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CHAOTIC MOTIONS OF IMPACT OSCILLATOR WITH KELVIN -VOIGT IMPACT MODEL INVESTIGATED BY NUMERICAL SIMULATION

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Abstract:. *Soft impact oscillator with one degree of freedom containing soft stop is considered. Ways of the system to a chaotic motion are dealt with. Various ways to this regime are presented with corresponding examples and commented in this contribution.*

Key words: piecewise linear systems, impact motion, numerical simulation, experiments, soft stop

1. Introduction

Ways to chaos of the single degree of freedom (SDOF) oscillator with Kelvin-Voigt impact model (Fig. 1) investigated by means of numerical simulation are dealt with in this contribution. Non-linearity is caused by impacts of mass *m* against the soft stop situated in a static distance *r*, while excited by harmonic force $F_0 \cos \omega t$ drives the system to different regimes characterised by impact number $z=p/n$ due to changes of parameters r, ω. The NON-1-SIM program (Černá, Čipera and Peterka 1995) was used for analysis.

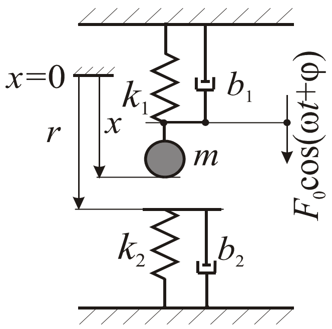

Figure 1. Scheme of the SDOF system with soft impact

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2. Mathematical model

The motion of the mechanical system in Fig. 1 is described by one of two dimensionless differential equations:

$$X'' + 2\beta_1 X' + X = \cos \eta\tau \qquad \text{for } X < \rho, \tag{1}$$

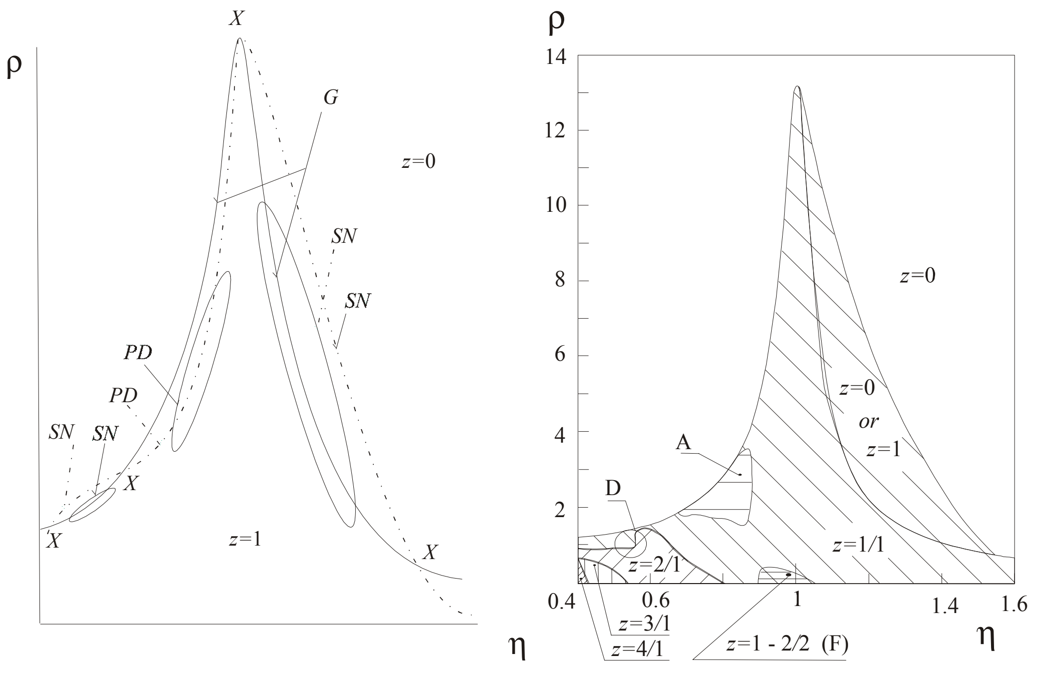
$$X'' + 2\beta_2 X' + X + \frac{k_2}{k_1}(X - \rho) = \cos \eta\tau \qquad \text{for } X > \rho, \tag{2}$$

where $\tau = \Omega t$, $\Omega = \sqrt{k_1/m}$, $\eta = \omega/\Omega$, $\beta_1 = b_1/2\sqrt{k_1 m}$, $\beta_2 = b_2/2\sqrt{k_1 m}$, $\rho = r/x_{st}$, $X = x/x_{st}$ $x_{st} = F_0/k_1$ are dimensionless time, excitation frequency, viscous damping, static clearance and displacement. The investigated system has stiffness ratio $k_2/k_1 = 13$ and damping $\beta_1 = 0.0375$, $\beta_2 = 0.1727$.

Three types of bifurcation were found in the impact oscillator:

1) Period doubling – a sequence of periodic motion bifurcation (Feigenbaum's cascade) where period doubling occurs. The eigenvalue for critical stability point is $\lambda = -1$.
2) Saddle-node bifurcation has he eigenvalue of stability critical point is $\lambda = 1$. (described in (Whiston 1987) in more detail).
3) Gazing bifurcation – a new impact occurs due to new grazing between mass and the stop during the motion period.

3. Investigation results



(a) (b)

Figure 2. (a) Scheme of the rigid and soft impact oscillator parametric subspace $\eta \times \rho$ map
(b) Global parametric subspace $\eta \times \rho$ map for soft impact oscillator

Soft and strong impact phenomenon introduced in (Peterka 2001) causes particular topology in state subspace $\eta \times \rho$. The following rule can be stated while rigid impact oscillator is considered: *Suppose grazing and stability boundaries intersecting in the parametric subspace* $\eta \times \rho$ *(see continuous and dashed line and points X in Fig. 2). Then period doubling occurs in the interval when stability boundary course is inside z=1/1 motion area. Subharmonic and chaotic motions lie between these boundaries. Hystereses occur while an inverse topology.*

However these motions topology creates separated islands in parametric space while soft impact oscillator is considered and don't reach the intersections (see continuous line and points *X* in Fig. 2). Contrary to rigid impact oscillator, where transition cross the grazing boundary is unstable but the points *X,* it is stable while soft impact oscillator behaviour is considered.

Ways to chaos

a) Feigenbaum cascade of period doubling

This way was found in motion *z*=2/2 hysteresis area to non-impact regime for $\eta = 0.7$ (area A in Fig. 2, detail in Fig. 3). The impact number *z* is not affected by this process.

While a system behaviour in parametric subspace is considered, the solutions properties are given by mutual bifurcations topology. When one solution area lies inside another one, the inner area solution is given by properties of all areas inside it lies, then.

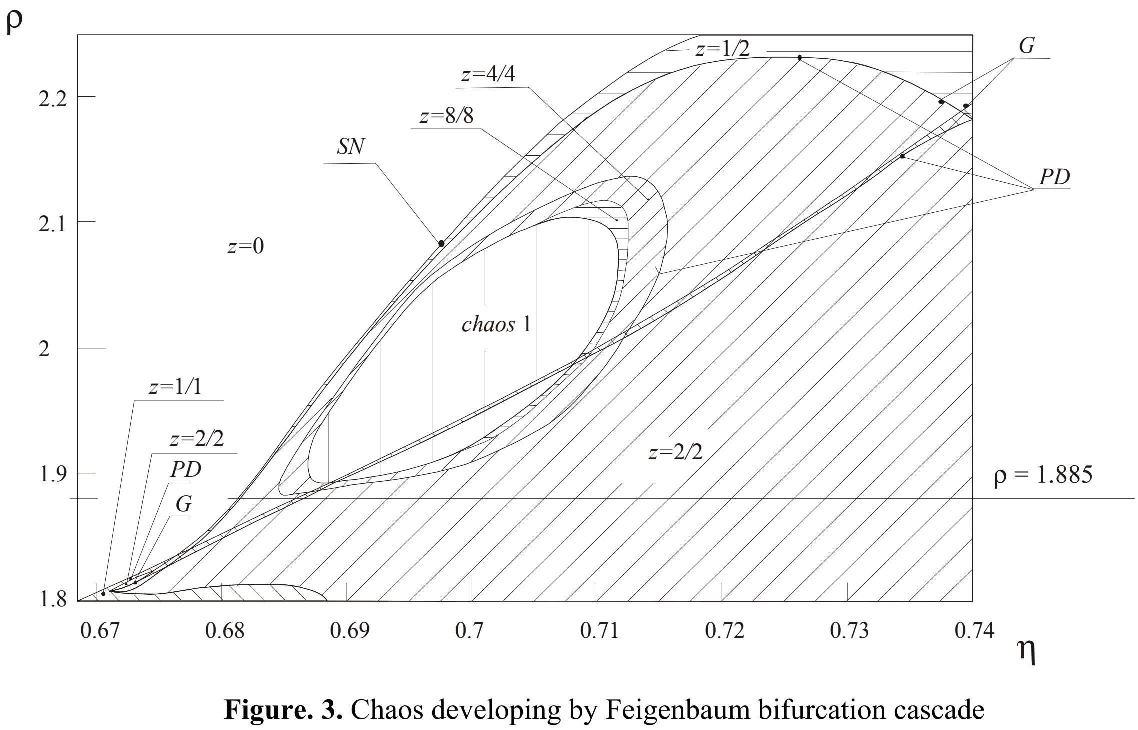


Figure. 3. Chaos developing by Feigenbaum bifurcation cascade

b) Feigenbaum period doubling cascade interrupted by grazing bifurcation

This way is known from the rigid impact oscillator behaviour, e.g. sub-harmonic resonances. The impact number *z* is affected in this case. The cascades with grazing and period doubling alternation were found in some cases of parameters variation for increasing static clearance at very slight damped system (area A and D in Fig. 2, Fig. 4 and 5).

Fig. 4 shows a complicated area in the frequency interval $\eta \in (0.81, 0.88)$. The chaotic regime occurs by interrupted Feigenbaum cascade from motion z=3/4 while parameter ρ is increased or parameter η is decreased. The motion z=3/4 can be set either from the motion z=1/2 cross the z=2/4 motion while parameter ρ is increased (i.e. cross *PD* and *G* boundaries), or from the motion z=2/2 cross the z=4/4 motion while parameter η is decreased (i.e. cross *PD* and inverse *G* boundaries). Comparing to the SDOF rigid impact oscillator, the areas of motions $z=p/n$ move for n>2 to the smaller values of non-dimensional static clearance ρ. Motion with n=3 can be identified only after the transient effect induced e.g. by jump in parameters. Chaos at the area F in Fig. 2 (Fig. 7) has the same developing scheme.

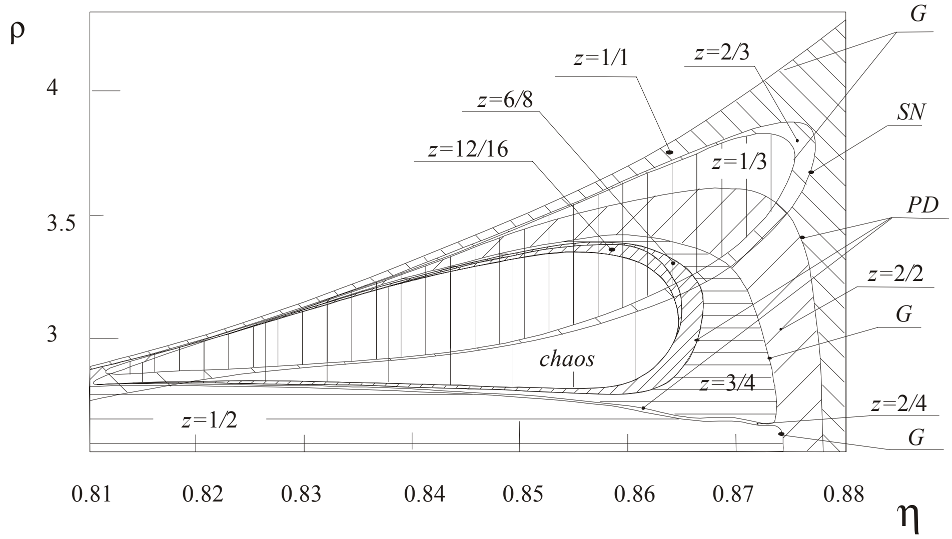


Figure 4. Chaos developing by interrupted Feigenbaum cascade and higher order subharmonic motions

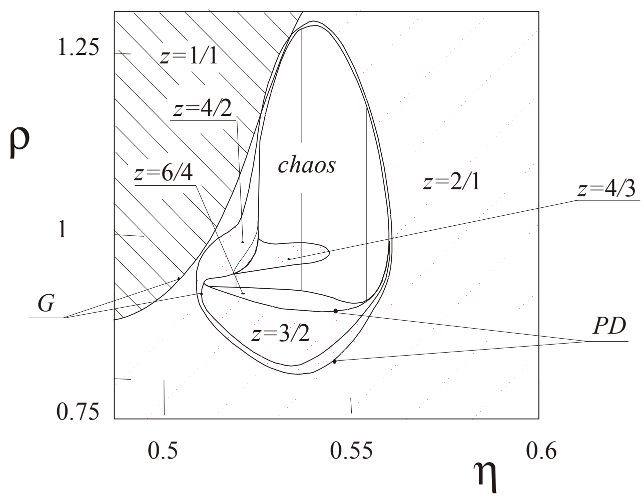


Figure 5. Chaos between motion z =2/1 and z =1/1 areas

c) Interruption of saddle-node instability development

Hysteresis area boundary (e.g. motion z=4/3 hysteresis area in Fig. 5) occurs in a tiny area inside more complicated structure.

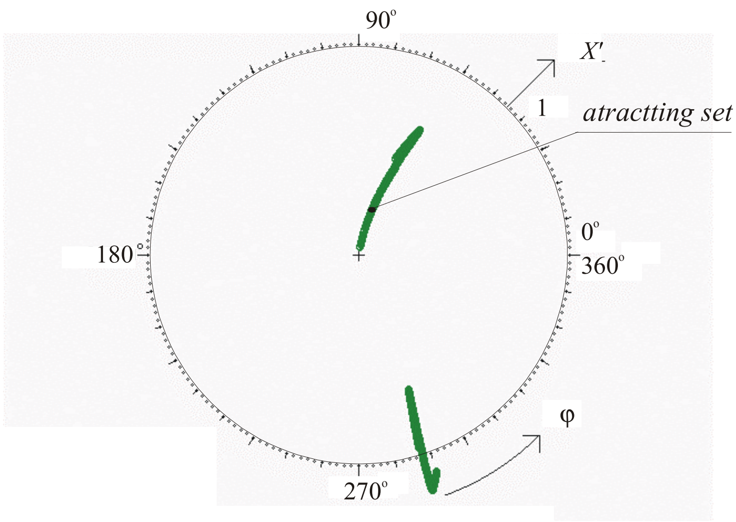


Figure 6. Chaos between motions z =2/1 and z =1/1 areas Poincare map (Fig. 5)

Chaos of this system in Poincaré map has particular features in the most of cases. The $X = \rho$ section at the parametric plane X'_-, φ (i.e. motion velocity and phase at the beginning of the penetration) was chosen for mapping. Fig. 6 shows chaos in the area between motions z =2/1 and z =1/1 developing via period doubling Feigenbaum cascade interrupted by grazing bifurcations (Fig. 5). This case is an example of the scheme (Fig. 2 a) while a basic multi-impact motion is considered. However, the Poincaré map of chaotic area for small ρ has another appearance (Fig. 8). Chaotic areas are parts of a complicated structures, which modified slightly compared to the rigid impact SDOF oscillator.

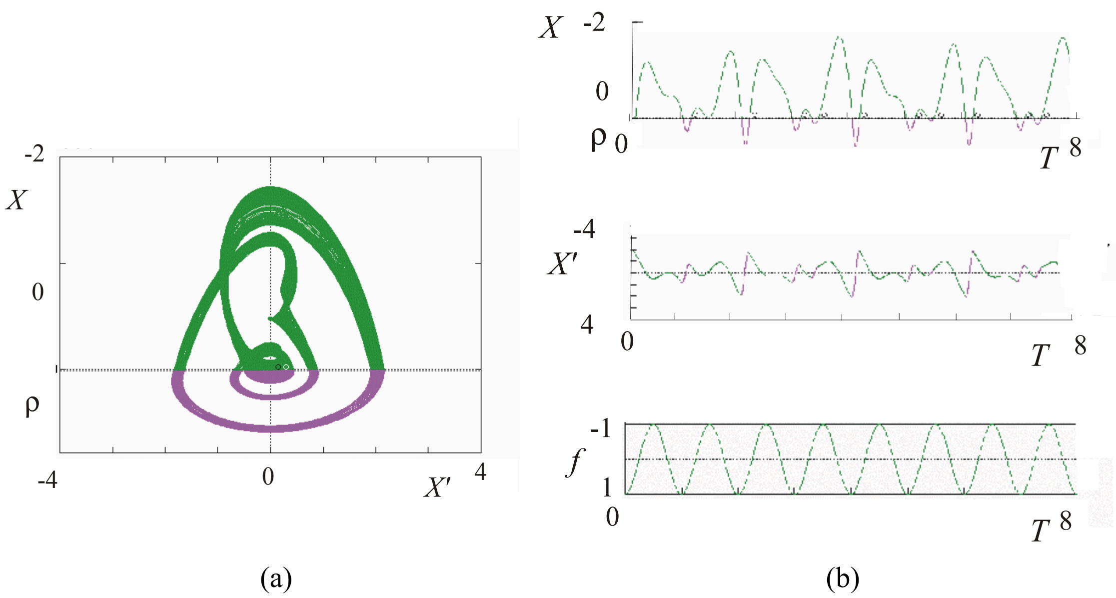


Figure 7. Phase trajectories (a) and time series (b) of the chaotic motion at point $\eta = 0.9558$, $\rho = 0.01$ of the area F in Fig. 2

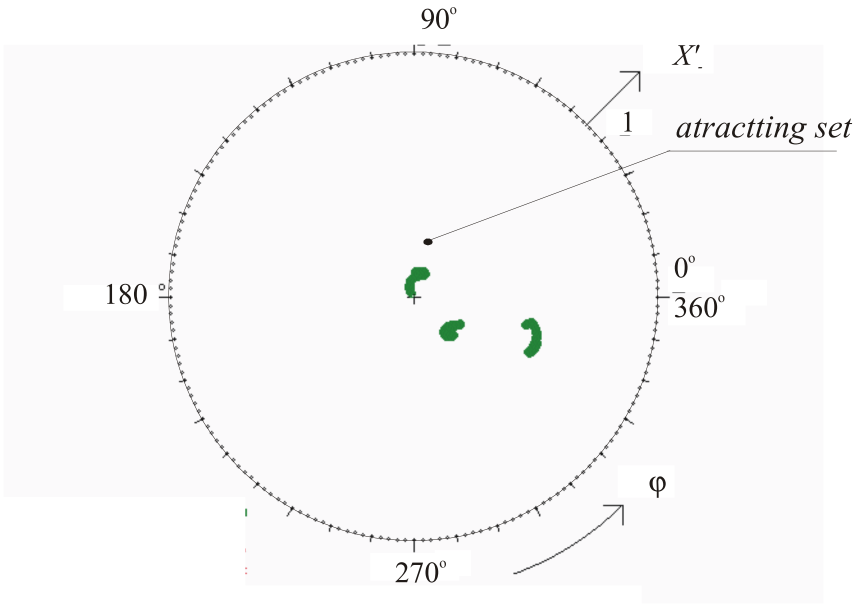


Figure 8. Chaos at motion z =1/1 area for parameters $\eta = 0.9558$, $\rho = 0.01$ at the area F in Fig. 2 - Poincare map

5. Conclusion

Basic schemes of ways to chaos (Peterka, Kotera 1996) were found in the behaviour of the SDOF oscillator with Kelvin-Voigt impact model. Five large chaos areas arising by three different ways were found and two phenomena of soft impact oscillator motion were stated. Group of higher harmonic motion, which can to be set by only a tiny initial condition interval (achieved e.g. by a transient effect), cannot be simply dealt with by the following the presented mechanisms.

6. Acknowledgement

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