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VISCO-PLASTIC MODEL FOR POLYCRYSTALLINE ROCKS

M. Mühlbauer, Z. Bittnar[1]
Czech Technical University,
Department of Structural Mechanics,
Thákurova 7, Prague 166 29
Czech Republic

Abstract

The long-term behaviour of a crystalline rock has been modelled as a visco-plastic problem. The Perzyna type of material models, which is based on so-called overstress, has been used. The constitutive law, known as Lemaitre's creep model is based on the use of internal state variables attached to specific phenomena, including hardening of material. The identification of model was carried out with aid of genetic algorithms and experimentally obtained data. The used genetic algorithm is based on differential evolution scheme and was adapted for fast model parameter identification.

Keywords: visco-plastic behaviour, material model, creep, polycrystalline materials, long-term behaviour.

1 Introduction

The mechanical time-dependent behaviour of anhydritic rock was scarcely investigated in the past and only few works exist on the topic, Sahores (1962), Müller W.H.& Briegel (1978), Müller P.&Siemens (1974). The behaviour of anhydrite, like many other geomaterials like rock salt (Aubertin, Hardy, 1998) or hard rock (Malan 1999), is characterized by delayed straining when subjected to constant or slowly changing loading.

The design of undeground structures, excavated in such strain-rate sensitive materials, requires the determination of a time-dependent constitutive law capable of predicting of long-term behaviour. This study analyses the convenience of Lemaitre's law, Lemaitre&Chaboche (1990), formerly used for long-term behaviour of metals, for crystalline rocks, in particular anhydrite. First of all, the time-dependent behaviour of anhydrite was examined in the laboratory and special testing program

[1]Ing. Michal Mühlbauer, prof.Ing. Z. Bittnar, DrSc., ČVUT, fakulta stavební, katedra stavební mechaniky, Thákurova 7, Praha 6, 166 29, michal@cml.fsv.cvut.cz, bittnar@fsv.cvut.cz

was designed for this purpose. A range of classical uniaxial and triaxial and physical identification tests was performed before a creep testing program has started. A multiple stress-level uniaxial creep tests were carried out to observe different creep phases.

2 Experimental investigation

The design of testing program was led with aim to cover all aspects of mechanical behavior of anhydrite. All specimens were taken from the drill holes in the vicinity of Modane (France) in alpine region, from the depth from 650 to 1150 m. Preliminary uniaxial compression tests were performed to find out the compressive strength of the material (σ_c =50-80 MPa) and other mechanical properties (E=40-90 GPa, ν =0.21-0.27). Triaxial compression tests, with various confining pressure, were designed to simulate probable conditions in the depth in rock mass.

A unixial compression test with very low strain-rate was carried out on device specially concieved for this purpose with strain-rates going down to $6.0{\cdot}10^{-9}s^{-1}$.

Creep tests were performed on dead-load cantilever device with rigid frame. The cantilever system transmits the force of dead load directly onto the cell unit with specimen and almost absolute constant load on specimen is thus assured. Creep test stress-levels were chosen to cover whole range of creep phases (attenuating creep, stable crack propagation resulting in steady-state creep, unstable crack propagation leading to tertiary creep).

3 Modelling of time-dependent phenomena

Presented visco-plastic model is suitable tool describing time-dependent behaviour with strain hardening of material. This model does not include a damage component. The choice of long-term parameters is discussed further on.

The model is based on the visco-plastic theory formulated by Perzyna (1966) for rate-sensitive plastic materials based on so-called over-stress concept. It gives incremental constitutive equations for the transient creep phase. The assumption is made that the strain rate tensor can be additively decomposed into an instantaneous reversible (elastic) part and an irreversible (inelastic) part,

$$\dot{\varepsilon}_{ij} = \dot{\varepsilon}^{e}_{ij} + \dot{\varepsilon}^{vp}_{ij}, \tag{1}$$

where irreversible strain rate combines viscous and plastic effects and is given by following relationship

$$\dot{\varepsilon}^{vp}_{ij} = \gamma \left\langle \Phi(F) \right\rangle \frac{\partial G}{\partial \sigma_{ij}}, \tag{2}$$

where γ is viscosity coefficient of the material, G is visco-plastic potential and Φ is function containing static yield function F. The function Φ is controlled by Macauly's brackets

$$\left\langle \Phi(F) \right\rangle = \frac{1}{2}\left[\Phi(F) + \left| \Phi(F) \right| \right]. \tag{3}$$

The yield function F includes the over-stress concept through effective stress f and strain hardening parameter κ, which depends on the updated accumulated visco-plastic strain as shown further on. Modified yield function F can be expressed in the following form

$$F\Big(\sigma_{ij}, \varepsilon_{kl}^{vp}\Big) = \frac{f\Big(\sigma_{ij}\Big)}{\kappa\big(\epsilon^{vp}\big)} - 1, \tag{4}$$

where ϵ^{vp} is considered as the internal state variable corresponding to strain hardening. To produce a practical tool, the power laws for the function Φ and for κ have been introduced similar to those proposed by Lemaitre. The relationships are summarised by equations (5) and (6)

$$\Phi(F) = (F+1)^n, \tag{5}$$

$$\kappa(\epsilon^{vp}) = (\epsilon^{vp})^{-m/n}. \tag{6}$$

Introducing (6) into (4) and then into (5) the function Φ reads

$$\Phi(F) = \Big(f\Big(\sigma_{ij}\Big)\Big)^n (\epsilon^{vp})^m. \tag{7}$$

The irreversible visco-plastic strain defined in (2) can then be formulated in the following manner

$$\dot{\varepsilon}_{ij}^{vp} = \gamma\Big\langle\Big(f\Big(\sigma_{ij}\Big)\Big)^n (\epsilon^{vp})^m\Big\rangle \frac{\partial G}{\partial \sigma_{ij}}. \tag{8}$$

If we define the constitutive law as associated, the yield function is equal to plastic potential. This assumption takes into account the fact that visco-plastic deformations develop without any volume changes. The derivative of yield function with respect to stress tensor components gives normalised deviatoric stress tensor

$$\frac{\partial G}{\partial \sigma_{ij}} = \frac{s_{ij}}{\|s_{ij}\|}. \tag{9}$$

The irreversible strain rate which combines viscous and plastic effects is given by the following relation

$$\dot{\varepsilon}_{ij}^{vp} = \gamma \|s_{ij}\|^{n-1} (\epsilon^{vp})^m s_{ij}. \tag{10}$$

If the simplifying hypothesis, that constitutive law is unidimensional one, is adopted, i.e. the material flows only in one direction, the formulating of constitutive law takes more concrete form. The only component of stress tensor is applied stress, $\sigma_{11} = \sigma$. Stress deviator component s_{11} is in one dimension problem taken as $s_{11} = 2/3\sigma$. The norm of stress deviator then equals to

$$\|s_{ij}\| = \sqrt{\frac{2}{3}}\sigma \tag{11}$$

Taking into account preceeding statements, the constitutive law is presented in the form

$$\dot{\varepsilon}^{vp} = A\sigma^n (\epsilon^{vp})^m, \tag{12}$$

where A is newly defined viscosity parameter

$$A = \gamma\Big(\sqrt{2/3}\Big)^{n+1} \tag{13}$$

The law (12) needs the identification of three parameters listed below

A viscosity coefficient of the material ($A > 0$),
n stress exponent $n \geq 1$,
m^* strain hardening exponent ($m^* = -m/n \geq 0$).

This law can be easily expressed in its integral form with aim to obtain the relation between strain and stress as a function of time. The left side is replaced by derivative of visco-plastic strain with respect to time and the following form is obtained

$$\frac{\mathrm{d}\varepsilon_{ij}^{vp}}{\mathrm{d}t} = A\sigma^n(\varepsilon^{vp})^m, \tag{14}$$

Integrating the equation (14) the power law for describing the visco-plastic behavior is obtained

$$\int (\varepsilon^{vp})^{-m} d\varepsilon_{ij}^{vp} = \int A\sigma^n dt, \tag{15}$$

$$\frac{1}{1-m}(\varepsilon^{vp})^{1-m} = A\sigma^n t + c. \tag{16}$$

At time $t = 0$ viscoplastic strain is zero and then $c = 0$. The power law is then in a form

$$\varepsilon^{vp} = \left[(1-m)A\sigma^n t\right]^{\frac{1}{1-m}} \tag{17}$$

The equation (17) is general expression for viscoplastic behavior of material. Total strain is than calculated from (1)

$$\varepsilon^{tot} = \frac{\sigma}{E} + \left[(1-m)A\sigma^n t\right]^{\frac{1}{1-m}}. \tag{18}$$

For constant stress deviator tensor the expression (18) describes the transient creep phase with decreasing visco-plastic strain rate. If $m = 0$ is put in (12), the secondary creep phase can be described with the same law.

4 Model parameters identification

The method of identification of parameters of Lemaitre's law is based on constant strain-rate test and creep test on anhydrite specimens.

4.1 Hardening parameter identification

During the quasi-static test, which is controlled by strain-rate and in which the strain rate is sufficiently low, the viscoplastic deformation can develop in real time. In other words the test is performed in such a way that visco plastic deformation has enough time to produce in real time. Supposing that it is true, the hypothesis that viscoplastic strain-rate tends to total strain-rate, can be pronounced, i.e. $\dot{\varepsilon}^{vp} \cong \dot{\varepsilon}$. Let us consider the equation representing the constitutive law (12) in the following way with respect to the previous paragraph and supposing that the deviatoric stress is in case of uniaxial loading the axial load $\sigma_1^{dev} = \sigma$. Since linear regression will be used to set up dependencies between the parameters, the logarithmic expression of (12) is needed to linearize the problem.

$$\ln \dot{\varepsilon} = \ln A + n \ln \sigma + m \ln \epsilon^{vp} \tag{19}$$

The equation is then tranformed to obtain $\ln q$ as a function of $\ln \varepsilon^{vp}$.

$$\ln \sigma = b - m^* \ln \epsilon^{vp} \tag{20}$$

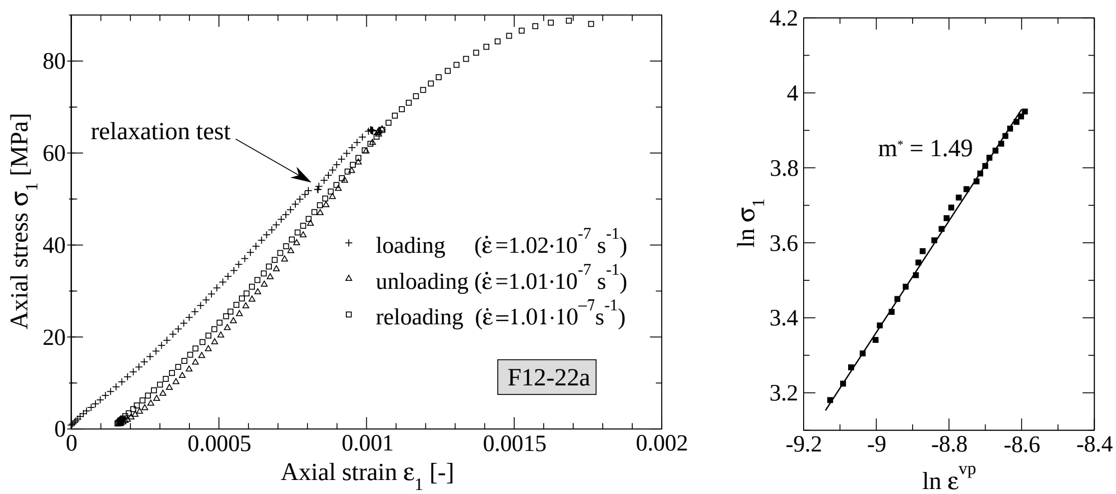
where

$$m^* = -\frac{m}{n} \tag{21}$$

and

$$b = \frac{1}{n}\Big(\ln \dot{\varepsilon} - \ln A\Big). \tag{22}$$

The multiplicator constant $m^* = -m/n$ is known as strain hardening exponent from equation 6. The ratio m^* can be accurately determined as the concavity of the curve in $(\sigma_1, \varepsilon_1)$ plot. In the $(ln(\sigma_1) - ln(\varepsilon^{vp}))$ plot the ratio m^* represents the slope of the curve, figure 1. The linearity of the curve is well verified in a strain section, between the elastic limit and the highly damaged zone. In the same diagram the intercept of the $\ln \sigma_1$-axis gives the viscosity parameter A which depends on n. Thus the parameters m and A are defined in relation to n. The method of least squares



(a) loading-unloading phases (b) determination of m^*

Figure 1: Identification of parameter m^* on specimen F12-22a.

is used to determine the abscisse b and the slope m^* of straight line described by equation (20) using the experimental data from quasi-static test with low strain rate. Once the parameters b and m^* are identified, the following expressions can be written from (21) and (22), which set the relation between parameters A and m to n.

$$m = -m^* \ n \tag{23}$$

$$A = exp(\ln \dot{\varepsilon} - bn) \tag{24}$$

A loading phase between 24 and 64 MPa was exploited for specimen F12-22a and gives value of hardening parameter $m^* = 1.49$, figure 1(b). The parameter was calculated also for reloading path but only slight difference from later was observed. It is also observed that for the same value of stress σ_1 we get more deformation for reloading phase in which accumulated deformation is already hidden. Boidy&Pellet

(2001) reffer that strain rate doesn't affect the ratio m^* much and for the purpose of our study, the value mentined above was adopted. The abscisse value b=16.7 was found for on the logarithmic plot.

4.2 Viscosity and stress exponent parameter identification

The identification of the two parameters of the model was performed with use of genetic algorithm. The use of genetic algorithms is suitable when the exact solution of scientific problem can't be determined. In indentification of model parameters, usually the analytical model prescription is known and the file of measured data obtained. The task is to find the best fit of model to experimental data, i.e. determine set of parameters which would satisfy previous condition. From mathematical point of view, we search for such parameters for which the sum of difference of least squares between experimental data and model values is minimal. Mathematical solution of such a problem demands at least some derivates of the function with respect to some parameters and treatment of experimental data file.

The great advantage of genetic algorithms is very liberal demand on optimalized function, i.e. the function doesn't need to be differentiative nor continuous which is not the case in regression methods. Neither they demand any special treatment of experimental data. The algorithm was adjusted to fast model fitting.

4.3 Algorithm SADE

For our purpose the algorithm SADE (Simplified Atavistic Differential Evolution) was used, Kučerová&Hrstka (2000). Its structure takes its origin in classical genetic algorithms. It keeps usual scheme of mutation, cross-over and selection. To be able to work with real numbers, the simplified cross-over operator (which has originally binary form) was modified according to differential evolution.

The algorithm SADE has in simplified C-language typing following form

```
void SADE ( void )
{
    FIRST_GENERATION ();
    while ( to_continue )
    {
        MUTATE ();
        LOCAL_MUTATE ();
        CROSS ();
        EVALUATE_GENERATION ();
        SELECT ();
    }
}
```

The first step is to generate random starting generation (function `FIRST_GENERATION`) of parameters sets. It is convenient to generate parameters within their logical range that can be determined from experience. The quastion is how many of new sets is to be generated. This number is first algorithm constant (called `pool_rate`) that must be determined.

In the function `MUTATE` a certain number of new sets is generated from starting generation. The number of newly generated sets is controlled with second constant radiation. This constant is usually kept low in order to prevent excessive dispersion of parameter sets and not ot slow down the convergency.

Operator LOCAL_MUTATE was created with aim to search for solution with bigger precision. It creates again new sets of parameters in close vicinity of existing sets. Third constant local_radiaton defines a number of new sets to be generated.

CROSS operator controls the number of newly created sets in such a manner that the total number is the double of initial starting generation. A new set is created from three randomly chosen sets according to equation 25, [ref.5].

$$D = A + cross\_rate * (B - C) \tag{25}$$

The fourth constant cross_rate influences the convergence of algorithm the most of all constants. Its magnitude is usually from 0.1 to 0.5.

Function EVALUATE_GENERATION evaluates all parameter sets. It calls the evaluating function fitness which for each set of parameters returns the value of solved function (material model prescription). In this concrete case the function EVALUATE_GENERATION returns the error defined as a sum of least squares between experimental data and solved function using actual parameters set.

Function SELECT reduces the total number of sets in file to half, i.e. the same number of sets as initial number of newly generated sets. This is done on the principle of tournament slection in reverse order. From two randomly chosen sets, worse is disqualified.

All constants must be carefully determined and testing runs must be performed to find out the sensibility of algorithm on constant values.

4.4 Fitting Lemaitre's viscoplastic model parameters using GA

Let us consider first the Lemaitre's law in the form given by equation (17). Three material parameters occur in model, A,n,m, with physical meaning mentionned peviously. Genetic algorithm, however, works with pure mathematical form of constitutive law and is very sensible on the form of fitted function. From the experiment record the viscoplastic deformation as a function of time was obtained.

The function with time variable t is examined. Obviously the independent parameter of the function is exponent $1/(1-m)$. This can be for the reason of simplicity replaced with new parameter d

$$d = 1/(1 - m). \tag{26}$$

Since the stress σ is a constant during any creep test phase and other parameters are dependent and can be considered constant for given material, the function can be simplified by introducing another coefficient a.

$$a = A\sigma^n \tag{27}$$

The constitutive law is then written in the following form which contains only two independent parameters and from which the relations for other parameters are easy to derive.

$$\varepsilon^{vp} = \left[\frac{1}{d} a\ t\right]^d \tag{28}$$

To be able to optimalize the parameters a, d, the intervals within which parameters can occur are required by genetic algorithm solver. They are derived from the margins set up initially for original parameters[1]: $\sigma \in (1, 100)$, $n \in (1, 20)$, $m \in (-5, 0)$ and $A \in (10^{-30}, 10^{-15})$. The margins for coefficients d and a are then $d \in (0.1, 1)$ and $a \in (10^{-30}, 10^{-1})$ according to expressions (26) and (27).

Proceeding algorithm SADE with function (28) the following problem was encountered: the margins for parameter a was too wide (range over order of tens) and the required precision of calculation (error of order 10^{-7}) was very high. These two requirements didn't match well and the function had to be modified to remedy this problem. This proved high exigency of algorithm on function prescription with respect to the margins and solution precision. A new parameter c was introduced to kill off the high exponent of parameter a margins, $a = 10^c$. New margins for parameter c were calculated, $c \in (-30, -1)$ and the final version of solving function has the following form

$$\varepsilon^{vp} = \left[\frac{1}{b} 10^c \; t\right]^b \tag{29}$$

To obtain conrete values of original parameters that figure in Lemaitre's viscoplastic law, it was necessary to transform found parameters on original ones. These are calculated from expressions for hardening exponent m, for stress exponent n and for viscosity parameter A.

$$m = \frac{d-1}{d} \tag{30}$$

$$A = 10^c (\sigma^n)^{-1} \tag{31}$$

Optimalized function in algorithm SADE was the sum function of least squares of differences between measured values and calculated values from model, which has following form

$$f = \sum_{i=1}^{p} \left[\left(\frac{10^c}{d} t_i \right)^b - \varepsilon_i^{vp} \right]^2 \tag{32}$$

where p is total number of experimental data points (t_i, ϵ_i^{vp}).

As conclusion on use of genetic algorithms for identification of material model parameteres it can be stated that one should have in mind that genetic algorithm is very sensible on function prescription. The introduced parameters must be lineary independent. In opposite case, there exists infinite number of solutions which perturbes the algorithm solver. The function must be therefore in the simplest form from the mathematical point view, even though it hides temporarily the physical representation of parameters.

In the table 1 the results from previous genetic algorithm fitting of the creep tests curves fro anhydrite specimen f31-33 are presented. Auxiliary parameters c, d are calculated for given stress-level with displayed error of least squares. Successively, the propre Lemaitre's model parameters m, n, A are obtained.

The figure 2(a) shows the result of one stress-level fitting of experimentally obtained data using genetic algorithm. The extrapolation of experimental data on longer time period is shown on figure 2(b). Both graphs are calculated for specimen F31.

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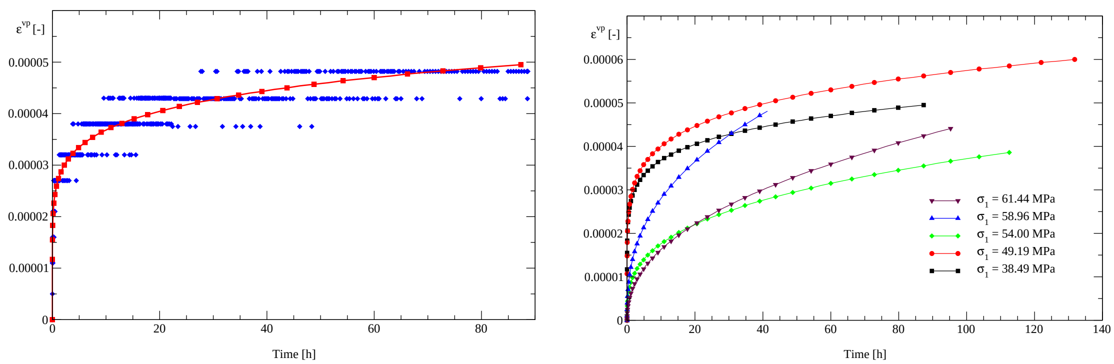
[1]Values are estimated with extremes wide enough for crystalic material at room temperature. Stress margins correspond to actual creep tests stress levels on anhydrite.

σ	c	d	error	A	n	m
38.49	38.385	0.1345	1.498e-07	5.706e-46	4.326	-6.433
49.19	33.236	0.1578	1.234e-07	4.891e-40	3.591	-5.339
54.00	19.339	0.3325	1.177e-07	2.101e-22	1.350	-2.007
58.96	16.820	0.3845	6.852e-08	1.993e-19	1.077	-1.601
61.44	15.869	0.4367	8.385e-08	3.798e-18	0.867	-1.290

Table 1: Results from GA calculation: model parameters m,n,A; σ is applied stress and error is calculated by means of least squares; calculated for $m^* = 1.487$



(a) experimental data and fitted curve

(b) extrapolation of experimental results for different stress-levels

Figure 2: Fitted experimental data results for specimen F31.

5 Conclusion

The visco-plastic material model was used to describe long-term behaviour of anhydritic rock. The model turned out to be a suitable tool for deformation prediction over long periods of time and fitted on experimental data is ready to be implemented ino finite element code.

It was proved that the simple mathematical tool as genetic algorithm can find its applicability in solving engineering problems. Linear regression was also used for fitting the experimental data, but special treatment of data was needed and presented more work that for the genetic algorithm.

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