Národní konference s mezinárodní účastí

INŽENÝRSKÁ MECHANIKA 2002

13. – 16. 5. 2002, Svratka, Česká republika

SUBHARMONIC MOTIONS OF THE OSCILLATOR WITH SOFT IMPACTS

František PETERKA and Aleš TONDL *

Abstract: *The excited one degree of freedom mechanical system with soft impacts, characterised by triangle hysteresis loop, is investigated using numerical simulation. Small viscous damping is assumed. Phenomena of subharmonic motions are explained by regions of their existence and stability in plane of dimensionless excitation frequency and static clearance. Bifurcation diagrams are evaluated during quasistationary changes of frequency by constant clearance.*

Keywords: impact oscillator, triangle model of impact interaction, numerical simulation, stability, chaos, bifurcation diagrams

1. Introduction

This contribution deals with the simulation analysis of the motion of oscillator with impacts (Fig.1). It is excited by harmonic force $F_0\cos\omega t$ and its mass m can impact against a soft stop situated in certain distance r from the mass equilibrium position. Elastic and damping characteristics $F(X)$ is assumed in piecewise linear form.

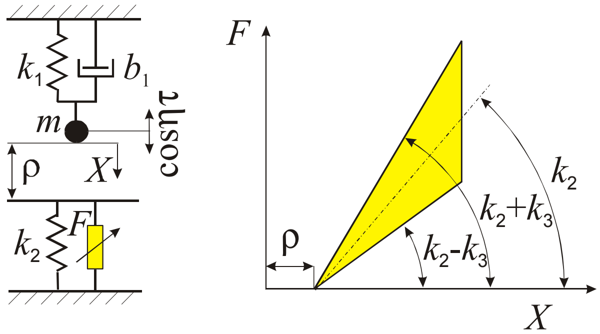

Figure 1. Scheme of the system and model of soft impact

Denoting dimensionless deflection $X=x/x_{st}$ ($x_{st}=F_0/k_1$) and using time transformation $\tau=\Omega t$, where $\Omega=(k_1/m)^{1/2}$, then the oscillator motion is described by equation

$$X'' + \beta X' + X + F = \cos\eta\tau, \qquad \text{where}$$

$$X'' = \frac{d^2x}{d\tau^2},\ X' = \frac{dx}{d\tau},\ \beta = b_1/\sqrt{k_1 m} = 0.01,\ F = \langle \begin{matrix} 0 \quad \text{for } X \le \rho,\ (\rho = r/x_{st}) \\ (X-\rho)(k_2 + k_3 \operatorname{sign} X')/k_1 \quad \text{for } X \rangle \rho\ . \end{matrix}$$

* Ing. František Peterka, DrSc., Doc. Dr. Ing. Aleš Tondl, DrSc., Institute of Thermomechanics, AS CR, Dolejškova 5, 182 00 Prague 8, E-mail: peterka@it.cas.cz, tondl@it.cas.cz

Results of simulation are presented in (ρ,η) plane showing the areas of different regimes of impact motion, which are characterised by impact number $z=p/n$, where p denotes the number of impacts and n denotes the number of excitation periods, during one impact motion period, respectively. $z=0$ symbolizes impactless motion, which exists for higher clearances ρ. It is bounded by grazing boundary g_0 (Figs.2-4), which corresponds to known frequency-amplitude characteristics of impactless motion.

This paper is the continuation of paper [1] and extends it by bifurcation diagrams along horizontal sections of impact motion regions (Figs.2-4), for constant clearances ρ and quasistationary changes of excitation frequency η. The behaviour of this system was also explained in [2], but for ten times higher viscous damping $\beta = 0.1$.

The ascertaining of the influence of damping on the system impact motions is therefore the aim of this contribution. The diversity of periodic and chaotic impact motions increases with decreasing viscous damping. Simultaneously hysteresis regions of impact motions increase over grazing bifurcation boundary g_0 (into the region $z=0$ of impactless motion). Several such hysteresis regions are shown in Figs.2,3. Saliences of these regions get to the infinity for zero viscous damping and they press down to grazing boundary g_0 with increasing damping.

2. Regions of system motions

Evaluated regions of periodic and chaotic impact motions for parameters $\beta=0.01$, $k_2/k_1=10$, $k_3/k_1=2$ are shown in Fig.2. They are labelled by impact numbers z. Impactless $z=0$ and fundamental motion $z=1/1$ exist over the resonance $\eta=1$ of impactless motion. Its amplitude-frequency characteristics g_0 represents also the grazing bifurcation boundary. Big hysteresis region exist between boundary g_0 and saddle-node stability boundary $SN_{1/1}$ of $z=1/1$ motion. Similar hysteresis region of fundamental motion $z=2/1$ exist over the subharmonic resonance $\eta=1/2$.

The main aim of this paper is the characterisation of subharmonic impact motions in subregion (Fig.3) of Fig.2, the next enlarged subregion of which is shown in Fig.4. There exist three types of region boundaries, which are marked by *g* or *PD* or *SN*. They characterise three typical bifurcations of the impacts oscillator motion.

g - grazing bifurcation corresponds to the state of periodic motion $z=p/n$, when the periodic touch of moving mass with the stop appears during quasistatic change of system parameters. The transition cross grazing boundary is reversible for the oscillator with soft impact. New periodic motion with almost zero before-impact velocity of appeared weak impact is characterised by $z=(p+1)/n$, so impact number z increases.

PD - period-doubling bifurcation appears on the stability boundary where $z=p/n$ motion splits on $z=2p/2n$ motion with double period. Transition cross this boundary is also reversible and mean value of impact number z does not change.

SN – saddle-node bifurcation corresponds to stability boundary, where one of impacts disappears and system stabilises after a jump transition in a new, usually periodic $z=(p-1)/n$, motion. This quantitative and qualitative change of the system motion is non-reversible and number z decreases. One exception exists in impact oscillator with soft impacts. It explains the existence of hysteresis regions of impact motions. The statement of reversible transition cross grazing bifurcation boundaries excludes the existence of hysteresis region. Nevertheless they exist. The explanation of this reality is based on the following phenomenon. After crossing the grazing boundary

g there appears motion with weak impact, marked e.g. $z=p/n_w$. This motion exists in narrow region along grazing boundary g and is limited by SN_w stability boundary. There arises a jump into the same type of $z=p/n_s$ motion, but with stronger impact. So, this quantitative and non-qualitative change is non-reversible and value z does not change. Only the motion with stronger impact exhibits the hysteresis and exists up to its stability boundary SN_s. The quantitative jump on this boundary can result in the return into $z=p/n_w$ motion (value z remains) or in the jump into $z=(p-1)/n$ motion. These problems are explained in more detail in [3].

3. Subharmonic motions of the order *n*

Subharmonic motions exist between neighbour fundamental motions $z=p/1$ and $z=(p+1)/1$, where $p = 0, 1, 2, \dots$. They are distinguished according to the order n – the number of excitation period $T=2\pi/\eta$ in the motion period. Motions of order n are derived from basic regimes $z = p/n$ ($n = 2, 3, 4\dots$) after period doublings ($z = 2p/2n$, $z = 4p/4n$, …), which can lied directly or through other derived motions into the chaotic impact motion. In the difference of the motion with rigid impacts, the regions of subharmonic motions with soft impacts can exist in separated energetic levels. The start of the system motion in the certain level depends in some cases on special ways during quasistatic changes of system parameters or on a special selection of motion initial conditions, according to the evaluation of basins of attraction [4]. These effects will be explained in more detail using bifurcation diagrams.

4. Bifurcation diagrams

The bifurcation diagram is the evaluation of characteristic quantities of the system motion during quasistatic change of system parameters through regions of different regimes of motion. Motion amplitudes $X_m(\eta)$ and before-impact velocities $X'_-(\eta)$ along horizontal section ***a***, ***b***, ***c*** of regions in Figs.2-4 are shown in Figs.5-9. It is necessary to mention, that X_m corresponds only to the lowest minimum of the motion, which appears in every period T and X'_- is included into diagram for every impact. The reason of this procedure is the possibility to ascertain values $z=p/n$ according to the number p of branches $z = p/n$ and the number n of branches $X_m(\eta)$ of a certain impact motion.

4.1 Bifurcation diagrams of subharmonic motions of order *n* = 2

The simplest diagram in the neighbourhood of points $a1$, $a2$, $a3$ of line ***a*** (ρ=3.3) in Figs. 3,4 is shown in Fig.5. Impactless motion $z = 0/1$ exists with increasing frequency η up to grazing boundary g_0 (point $a1$), where $z = 1/1$ motion continuously arises and is stable up to its period doubling $PD_{1/1}$ (point $a2$). The inverse $PD_{1/1}$ appears in point $a3$ and $z = 1/1$ motion stabilizes. The transition through regions of motion $z = 0/1$, $z = 1/1$, $z = 2/2$ cross boundaries g_0 , $PD_{1/1}$ is reversible, without hysteresis phenomena.

Bifurcation diagrams (Fig.7) along line ***b*** (ρ=1.9) are more complex and they were evaluated separately for increasing and decreasing η. Impactless motion transits on grazing boundary? g_0 (point $b1$) again in $z = 1/1_w$ motion, but its stability ends on $SN_{1/1w}$ boundary. New $z = 1/1_s$ splits two times and $z = 4/4_s$ stabilizes. This motion transits through the inverse Feigenbaum cascade into $z = 2/2_s$ motion (point $b2$). This cascade up to $z = 1/1$ motion is interrupted by the transition of $z = 2/2_s$ motion into $z = 1/2$ motion on stability boundary $SN_{2/2s}$ (points $b5$). The second impact appears in $z = 1/2$ motion on its grazing boundary $g_{1/2}$ (points $b6$) and the inverse Feigenbaum cascade ends in points $b7$. The course of bifurcation diagram in Fig.7(b) is the same, except two hysteresis phenomena between points $b4$, $b5$ and $b1$, $b3$. The chaotic motion arises at the end of the period doubling cascade of motion $z = 2/2_s$ and it escapes in point $b3$ into impactless motion.

Bifurcation diagram between points $c8$ and $c5$ of line c ($\rho = 3$) in Fig.4 is shown in Fig.6 right down. It explains next type of the interruption of the Feigenbaum cascade of motion $z = 1/1$, which starts with decreasing η in point $c5$. The $z = 2/2$ motion splits again near point $c6$ on $z = 4/4$ motion, but the weakest impact disappears and z 3/4 motion arises. Only Feigenbaum´s cascade of $z = 3/4$ ($z = 6/8$, $z = 12/16$, …) motion ends in the chaotic motion, which escapes into $z = 1/3$ motion in point $c8$. This is precisely one of ways from the level of subharmonic moitons of order $n = 2$ into the level $n = 3$.

4.2 Bifurcation diagrams of subharmonic motions of order $n = 3$

Subharmonic motion $z = 1/3$ and other derived from it motions exist in a large frequency interval between points $c12 – c4$ of line $\boldsymbol{c}$ (Figs.3,4). Motion $z = 1/3$ appeared in point $c8$ is stable between points $c9$ and $c3$, which belong to grazing boundary $g_{1/3}$. There arises z=2/3 motion, which losses the stability in point $c4$ ($SN_{2/3w}$) and jumps into $z = 1/1$ motion. Near grazing boundary $g_{1/3}$ (point $c9$) exists stability boundary $SN_{2/3w}$, where $z=2/3_w$ motion jumps into $z = 2/3_s$ motion and it returns into $z = 1/3$ motions in point $c2$ on boundary $SN_{2/3s}$. Maximum displacements and impact velocities of $z = 2/3$ motions exist along boundary $SN_{2/3s}$. This phenomenon is connected with the resonance of $z = 2/3$ motion and big hysteresis region $z = 2/3$ in Fig.3. On left boundary $PD_{2/3}$ (points $c10$ in Figs.3,6) begins the Feigenbaum cascade ($z = 2/3$, $z = 4/6$, $z = 8/12$, …), which ends in point $c11$ by the transition into chaos. Its escape into impactless motion $z = 0/1$ appears in point $c12$.

It is important to mention, that region of $z = 2/3_s$ impact motion has the sharp salience created by two different stability boundaries $SN_{2/3s}$ (Fig.4). The intermittency chaos appears along a part of lower (horizontal) boundary $SN_{2/3s}$. This phenomenon was ascertained and in more detail explained in [5] for the oscillator with rigid (Newton´s) impacts. Only the difference is that intermittency chaos exhibits hysteresis into region of periodic $z = 2/3_s$ impact motion.

One of other possible ways into level $n = 3$ of subharmonic motions is the faster transition from impactless motion z=0 cross grazing boundary g_0 (points $c1$ in Figs.4,6).

Next examples of subharmonic motions of order $n = 3$ along line $\boldsymbol{a}$ (ρ=3.3) of Figs.3,4 are shown in Fig.9. They cannot be received by quasistatic changes of frequency η, but by other means, e.g. by the start from certain motion initial conditions or from other parts of their regions of existence and stability. For example $z = 1/3$ motion just exists in the single horizontally shaded subregion $z = 1/3$ in Fig.4. Bifurcation diagram z=1/3 between points $a13$ and $a11$ in Fig.9 can be obtained after increasing ρ to line $\boldsymbol{a}$ (ρ=3.3). Motion z=1/3 losses the stability ($SN_{1/3}$) in point $a13$, then jumps into $z = 1/1$ motion (Fig.9) and transits into $z = 2/3$ motion on grazing boundary $g_{1/3}$ in point $a11$.

The $z = 2/3_s$ motion exists between points $a7$, $a8$ (Figs.3,9). This motion losses the stability in point $a7$ ($SN_{2/3s}$) and jumps into impactless motion. The period doubling cascade of $z = 2/3_s$ motion begins in point $a8$ ($PD_{2/3s}$), ends in point $a9$ in chaotic motion, which escapes into impactless motion in point $a10$.

4.3 Bifurcation diagram of subharmonic motions of order *n* = 4 and *n* = 7

These subharmonic motions should be obtained also by a special way. Regions of z = 1/4 and z = 2/4 motions are shown in Figs.3,4. Bifurcation diagram of z = 2/4 and derived (z=4/8, z=8/16, …, chaos) motions between points a4 and a6 of line ***a*** are shown in Figs.8,9. Motion z = 2/4 losses stability in point a4 and transits on z = 1/1 motion, while chaotic motion escapes into impactless motion in point a6.

Regions of subharmonic motions of order n = 7 were observed by chance and their existence regions are not included into Figs.2-4. Bifurcation diagram of motions z = 3/7, z = 6/14 and chaos along line ***a*** (ρ = 3.3) is in Fig.9.

5. Conclusion

Small damping of impactless motion introduces large diversity of periodic and chaotic impact motions. There were ascertained and explained, using numerical simulations, several phenomena of subharmonic motions with soft impacts:

- grazing, period doubling and saddle-node bifurcations,
- reversible transitions cross grazing and period doubling bifurcation boundaries,
- two types of quantitative jumps by saddle-node instabilities,
- separated levels of different order subharmonic motions,
- interrupted Feigenmaum´s cascade into the chaos
- interrupted development of saddle-node instability, which leads into the intermittency chaos.

Acknowledgement

This investigation is financially supported by the Grant Agency of the Czech Republic, Project No. 101/00/0007.

References

[1] Tondl A., Peterka F.: To the Dynamics of Oscillator with Soft Impacts, *Proc. 5th International Conference on Vibration Problems*, Moscow, 2001.
[2] Peterka F., Tondl A.: Dynamics of Oscillator with Piecewise Model of Soft Impacts. *Engineering Mechanics 2001*, Svratka, Czech Rep., pp.209-210, CD ROM.
[3] Peterka F.: Dynamics of Oscillator with Soft Impacts. Proc. DETC'01, Sept. 9-12, 2001, Pittsburgh, Pennsylvania, USA, Paper DETC 2001/VIB-21609, CD ROM.
[4] Peterka F., Čipera S.: Regions of Subharmonic Motions of the Oscillator with Hertz´s model of Impact. *Proc. Dynamics of Machines 2002*, Institute of Thermomechanics AS CR, Prague, pp. 145-152.
[5] Peterka F., Čipera S., Kotera T.: Additional impact causes the intermittency chaos of unstable subharmonic motions of impact oscillator. ICTAM 2000, Chicago, USA, August 27- September 2, 2000, International Congress of IUTAM, Abstract Book, pp. 144-145.