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**COMPARATIVE NUMERICAL EXPERIMENTS WITH LATERAL PRESSURE  
OF THE NONCOHESIVE SAND**

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*ABSTRACT: Two medium-term experiments with lateral pressure of flowing loose sand acting on a retaining wall were performed. The experiments showed some rather unexpected behaviour of the granular mass, especially its deformations and failures during three different wall movements. The measurements included both components of the pressure of the mass. Two analogous numerical model experiments were made, based on the General Lateral Pressure Theory (GLPT).*

*The paper presents the main results of a detailed numerical analysis of the second numerical experiment using an advanced FORESTER programme. The analysis involves all three basic types of active wall movements. The results are compared with the results of the respective phases of the physical experiment.*

**KEYWORDS:** Earth pressure theory, general earth pressure, retaining structure, physical experiment, numerical model.

## 1 Introduction

The actual earth pressure theory is based particularly on the works of Ohde, Terzaghi, Caquot-Kerisel, Ehrenberg, Jáky, de Wett, Sowada, Siedek, Myslivec, Pruška, Janbu, Brooker-Ireland, Morgenstern, Eisenstein, Gudehus {1980} and others. Very advanced actual studies, analyses and experiments were presented at the XIIth EC SMGE in Amsterdam 1999 and at the jubilee IS JGS in Tokyo 1999, which used very advanced technologies, and in the latest versions of such programs as Ariizumi et al., Korte et al., Onishi & Sugawara, Powderham, Siemer et al., Uchiama and others. However, the ancient original idea (probably of French and Belgium fortification engineers) on the effect of solid earth wedges, followed by the idea on the possibility of the effect of the *particular type* of earth pressure against the *whole* retaining structure and the idea of the *particular stress/strain state (mobilization of shear strength) in the whole respecting part* of the mass in dependence on the movement of the toe or the top of the structure have persevered in theory and practice.

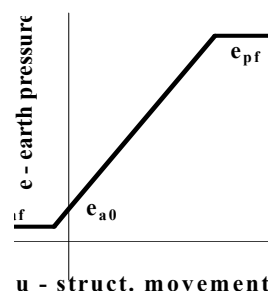


Figure 1: Elastic-plastic relation of the Dependent Pressure Method (DPM).

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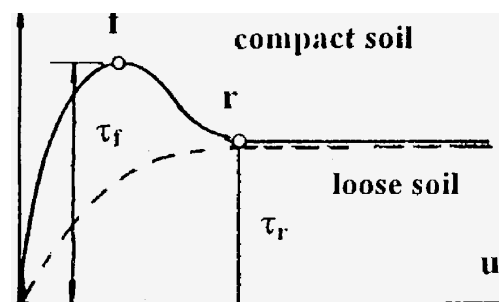
The earth pressure computation models use a scale of very different algorithms from the simplest, which have been used dozens of years, to very advanced algorithms using FEM or BEM. Probably the most widespread method is the Depended Pressures Method (DPM), which was used apparently first by Zapletal 1980 and was based on the simple elastic-plastic relation shown in Fig.1. The greatest weak points of the DPM are the uncertainty of defining procedures of the elastic constant and the course of the plastic areas.

## 2 Objections to present theory

Despite the great progress of technology, practice and computer analysis the present theory contains several discrepancies, which are more or less known or obvious, but have not received due attention either in theory or in practice. The fundamental objections to this theory include the following points:

- In the area of zero or very small movements of the retaining structure only a single value of the (active) pressure at rest is considered (Jáky 1944), although the theoretical existence and the approximate value of the passive pressure at rest have been known for over the past 25 years (Pruška 1973).
- The conditions of a single (mostly plane) shear or slip surface in the mass, the full mobilization of the shear strength on it and not deforming soil mass as of the *general* effect of extreme values of active (minimal) or passive (maximal) pressure affecting the *whole* retaining structure are unrealistic particularly for geometric, but also for other reasons (see further on).
- In the area of current movements of the retaining structure only the values of active or passive pressure (extreme values during the shear strength peak mobilization) are considered. However, it is generally known that during shear tests after the respective peak displacement has been exceeded, the shear stress drops to the residual value. The residual strength is significantly lower than the peak strength, which is illustrated by Fig. 2. Thus, this assumption is very optimistic and therefore risky. This applies not only to the limit equilibrium methods but also to DPM, which is based usually on the linear elastic-plastic relation according to Fig.1 and takes into account pressure/movement relation.

Figure 2: Relation of shear strength to displacement for compact soil (solid) and loose soil (dashed).



- The idea of general validity of certain values of maximum extreme relative movement of the structure, directly dependent on its height, which assure the full mobilization of the peak shear strength and, consequently, the action of active or passive earth pressure along its whole height *is not adequate*, even if the three basic cases of movements of the retaining structure are differentiated. The extreme movement values specified in various standards and codes differ substantially which also testifies to the fact that the theory is not convincing.
- The assumption of curved shear surfaces in lateral pressure computations by the ultimate equilibrium method (except for the surfaces of revolution) not taking into account the geometrically necessary deformations of the granular mass, is incorrect. The unconsidered and inevitable deformation of the granular mass probably must

exercise a non-negligible influence on the computation results. The same, and even more strongly, applies to the assumption of shear surfaces with a polygonal directrix, unless consequent predestined surfaces are created.

- f) The solutions of granular masses by the FEM and BEM, known so far, are based on much too simplified (elastic, elastic-plastic, simple non-linear) relations which cannot characterize truly the complex behaviour at the granular mass/structure contact surface.

### 3 Concept of experiments

The purpose of the research of lateral pressure of multiphase granular materials (comprising also soils), proceeding since 1997, is in particular to clarify in greater detail the behaviour of granular masses at the contact with the retaining structure and the deformation of the actual mass as well as the verification of a highly advanced numerical model based on a complex non-linear dependence for the contact as per Fig. 3 (see Koudelka 1996, 1998, 1999, 2000). In the course of 1998 and 1999 two medium-term physical experiments E1 and E2 with an ideally loose material were performed which brought about an extraordinarily great number of results some of which have been published previously (Koudelka 2000a, 2001, P. Koudelka&T. Koudelka 2001) and previous). The analyses of further results of the physical experiments and further experiments are under preparation.

Parallely with the physical experiments the previously started works on the development of the algorithm of an advanced numerical model and numerical analysis by means of the FORESTER programme based on this model were in progress. A numerical analysis of the values of standard relative extreme movements according to EUROCODE 7-1 and the Czech Standard ČSN 73 0037 for shear strength mobilization in soil masses was made (Koudelka 1998). In the last phase of numerical research performed so far both physical experiments based on corresponding numerical models were made. The numerical experiments were marked N1 and N2. The object of this paper is the report on the numerical experiment N2 and its comparison with the physical experiment E2.

### 4 Principal postulates of GLPT

The General Lateral Pressure Theory (GLPT) of soils can be characterized by the following postulates which also form the basis of the numerical models of the FORESTER computer programme :

I In its initial state before any movement of the structure and the deformation of the rock or soil mass the pressure at rest acts in all points of contact. Its values  $e_0$  depend in every point on the mechanical history of the origin of the mass and of the construction of the structure. In usual cases the values of  $e_0$  of the points of contact are within the appropriate intervals of the pressure at rest.

II An imperceptible or small movement of the given *place* of the retaining structure (in contact) to the active (away from the mass) or passive (into the mass) side results in a steep pressure decrease or increase respectively in this place to the very limit values of the pressure at rest interval, depending also on the direction and the magnitude of the movement of the other places of the structure, i.e. on the form and magnitude of deformation of the activated zone of the mass pertaining to the given place of contact.

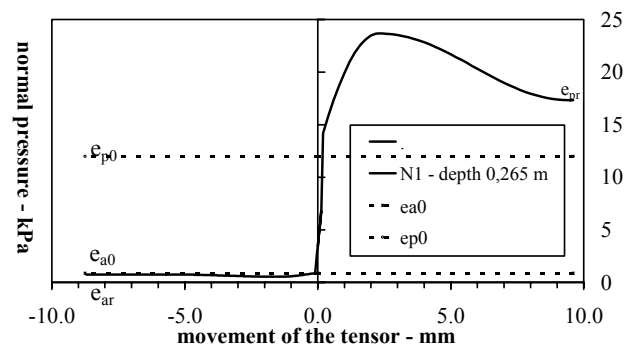
III During the further movement of the given *place* of the retaining structure (in contact) to the active or passive side respectively the value of lateral pressure in this place changes also in mutual dependence on the direction and the magnitude of other places of the structure or in dependence on the form and magnitude of the deformation of the activated zone of the mass pertaining to the given place of contact and influenced also by the movements of other place of contact. This deformation is determined particularly by the geometric and static conditions and by the geotechnical characteristics of the mass. In case of an unsupported structure a further movement to the active or passive side usually decreases or increases respectively the pressure until it attains the critical value, i.e. to the minimum or maximum value respectively when the critical values of the movement of the given place of the retaining structure have been attained.

IV During the movement of the given *place* of the retaining structure (rear face) which is higher than its critical value for the given place, the value of lateral pressure in this place generally *is not constant* and continues to change also in mutual dependence on the direction and the magnitude of the movement of other places of the structure. That means that the pressure depends, apart from the direction and the magnitude of the movement of the given place, also on the form and the magnitude of the deformation of the activated zone of the mass. This deformation is determined particularly by the geometric and static conditions and the geotechnical characteristics of the mass. In case of an unsupported structure further movement to the active or passive side usually increases or decreases the pressure respectively to the appropriate residual values. The residual values do not change much under higher displacement values, as a rule.

V *Under constant external conditions* the lateral pressure of granular multiphase materials *is not constant* and changes in the course of time in dependence on the previous deformation of the activated part of the mass. In case of unsupported structures and in certain parts of the contact of supported structures the pressure increases successively, as a rule.

The basis of the GLPT is defined by the constitutive relations of the individual places of the retaining structure an example of which is shown in Fig. 3.

Figure 3. Numerical model N2 relation between the normal component of lateral pressure and the structure rear face point in the depth of 0.265 m, resp. in the location of the tensor no.2.



## 5 Experiment N2

The numerical experiments were modelled in accordance with the present state of the general lateral pressure theory (GLPT), based on fundamental constitutive relations an example of which is shown in Fig. 3. The numerical experiment N2 copied accurately (like by experiments N1 and E1) the progress of the physical experiment E2 without re-consolidation between individual stages and consisted, consequently, of four stages:

- Phase 0 - pressure at rest – translative motion to the *passive* side of 0,5 mm (experiments N1 and E2 rotation about the toe of about 0,2 mm),

- Phase 1 - rotation about the toe to the *active* side up to the top displacement of 8.75 mm,
- Phase 2 - rotation about the top to the *active* side up to the toe displacement of 8.75 mm,
- Phase 3 - further translative motion of the retaining structure to the *active* side by 8.75 mm, i.e. to the total displacement of 17.5 mm.

The numerical experiment N2 observed the same progress of wall movements incl. stepping to achieve maximum similarity with the physical experiment E2. Numerical modelling did not include the periods of re-consolidation between individual stages; however, the re-consolidation results were included also in the numerical models for Phases 2 and 3. This concept necessitated 561 computations of the retaining wall positions for model N1 (and 559 computations for model N1). Numerical models, consequently, are based on 1120 computations of retaining wall positions each of which yielded the data on 5 (five) sensors. The computation database of both models, consequently, comprised 5600 data files – the same as the database of physical models except for the observations during re consolidation periods.

### 5.1 Experimental Model

The numerical 2D model (like the physical model) consists in a granular mass and a retaining wall which can perform the movements of all three above mentioned basic types with an accuracy lower than 0.024 mm. The wall is 1.0 m high and perfectly stiff, without any deformations of its own.

The granular body is 1.5 m long and 1.2 m high and consists of an ideally loose material. Five measuring points are situated at the granular mass/retaining wall contact

PROPERTY	SYMBOL	UNIT	VALUE			
			phase 0	phase1	phase2	phase3
<u>Model</u>						
Basic properties :						
- unit weight	$\gamma$	kN/m <sup>3</sup>	14,88			
- water content	w	%	0,04			
- compaction	-	-	loose			
Shear strength :						
- angle of top shearing resistance (ef.)	$\phi_f'$	°	48,7			
- top cohesion (effective)	$c_f'$	kPa	0			
- angle of residual shearing resist. (ef.)	$\phi_r'$	°	37,7			
- residual cohesion (effective)	$c_r'$	kPa	0			
Deformation characters :						
- deformation modulus	$E_{def}$	MPa	12			
- Young's modulus	E	MPa	35			
- Poisson's ratio	$\nu$	-	0,35			
- coefficient of structure strength	m	-	0,1			
Contact characters :						
- top angle of wall friction	$\delta_f$	°	18,74	18,74	10,42	37,35
- top adhesion of wall to granul. mass	$a_f$	kPa	0	0	0	0
- residual angle of wall friction	$\delta_r$	°	12,45	12,45	21,49	27,04
- residual adhesion of wall to mass	$a_r$	kPa	0	0	0	0

Table 1: Properties of the granular mass and the contact of model used for experiment N2.

surface 0.065 m, 0.265 m, 0.465 m, 0.665 m and 0.865 m deep. The numerical model included also the depth of 1.0m.

## **5.2 Constitutive Relations**

The constitutive relations of numerical model N2 (like N1) were computed according to the GLPT by the programme FORESTER 2.8 on the basis of the above mentioned input data for the position of every sensor separately and for the toe of the retaining wall. The relation for the wall top is trivial (zero) and was not computed. The relations are expressed by such diagrams as that in Fig. 3.

## **5.3 Processing of Results**

The computation results of numerical model N2 for every sensor were processed in graphic form. The diagrams in Figs. 4 and 5 show the dependence on the movements of individual sensor together with the corresponding results of the physical model E2. According to E1 the experiment E2 used more sensitive sensors. The analysis must take into account that the more sensitive sensors of the model E1 were placed in positions Nos. 4 and 5 and the very sensitive new sensors for lower pressures in positions Nos. 1, 2 and 3.

## **5.4 Passive Pressure at Rest and Rotations to Active Side**

The analyses of the rotation about the toe (Phase 1) and about the top (Phase 2) are represented together and contain also the modelling of the passive movements in the region at rest (Phase 0), as in spite of the re-consolidation periods between individual stages these parts of the experiment were mutually connected and the initial state for the rotation about the top could not be determined objectively. For this reason a single numerical model was used in the numerical experiment for all these first three stages which, however, is based always on the initial data of the zero wall position, i.e. the model for the rotation about the toe. As it follows out of the input data, the mutual relations of the numerical and the physical models show the influence of the application of the mean values of the pressure at rest characteristics for the whole mass and the more sensitivity of the sensor used in E2 experiment. An example of the behaviour of the numerical model in comparison with the behaviour of the physical model is shown in Fig. 4.

## **5.5 Translative Motion to Active Side**

The computation of the translative motion to the active side (3 stages) was based on a separate model which, however, could take into account the changed parameters of the mass after the preceding two stages and re-consolidations. Also in this case the presented mutual relations of numerical and physical models show the influence of the use of the mean values of the pressure at rest for the whole mass and the sensitivity of the sensor used in E2.

## **6 Evaluation**

The diagrams shown as in this paper as previously and others not shown hitherto give such distinct and comprehensive information about the relation of the numerical model in individual stages of the experiment to the results of physical experiments that they do not require too detailed explanation. Therefore, the evaluation is limited to the statement of the basic facts arising from comparative analysis.

## 6.1 The Region of the Pressure at Rest

There is a substantial difference between the experiments E1 and E2 in the magnitude of pressure increments at the beginning of the movement to passive side within the pressure at rest interval; the increments in E1 are much higher than those in E2. As there are no substantial differences between both experiments in the material and the preparation of the masses, only one substantial difference remains, viz. the different types of the retaining wall movement in this region: rotation about the toe in E1 and translative motion in E2 (see Koudelka 2001). Hence it logically follows that the passive pressure at rest in a perfectly non-cohesive loose mass increased during the rotation about the toe faster than in case of the translative motion of the wall. The character of the relation in E1 is almost linear (elastic), in E2 it is less linear.

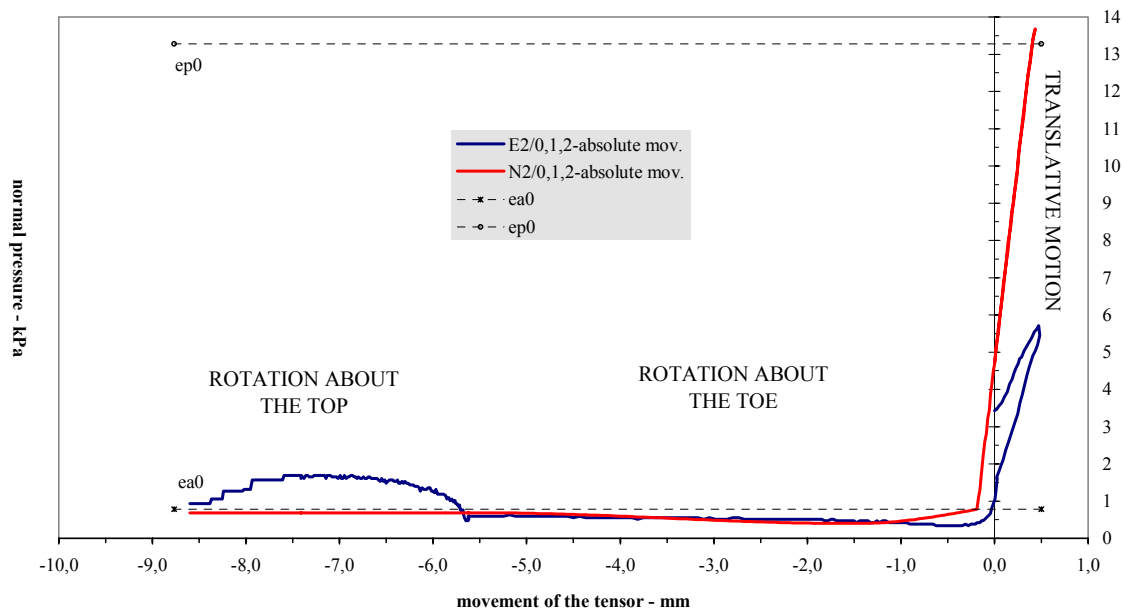


Figure 4: N2 – E2/0,1,2 – Rotation about the toe to the passive and the active sides and rotation about the top to the active side -Sensor No.2, depth 0.265 mm, normal lateral pressures of the physical and numerical models in dependence on the actual sensor movement (not on the movement of the wall top).

The magnitude of the pressures measured during the movement to the passive side did not exceed the values of the passive pressure at rest either during the rotation of the wall top by some 0.2 mm (E1) or during the translative motion by some 0.5 mm (E2). The values high above the extreme value of the passive pressure at rest were attained only in Sensor No. 1 in E1 (not in Sensor No.1 in N2); however, in this sensor the high values existed already before the start of the experiment as initial stress, which can be explained as residual stress after the compaction of the mass (the contact with Sensor No. 1 was compacted last) or as a technical error of the recording equipment.

The agreement of numerical model N2 with the physical model E2 appears very good, if we take into account the middle sensitivity of Sensors Nos. 4 and 5 and their initial value adjustment. The gradients of the lines are practically identical.

## 6.2 Rotation about the Toe to Active Side

Fig. 4 shows plainly that the respective lines proceed almost parallelly and with the afore mentioned reservation the differences in values are not great, either. The differences can be ascertained at closer observation in the values of the displacements

under extreme active pressure (peak pressure) which occurred in physical experiments nearer the area at rest. The agreement of numerical models with physical models with reference to the history and values can be assessed as very good.

The numerical models consider the values of extreme pressures according to CSN 73 0037 (both active and passive pressures). Also the computation of the pressure values at the time of the shear stress drop in the mass to residual value was based on the methodology of the same standard. The progress of the physical experiment E2, where the sensitivity is sufficient to provide the possibility of comparison, however, has shown certain differences of minimum values, in most cases lower in E2 than the standard values according to N2. They are on the conservative side, but there are also opposite cases.

### **6.3 Rotation about the Top to Active Side**

Fig. 4 and diagrams of others sensors (not shown) here reveal that during this type of wall movement the respective lines of most more sensitive sensors are considerably different from their progress during the rotation about the toe, and after an initial drop they proceed to the values exceeding considerably the active pressure values according to CSN 73 0037 as well as the residual values represented by the lines according to N1 and N2. At the end of rotation, i.e. after absolute summary movement of some 8.75 mm, however, the pressures drop again to the proximity of residual values.

The lines of the diagrams of physical and numerical models not shown here proceed almost parallelly and with the afore mentioned reservations the differences in the values are not very great, either. The differences can be ascertained at closer sight in the values of the displacement of the extreme active (peak) pressure which took place in physical experiments nearer the rest area. Taking into account the initial deviations of Sensor No. 1 in E2, Sensors Nos. 4 and 5 as well as the sensitivity of individual sensors, it is possible to assess the agreement of numerical models with reference to both history and values as very good.

Numerical models consider the earth pressure values according to CSN 73 0037 (both active and passive). Also the computation of the pressure value at the time of the shear stress drop in the mass to residual value was based on the same standard. The progress of physical experiment E2, the sensitivity of which is sufficient to enable comparison, however, has shown certain differences of the minimum values, lower in E2 than in the standard values of N2 in most cases. They are on the conservative side, but there are also opposite cases.

### **6.4 Translative Motion to Active Side**

According to Fig. 5 and further diagrams of all E2 sensors not shown here and in opposite to E1 Sensors Nos. 1 and 2, the final pressure values are not higher than the extreme (maximum) values in the proximity of the start of the motion. The pressure history in Sensor No. 4 (sensitive) in E1 corresponds well with the pressure history according to N1, but its values are lower. The values of Sensors Nos. 1 and 2 exceed relatively highly the N1 values.

Good agreement of both history and values between E2 and N2 can be observed in Sensors Nos.1 and 2 (highly sensitive); good agreement can be observed also in Sensor No. 3 (highly sensitive). The sensitive Sensors Nos. 4 and 5 do not show agreement in history, as at the end, after pressure increase in the major part of the interval, the pressure dropped again. In Sensor No.4, however, it remained above the extreme active



pressure level. The differences of values of these sensors are not relevant because of the zero setting at the beginning of E2 experiment.

The numerical models consider the extreme pressure values according to ČSN 73 0037 (both active and passive). Also the computation of the pressure value at the time of shear stress drop in the mass to residual value was based on the same standard. The history of the physical experiment E2, where the sensitivity is sufficient to enable comparison, however, has shown certain differences of minimum values, in most cases lower in E2 than the standard values of N2. They are on the conservative side, but there are also opposite cases.

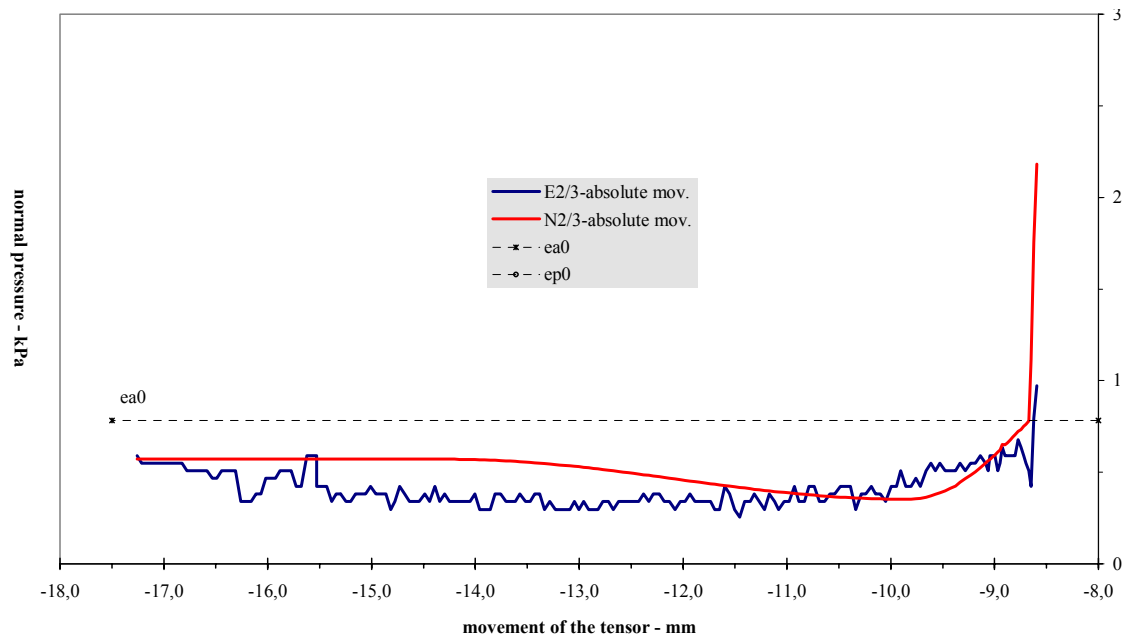


Figure 5: N2 – E2/3 Translative movement to active side - Sensor No. 2, depth 0.265 m, normal lateral pressure of the physical and the numerical models in dependence on sensor movement.

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## 7 Conclusion

The behaviour of the numerical model according to the results of experiment N2 appeared considerably similar to the behaviour of the actual physical model in experiment E2. Both experiments (numerical and physical) have shown very good agreement, if we consider all technical circumstances which influenced the physical experiment E2. The numerical model makes it possible to make this agreement substantially more accurate, should it take into account the history of the pressure at rest of the individual sensors according to the results of experiment E2. However, data of such accuracy are not available in engineering practice, as a rule, so that the standard of

agreement of the numerical and physical models corresponds approximately with practical conditions.

Therefore, with reference to the present state of knowledge, the results of both experiments justify the conclusion that the postulates of the General Lateral Pressure Theory and the concept of the numerical programme FORESTER appear correct.

## 8 ACKNOWLEDGEMENT

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