa relates to the x coordinate of element centers. **Note:** We plotted separate figures for the particular rows of elements because of the different behaviour in the middle and the upper element row, e.g. on the front line, the middle row stresses are tensile, while the upper (and lower) row stresses are compressive. Analogously on the back line, the middle row stresses are compressive, while the upper (and lower) row stresses are tensile.

Torsion load: In this case there is no fundamental difference among the element rows as in the previous case; the "averaged" (least squares with the third order polynomial) t_{22} is plotted in Figure 4.

The influence of the "micropolarity" on the stresses is clearly visible; more on this example can be found in [10].

4 Conclusion

In this article we have presented a weak formulation of problems involving micropolar continuum using isotropic and (macroscopically) orthotropic material relations. We have compared numerical results of simple examples (a block specimen of bone material, a femur bone with a steel nail) for micropolar isotropic and orthotropic cases.

Simple example (a block specimen of bone material under torsion) was proposed according to the study presented in [6]. There are apparent differences between isotropic and orthotropic cases. Orthotropic material is more "stiff" in torsion then isotropic material due to higher orthotropic material parameters chosen in this study.

The femur example was studied in more detail in [9] for the isotropic case. Here we have presented similar results for the orthotropic case, namely the (tangent) stress curves along the hole in the bone and macroscopic displacements for bending and torsion loads. The micropolar isotropic data correspond to linear elastic ones given in [6] (human femur), while the micropolar orthotropic data come from [5] (bovine femur). We have seen that for the bending case, the displacements diminish in the orthotropic case, while, for the torsion load, they increase. Hence the orthotropic data lead to a "stiffer" bone for bending load modes then the isotropic data, and "softer" for torsion loads.

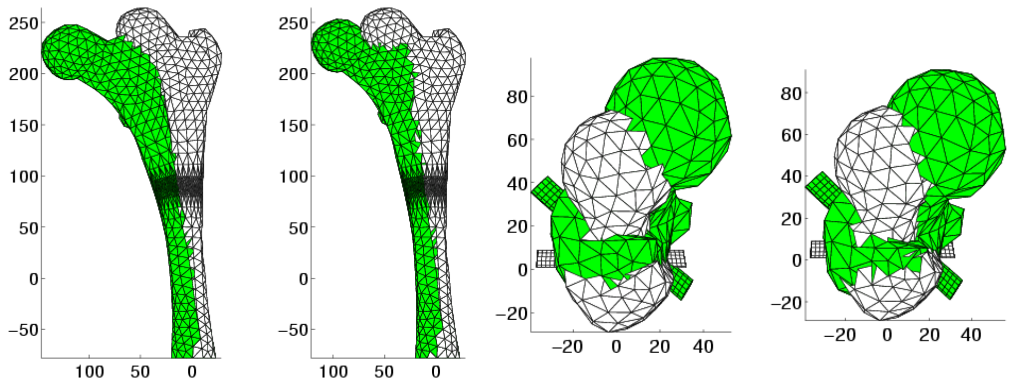

Figure 3: Displacements (300× magnified) in bending and torsion.

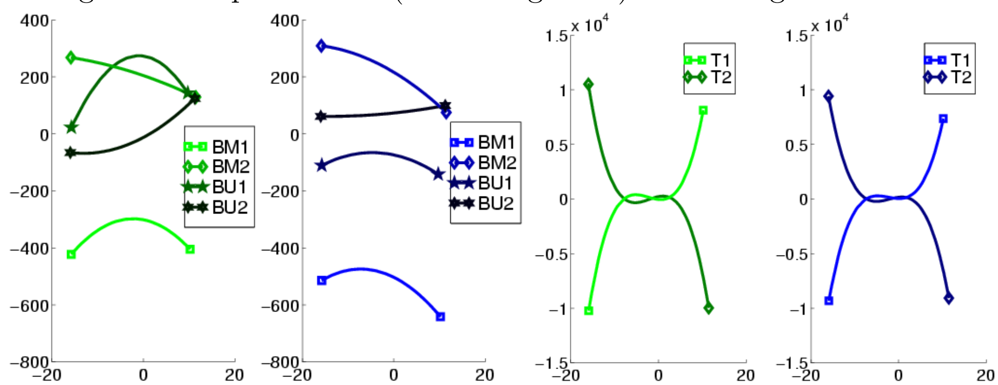

Figure 4: Stress component t_{22} and t_{33} along the lines. P — bending, T — torsion, M — middle row, U — upper row.

This study should be considered as a starting effort for introducing an anisotropy into the micropolar bone model. Further research should involve a material identification procedure based on sensitivity analysis and experiments.

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