Národní konference s mezinárodní účastí

INŽENÝRSKÁ MECHANIKA 2002

13. – 16. 5. 2002, Svratka, Česká republika

Peculiarities of the elastohydrodynamic lubrication of crowned roller

M.Ya. Panovko[1], E.L. Airapetov[2]

Method and results of numerical simulation of the elastohydrodynamic (EHD) lubrication of the profiled roller of finite length are presented. Peculiarities of the influence of crowning geometry on pressure and film thickness distributions in the EHD contact are shown. In particular, it is shown that the film thickness at the centre of contact is near-constant value but the minimum film thickness decreases appreciably when the parameter of crowning increases, other conditions being equal.

Keywords: elastohydrodynamic lubrication, profiled roller, numerical simulation

1 INTRODUCTION

Elastohydrodynamic (EHD) problems arise when processes in lubricated contacts of the elastic solids are analysed. Similar contacts take place in various machine components. EHD lubrication is characterized by thin film thickness between contacting solids (it is supposed that the film thickness exceeds the roughness in several times) and elastic deformation of the solids in contact zone. Rolling-contact bearings and gears are the typical examples of friction units having concentrated contacts operating under EHD lubrication conditions.

Equations describing the flow of lubricant in the EHD contact are derived, taking into account a number of simplifying assumptions (see, for example, [1-3]), from hydrodynamic, heat transfer and elasticity equations. The main assumptions are: the film thickness is very small compared with the effective radius of curvature of contacting solids, friction force is very large compared with the inertia force, the contacting solids are replaced by semi-spaces. It is assumed that the reological model is given and physical properties of lubricant and solids are known. If micro-EHD lubrication is considered the surface topography is also given. The EHD equations are complemented by initial-

[1] Mechanical Engineering Research Institute of the Russian Academy of Sciences, Maliy Kharitonievsky Lane 4, 101990 Moscow, Russia; e-mail: panovko@imash.ac.ru
[2] Mechanical Engineering Research Institute of the Russian Academy of Sciences, Maliy Kharitonievsky Lane 4, 101990 Moscow, Russia; e-mail: airapetov@imash.ac.ru

boundary conditions. The EHD problems are related to the type of nonlinear integrodifferential equations with free boundary. The distributions of pressure, temperature, film thickness and free (outlet) boundary location in the contact are determined by solving of the governing equations. The solution obtained can be used for calculation of the frictional and subsurface stresses in the EHD contact.

Nowadays, the majority of numerical results were obtained for the line and point EHD contacts (see summarizing paper [4]). At the same time a lubrication of the rollers of finite length in rolling-contact bearings is of great interest for machinery. It is necessary to mention that from the point of view of the bearing life special attention should be paid to analysis of the stresses at the roller edge. To diminish the stress growth at the roller edge the crowning is often applied. It is well known that similar profiling is an effective way of decreasing or eliminating entirely this growth. A number of numerical results obtained for dry contact demonstrate the influence of crowning (see, for example, [5-8]).

As a distinct from the EHD lubricaton for line and point contacts the EHD lubrication for short rollers was investigated to a less extent. The main cause of this is a computational instability particularly for heavily loaded EHD contacts. A number of results for cylindrical roller having the edge relief in the form of a circular arc were obtained at moderate load in [9].

The goal of the present paper is to study the EHD lubrication for roller having the generatrix made of two circular arcs. These arcs have a smooth junction. Radii of these arcs differ from each other in several orders: the greater one describes the central part of the roller, the smaller one - the roller edge. Influence of the parameters of crowning on the EHD lubrication was studied by numerical method.

2 PROBLEM FORMULATION

The steady state EHD contact problem for short roller having a barrel-type form under isothermal conditions is considered. For instance, such contact is formed under rolling/sliding of the elastic profiled roller on the smooth elastic semi-space, moreover the contacting bodies are separated by layer of the lubricant (see Fig. 1).

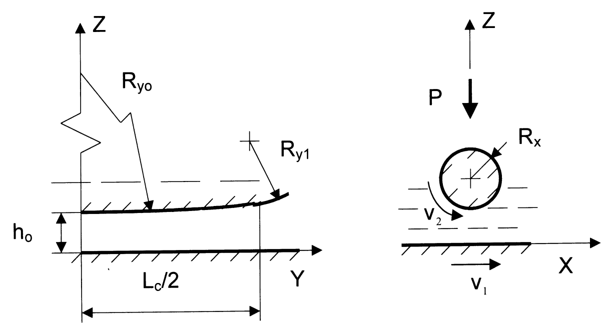

Fig. 1 Schematic description of lubricated contact

It is assumed that the lubricant behaves like a Newtonian incompressible fluid. The external load is given. Typical assumptions of the EHD lubrication theory are used in solving the problem. The following dimensionless variables are introduced

$$(x', y') = \frac{(x, y)}{a_H},\ \ l_c' = \frac{l_c}{a_H},\ \ p' = \frac{p}{p_H},\ \ h' = \frac{h}{h_0},\ \ \mu' = \frac{\mu}{\mu_0},\ H_0 = \frac{2R_x h_0}{a_H^2}$$

$$\varepsilon_0 = \frac{R_x}{R_{y0}},\quad \varepsilon_1 = \frac{R_x}{R_{y1}},\ V = \frac{24\mu_0|\mathbf{v_1}+\mathbf{v_2}|R_x^2}{p_H a_H^3},\ P' = \frac{P}{p_H\, a_H^2},\ \mathbf{v} = \frac{\mathbf{v}_1+\mathbf{v}_2}{|\mathbf{v}_1+\mathbf{v}_2|}$$

where *x*,*y* – Cartesian coordinates in the contact plane; R_x, R_{y0}, R_{y1} - radii curvature; $\varepsilon_0, \varepsilon_1$ - parameters of crowning; *p* – lubricant pressure; a_H - semi-width of Hertzian line contact; p_H - maximum Hertzian pressure; *P* - external load; *h* - film thickness (gap); h_0 - film thickness at the origin of coordinates; μ - viscosity of lubricant; μ_0 - viscosity at ambient pressure; $\mathbf{v}_1(\mathrm{v}_{1x}, \mathrm{v}_{1y}), \mathbf{v}_2(\mathrm{v}_{2x}, \mathrm{v}_{2y})$ - vectors of the surface velocities; H_0 - dimensionless film thickness at the origin of coordinates; *V* - load-velocity parameter; l_c - longitudinal coordinate of the point of junction for two arcs.

Equations of the EHD contact in dimensionless form can be written as (hereinafter without primes)

$$L(p) = \nabla \cdot \left(H_0^2 \frac{h^3}{\mu} \nabla p - V\mathbf{v}h \right) = 0 \tag{1}$$

$$h(x, y) = 1 + \frac{x^2 + \varepsilon_0 y^2 + (\varepsilon_1 - \varepsilon_0)\theta(|y|)(|y| - l_c/2)^2}{H_0} + \frac{1}{\pi H_0} \iint_{\Omega} G(x, y, \xi, \eta) p(\xi, \eta) d\xi d\eta \tag{2}$$

$$M(p) = \frac{\pi}{2} l_c - \iint_{\Omega} p(\xi, \eta) d\xi d\eta = 0 \tag{3}$$

$$p|_C = \left.\frac{\partial p}{\partial n}\right|_{C_e} = 0 \tag{4}$$

$$\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y} \right), \mathbf{v} = (\mathrm{v_x}, \mathrm{v_y}),\ G(x, y, \xi, \eta) = \frac{1}{\sqrt{(x-\xi)^2 + (y-\eta)^2}} - \frac{1}{\sqrt{\xi^2 + \eta^2}}$$

$$\theta(|y|) = \begin{cases} 1 \text{ при } |y| \geq l_c/2 \\ 0 \text{ при } |y| < l_c/2 \end{cases}$$

where C – boundary of the contact domain Ω, C_e - outlet (free) boundary (part of the boundary where $(\mathbf{v} \cdot \boldsymbol{n}) > 0$), $\boldsymbol{n}$ – single vector of the outward normal. It is assumed that the applied load P is $P = \pi p_H a_H l_c / 2$.

The system (1)-(4) is nonlinear system of integrodifferential equations: Reynolds equation (1), equation of elasticity (2), force balance equation (3), i.e. the integral over the pressure equals to the external applied load, and boundary conditions (4).

The complementarity conditions [10] are used to determine the outlet boundary which separates the pressure-positive region from cavitation region. These conditions are written in the following form

$$
\begin{aligned}
&L(p) = 0, \quad p > 0 \quad \text{in lubricant domain} \\
&L(p) < 0, \quad p = 0 \quad \text{in cavitation domain}
\end{aligned}
\tag{5}
$$

The shape of the inlet boundary, load-velocity parameter V, vector of velocity $\mathbf{v}(\mathrm{v}_x, \mathrm{v}_y)$, parameters of crowning $\varepsilon_0 = R_x / R_{y0}$, $\varepsilon_1 = R_x / R_{y1}$ and Barus viscosity-pressure relationship $\mu = \mu_0 \exp(Qp)$, where Q – piezoviscous parameter (for heavily loaded contacts Q>>1, V<<1), for system (1)-(5) are given.

The solution of system (1)-(5) is represented by the pressure $p(x,y)$ and gap $h(x,y)$ distributions, outlet boundary $x_e(y)$, and film thickness at the origin of coordinates H_0.

3 NUMERICAL METHOD

The computational domain in the plane (x,y) is given as a rectangle $\{x, y : x1 \le x \le x2, y1 \le y \le y2\}$ and covered by nonuniform staggered grid with nodes $(x_i, y_j), (x_{i-1/2}, y_{j-1/2})$. The pressure is calculated in nodes (x_i, y_j), the film thickness – in nodes $(x_{i-1/2}, y_{j-1/2})$. After integrating equation (1) over domain $\Delta\Omega_{ij}$ of the cell (i,j) and transforming the double integral to the line integral over the contour of the cell (l) we obtain

$$
L_1(p) = \oint_{(l)} \left[H_0^2 \frac{h^3}{\mu} (\nabla p \cdot \boldsymbol{n}) - V(\mathbf{v} \cdot \boldsymbol{n}) h \right] dl = 0 \tag{6}
$$

In conditions (5) the operator $L(p)$ is replaced by $L_1(p)$. The solution of the EHD equations is carried out by Newton method. After linearization near solution $(p(x,y), H_0)_k$ the equations (3), (4) and (6) are used for developing the difference scheme. The system of the finite difference equations may be written as

$$
\begin{Vmatrix} (a_{qr})_{nn} & (b_q)_n \\ (c_r)_n^T & 0 \end{Vmatrix}_k \begin{Vmatrix} (\Delta p_r)_n \\ \Delta H_0 \end{Vmatrix}_k = - \begin{Vmatrix} (L_1(p_r)_n \\ M(p_r) \end{Vmatrix}_k
$$

where the Jacobian matrix of order $(n+1)$ contains the full square matrix $(a_{qr})_{nn}$ of order n, column-vector $(b_q)_n$ and row-vector $(c_r)_n^T$ of length n (q, r = 1, …, n). The value n is a number of grid nodes where the presure is calculated. This number depends on the outlet boundary location (according to the complementarity conditions), k is the number of iteration step.

One iteration step includes the solution of difference equations for $\Delta H_{0,k+1}$, $\Delta p_{k+1}(x_i, y_j)$ by Gaussian elimination with partial pivoting, calculation of $H_{0,k+1} = H_{0,k} + \omega\Delta H_{0,k+1}$, $p_{k+1} = p_k + \omega\Delta p_{k+1}$ taking into account the variable coefficient of relaxation ω ($0.05<\omega<1$), calculation of $h_{k+1}(x_{i-1/2}, y_{j-1/2})$ using relationship (2) and determination of the outlet boundary $x_{e,k}(y_j)$ according to complementarity conditions (5). Integral in the expression (2) is calculated according to cubature formula applying for calculation of the singular integrals [11]. Iteration procedure is finished when the given relative accuracy of a solution δ satisfies the following condition $\max\left(\left|H_{0,k+1}/H_{0,k} - 1\right|, \left|p_{k+1}/p_k - 1\right|, \left|h_{k+1}/h_k - 1\right|, \left|x_{e,k+1}/x_{e,k} - 1\right|\right) < \delta$.

4 NUMERICAL RESULTS

The numerical simulation of the EHD contact for shaped roller at V=0.1, Q=3, l_c=40, $\mathrm{v}_x = 1, \mathrm{v}_y = 0$, $\varepsilon_0 = 0.0 \div 0.005$, $\varepsilon_1 = 0.1 \div 0.5$, δ=0.001 was carried out. Taking into account the symmetry the calculations on the grid 22×92 for a half of the roller was carried out.

The pressure $p(x,y)$ and film thickness distributions $h(x,y)$ in the EHD contact at ε_0=0.0005 and ε_0=0.005 for parameters ε_1=0.1, ε_1=0.3, ε_1=0.5 are shown in Fig. 2 and Fig. 3 respectively. Plot contours for these distributions are shown in Fig. 4 and Fig. 5 respectively.

One can see that the pressure and gap distributions in the central part of roller are like the distributions for line contact, i.e. in the vicinity of outlet the pressure ridges and gap constrictions are observed. Also, the flattening effect takes place (see $h(x,y)$ distributions). However, in the vicinity of junction $y = l_c/2$ the spatial effects are displayed - the pressure increases and gap decreases. These phenomena reduce when parameter ε_0 increases and parameter ε_1 decreases (see Fig. 2 and 3).

The influence of parameters ε_0 and ε_1 on the film thickness at the origin of coordinates H_0, minimum film thickness $h_{\min}$ and pressure maximum $p_{\max}$ is shown in Fig. 6-8. One can see that the film thickness H_0 is near-constant value in the cases when $\varepsilon_0 \leq 0.001$ (see Fig. 6). It should be noted that according to numerical simulation for line contact (when V=0.1, Q=3) H_0=0.1622 [12]. As a distinct from H_0 the minimum film thickness $h_{\min}$ and pressure maximum $p_{\max}$ alter appreciably when parameter ε_1 increases.

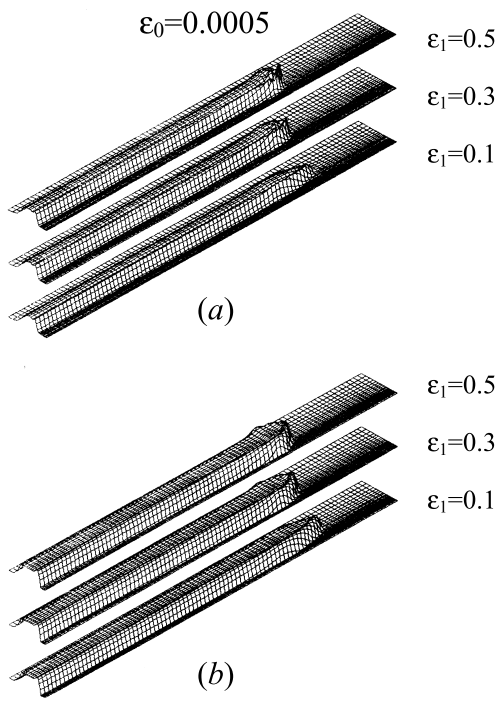

Fig. 2 Pressure (*a*) and gap (*b*) distributions

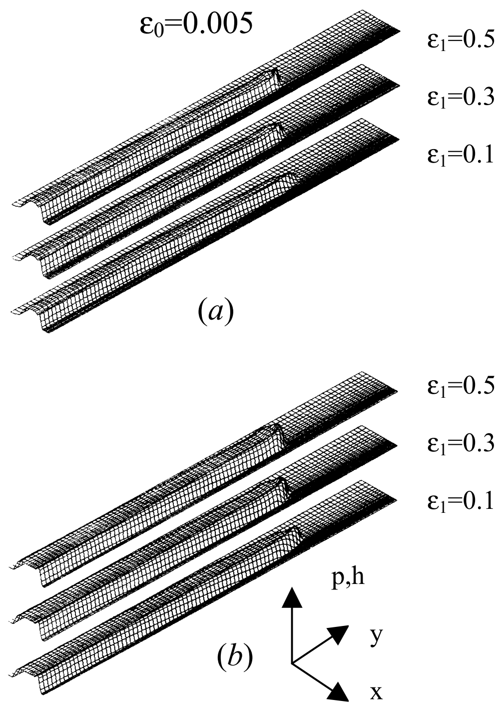

Fig. 3 Pressure (*a*) and gap (*b*) distributions

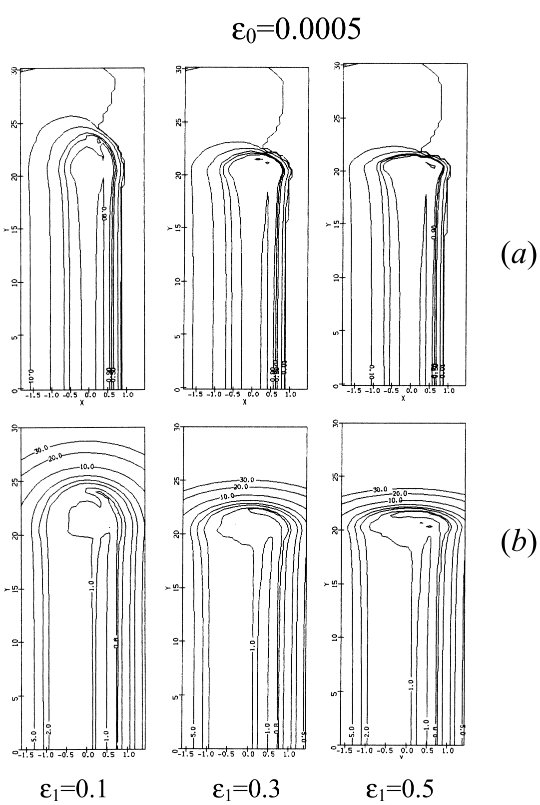

Fig. 4 Pressure (*a*) and gap (*b*) plot contours

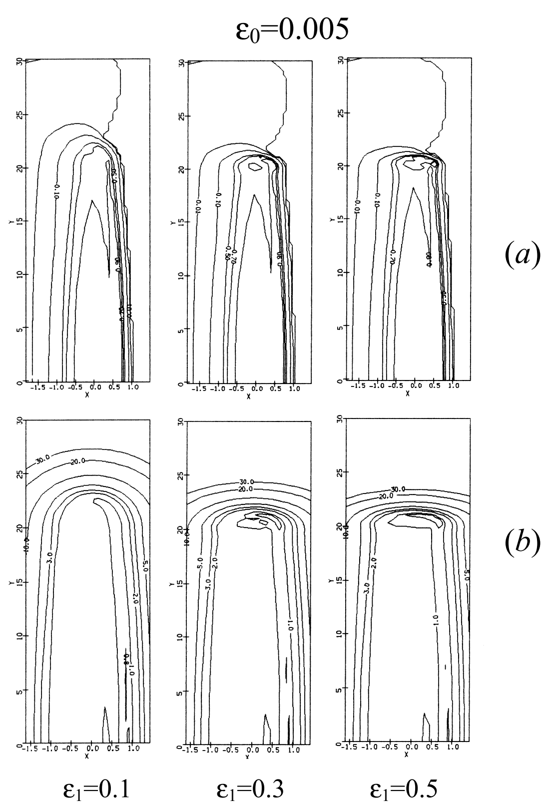

Fig. 5 Pressure (*a*) and gap (*b*) plot contours

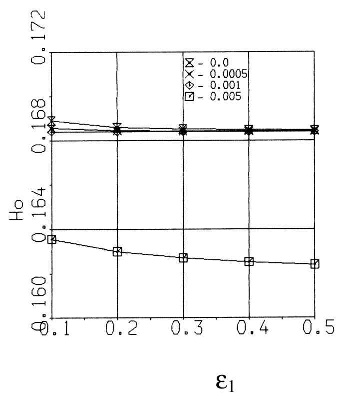

Fig. 6 Variation of H_0 with ε_1

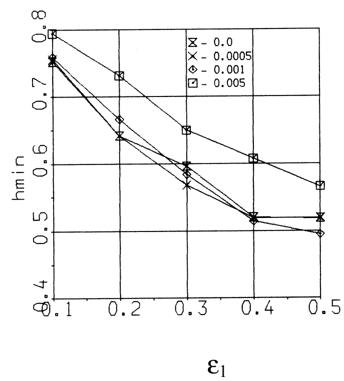

Fig. 7 Variation of h_{min} with ε_1

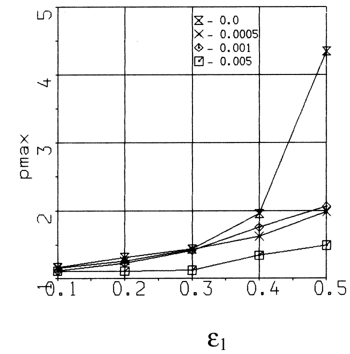

Fig. 8 Variation of p_{max} with ε_1

5 CONCLUSION

The numerical algorithm for solving of the EHD lubrication problems for shaped roller was described. The results of numerical simulation demonstrates a marked influence of the crowning on pressure and gap distributions in the EHD contact. The results of the simulation may be used for further numerical analysis of friction in the EHD contact and stress tensor field in the subsurface layer.

6 REFERENCES

1. Galakhov, M.A., Gusyatnikov, P.B., Novikov, A.P. Mathematical models in elastohydrodynamics, Nauka, Moscow, 1985.
2. Kodnir, D.S. Elastohydrodynamic lubrication of machine components, Mashinostroenie, Moscow, 1976.
3. Dowson, D., Higginson G.R. Elastohydrodynamic lubrication, Pergamon Press, Oxford, 1966.
4. Dowson, D., Ehret, P. Past, present and future studies in elastohydrodynamics. Proc. Instn. Mech. Engrs. Part J., J. Engineering Tribol., 213 (1999), J5, 317-333.
5. Hartnett, M.J. The analysis of contact stresses in rolling element bearings. Trans. ASME. J. Lubric. Technol. 101 (1979), 1, 105-109.
6. Kannel, J.W., Hartnett, M.J. Theoretical and experimental evaluation of edge stresses under severe edge loads. ASLE TRANSACTIONS. 26 (1983), 1, 25-30.
7. de Mul, J.M., Kalker, J.J., Fredriksson, B. The contact between arbitrarily curved bodies of finite dimensions. Trans. ASME. J. Tribol., 108 (1986), 1, 140-148.
8. Natsumeda, S. Application of multi-level multi-integration to contact problems. Part 1: non-Hertzian contact in rolling bearings. Proc. Instn Mech. Engrs. Part J., J. Engineering Tribol., 213 (1999), J1, 63-80.

9. Mostofi, A., Gohar, R. Elastohydrodynamic lubrication of finite line contacts. Trans. ASME. J. Lubric. Technol., 105 (1983), 4, 82-88.
10. Oh, K.P. The numerical solution of dynamically loaded elastohydrodynamic contact as a nonlinear complementarity problem. Trans. ASME. J. Tribol., 106 (1984), 1, 88-95.
11. Belotserkovsky, S.M., Lifanov, I.K. Numerical methods in singular integral equations, Nauka, Moscow, 1985.
12. Airapetov., E.L., Kudish, I.I., Panovko, M.Ya.. Numerical solution of heavily loaded elastohydrodynamic contact. Soviet Journal of Friction and Wear, 13 (1992), 6, 957-964.