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TRACTOR DYNAMICS BY MEANS OF THE STRUCTURAL SYNTHESIS OF THE RECEPTANCE.

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Summary: *The traditional method of motion description of heavy machines are differential equations. In this paper for the description of tractor motion was used the method of structural analysis receptance which is described in references [1] and [2]. This method consists in dividing of system model into individual subsystems and then in structural connecting receptances of individual subsystems. It is very useful when some of this subsystems are difficult for analytical description. In this case an element or the whole subsystem is experimentally described. An example of this description is determining tractor tyre stiffness in deep mud.*

Keywords: *receptance, heavy machines, structural diagram, dynamics*

1. MATHEMATICAL MODELLING

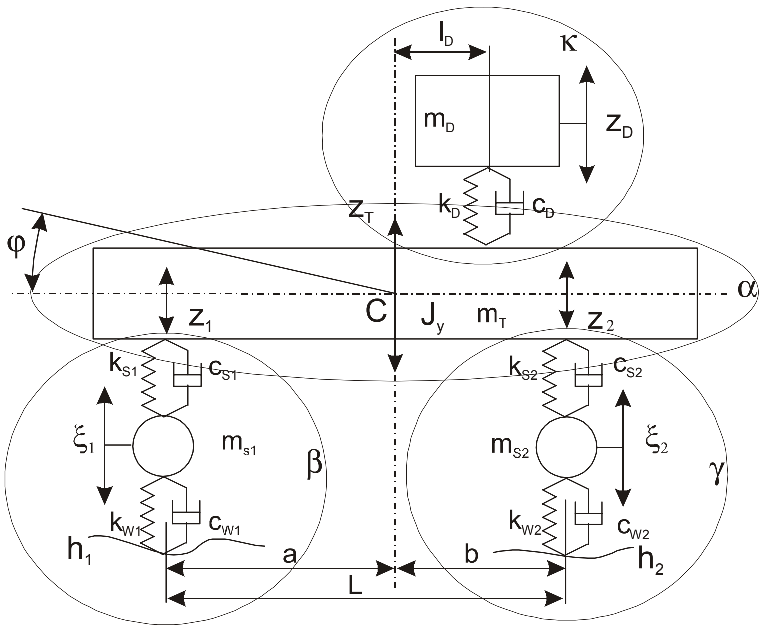

Fig. 1. Tractor dynamic model

Figure 1 shows schematic diagram of tractor dynamic model which was considered. The motion of this system consists in translational motion of the mass centre and

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rotational motion round mass centre. Mathematical modelling of the complete system is quite complicated.

Differential equations above presented model are following:

$$m_D \cdot \ddot{z}_D + c_D\left[\dot{z}_D - \frac{b-l_D}{L}(\dot{z}_1 - \dot{z}_2)\right] + k_D\left[z_D - \frac{b-l_D}{L}(z_1 - z_2)\right] = 0 \tag{1}$$

$$M_1\ddot{z}_2 + M_3\ddot{z}_1 + 2c_{s2}(\dot{z}_2 - \dot{\xi}_2) + 2k_{s2}(\dot{z}_2 - \dot{\xi}_2) = 0 \tag{2}$$

$$M_1\ddot{z}_2 + M_3\ddot{z}_1 + 2c_{s2}(\dot{z}_2 - \xi_2) + 2k_{s2}(\dot{z}_2 - \xi_2) = 0 \tag{3}$$

$$m_{s1}\dot{\xi}_1 - 2c_{p1}(\dot{z}_1 - \xi_1) + 2c_{w1}(\dot{\xi}_1 - \dot{h}_1) - 2k_{w1}(\xi_1 - h_1) = 0 \tag{4}$$

$$m_{s2}\dot{\xi}_2 - 2c_{p1}(\dot{z}_2 - \xi_2) + 2c_{w2}(\dot{\xi}_2 - \dot{h}_2) - 2k_{w1}(\xi_2 - h_2) = 0 \tag{5}$$

where: $M_1 = (b^2 + \rho^2)\frac{m}{L^2}$; $M_2 = (a^2 + \rho^2)\frac{m}{L^2}$; $M_3 = (ab - \rho^2)\frac{m}{L^2}$ (6)

m_T denotes springing mass of tractor,
m_D – mass of driver and his seat,
m_{s1} – no-springing mass referred to leading axle,
m_{s2} – no-springing mass referred to back axle,
φ – angular co-ordinate of springing mass,
z_T – vertical co-ordinate of springing mass,
z_1 – vertical co-ordinate of no–springing mass referred to leading axle,
z_2 – vertical co-ordinate of no–springing mass referred to back axle,
k_D, k_{s1}, k_{s2}, k_{w1}, k_{w2} – spring constants, suitably driver seat, leading suspension, back suspension, leading wheel, and back wheel,
c_D, c_{s1}, c_{s2}, c_{w1}, c_{w2} – viscous friction coefficients, suitably driver seat, leading suspension, back suspension, leading wheel, and back wheel,
a, b, L, l_B – geometrical parameters,
J - moment of inertia
h_1, h_2 – accidental inputs (soil level).

2. Structure diagram of dynamic receptance

Above the shown set of differential equations (1 – 6) was changed to the structure diagram in which the individual blocks were presented by the dynamic receptances of the subsystems α, β, γ. For simplicity of the system the subsystem κ was omitted (small influence of driver mass on system vibration).

System response is obtained from the equation

$$\begin{bmatrix} z_T \\ \varphi_T \\ \xi_1 \\ \xi_2 \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} & G_{13} & G_{14} \\ G_{21} & G_{22} & G_{23} & G_{24} \\ G_{31} & G_{32} & G_{33} & G_{34} \\ G_{41} & G_{42} & G_{43} & G_{44} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ F_1 \\ F_2 \end{bmatrix} \tag{7}$$

In shorten notation equation (7) has form

$$\mathbf{Y} = \mathbf{GF}$$

where: **Y** is column vector including outputs z_T, φ, ξ_1, ξ_2, $\mathbf{G} = [G_{ij}]$ is matrix of receptances of the whole system after synthesis the subsystems, and **F** is column vector of input forces.

Input forces where calculated from following equations:

$$F_1 = h_1(t)k_{w1} + \dot{h}_1(t)c_{w1} \tag{8}$$

$$F_2 = h_2(t)k_{w2} + \dot{h}_1(t)c_{w2} \tag{9}$$

Instead this traditional method of motion description the method of structural analysis receptance is used.

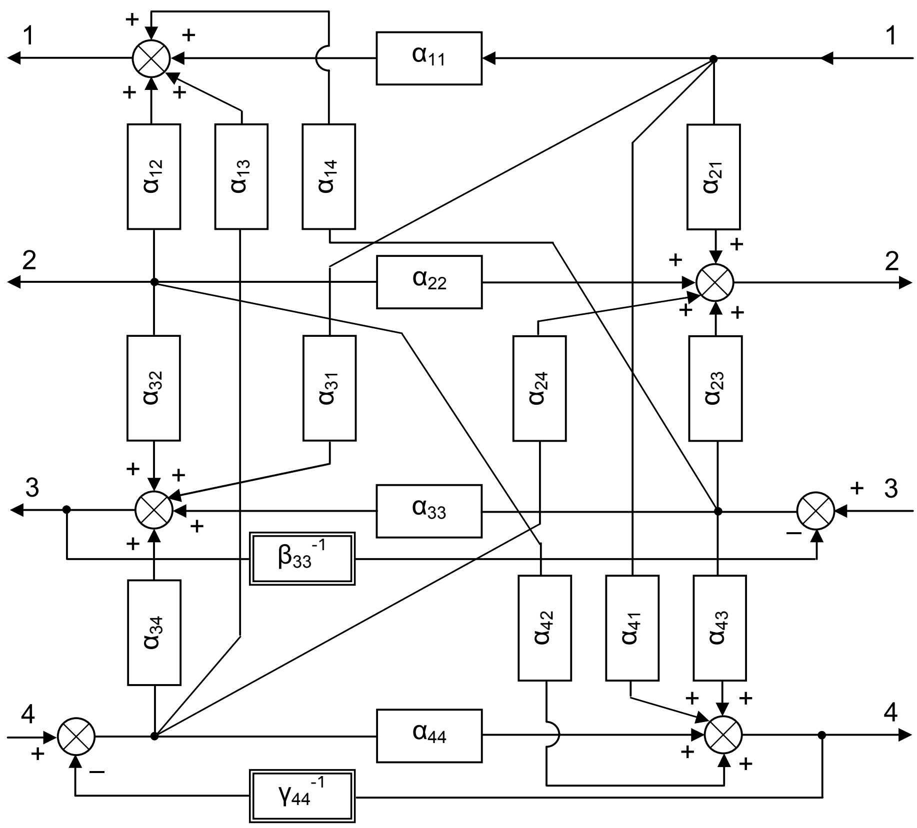

Fig.2. Structural diagram of dynamic receptance synthesis

The set of receptances we obtain from above presented structural diagram (Fig. 2). For example we find receptance $G_{13}(i\omega)$. The structural diagram of this receptance is presented in Figure 3. It was obtained from main diagram (Fig. 2) which was presented above. This method consists in dividing of system model into individual subsystems and then in structural connecting receptances of individual subsystems.

It is very useful when some of this subsystems are difficult for analytical description like a tractor tyre stiffness or dumping in deep mud. This parameters were experimentally determining (see Table 1).

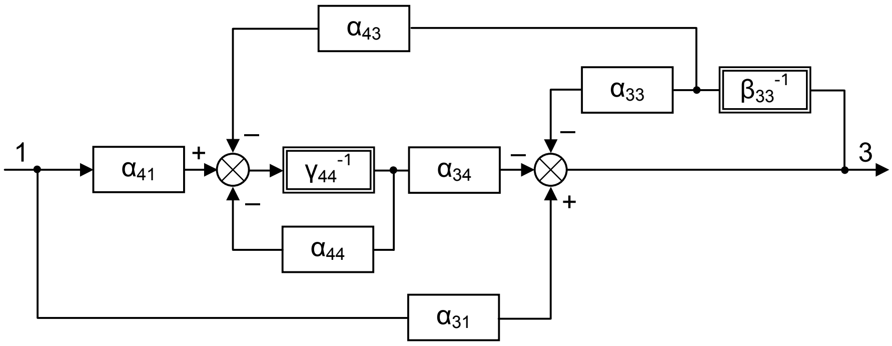

Fig.3. Structural diagram of receptance $G_{31} = G_{13}$

Solving above presented diagram we obtain:

$$G_{13}(i\omega) = \frac{\alpha_{13}\beta_{33}(\gamma_{44}+\alpha_{44})-\alpha_{34}\alpha_{41}\beta_{33}}{(\gamma_{44}+\alpha_{44})(\alpha_{33}+\beta_{33})-\alpha_{34}^2} = \frac{L_{13}}{M(i\omega)} \tag{10}$$

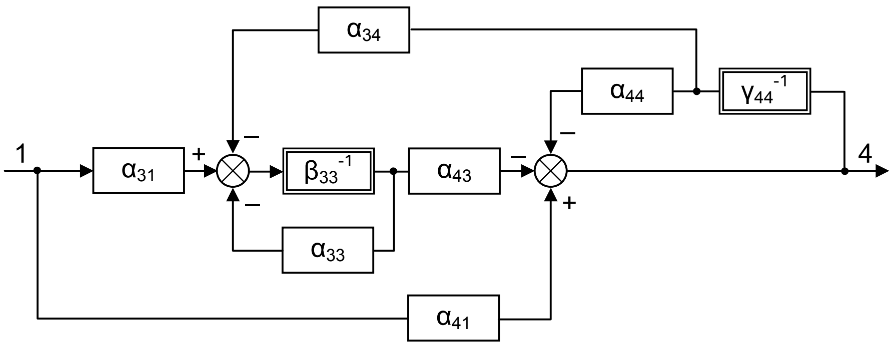

Fig.4. Structural diagram of receptance $G_{41} = G_{14}$

Solving above presented diagram we obtain:

$$G_{14}(i\omega) = \frac{\alpha_{41}\gamma_{44}(\beta_{33}+\alpha_{3})-\alpha_{43}\alpha_{31}\gamma_{44}}{(\gamma_{44}+\alpha_{44})(\alpha_{33}+\beta_{33})-\alpha_{34}^2} = \frac{L_{14}}{M(i\omega)} \tag{11}$$

We can see that denominators of equation (10) or (11) are same. This denominator $D(i\omega) = (\gamma_{44}+\alpha_{44})(\alpha_{33}+\beta_{33})-\alpha_{34}^2$ defines free vibrations (characteristic frequency) of the considered system.

Compare denominator of equation (10) or (11) to zero we obtain equation of system. Occurred in equation receptances α, β, γ are subsystem receptances, for example:

$$\beta_{33}(s) = \frac{1}{m_{s1}s^2 + c_{w1}s + k_{w1}} \tag{12}$$

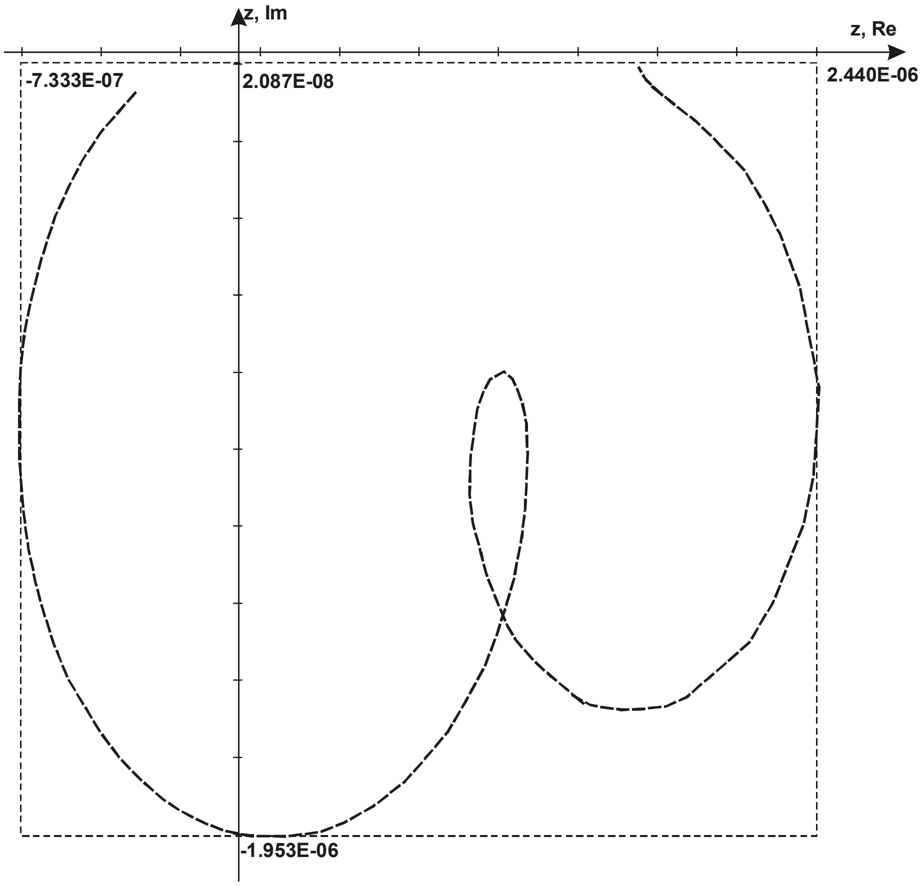

Fig. 5. The receptance G_{13} on complex number plane

How was stated above it is completely indifferent when occurred in structural diagram particular receptances are found by theoretical analysis or by experimental measures.

3. RESULTS

Experimental testing was curried out for tractor MTZ–80.

Figure 5 shows the example of calculation. There is receptance G_{31} calculated by using of this method.

In table 1 are presented experimentally obtained parameters of tyres (stiffness and damping), necessary to qualify their influence on characteristic frequency of system.

Table 1. Experimentally obtained stiffness and damping for testing tyres in deep mug.

	$2k_{01}$ [kN·m^{-1}]	$2k_{02}$ [kN·m^{-1}]	$2c_{01}$ [kN·s·m^{-1}]	$2c_{02}$ [kN·s·m^{-1}]
1	450	580	4,32	6,97
2	450	580	6,48	10,44
3	675	870	4,32	6,97
4	900	1160	8,64	13,04

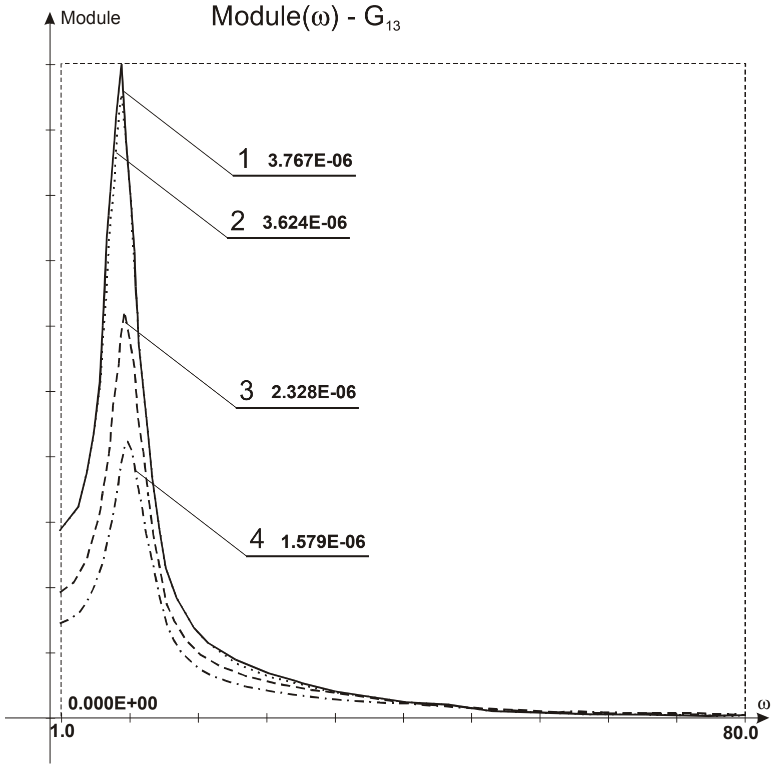

Fig. 6. Influence of tyre parameters on receptance G_{13}

Figure 6 shows calculation results illustrated the influence of the tyre parameters (stiffness and damping) on the receptance $G_{13}(i\omega)$.

4. CONCLUSIONS

– Performed analysis confirms usefulness of the presented method for testing of complicated mechanical systems.

– Without doubt the advantage of this method is a possibility of identification of unknown parameters experimentally.

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