Národní konference s mezinárodní účastí

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# SIMULATION OF ROTOR-TO-STATOR RUB-RELATED PHENOMENA

**Pavel Hynek**[1]

## Abstract

*This paper describes three modelling technique for rub related phenomena in rotor systems and compares their suitability for numerical simulation. Numerical tests are conducted on a simple Jeffcott rotor.*

## 1 Introduction

A critical requirement for optimum performance of many rotating machines is the minimization of radial clearances e.g. the performance of turbomachinery is very dependent on maintenance of minimum rotor/stator radial clearance of labyrinth seals. This, along with an increasing trend towards lighter and more flexible rotors running at higher speeds has significantly increased the probability of rotor/stator rub. Rub between rotor and stator has long been recognized as a major contributor to excessive wear in a wide variety of industrial plants, namely gas and steam turbines, pumps, compressors, motors, generators, centrifuges etc. Although many such machines are designed to sustain rubbing occasionally, or systematically, the rotor/stator rub can occasionally lead to a catastrophic system failure. Reducing rubbing may extend lifespan.

These problems have attracted researches attention for past few decades. The early works were mainly analytical studies of the rub phenomenon. (Johnson, 1962) analysed the interaction of a rotor with a rigid stator at an annular frictionless clearance space by using model of Jeffcott rotor system with one additional bearing having clearance. (Ehrich, 1969) studied a dynamic stability of rotor/stator rub and derived stability criteria for case when vibration amplitude is much larger than bearing clearance. (Billett, 1965) showed that the frequency of reverse whirl induced by dry friction cannot exceed the natural frequency of the rotor. However, the rotor/stator rub is very complicated non-linear vibration phenomenon and its analytical analysis is possible only with some simplifying assumptions. Thus many researches turned their attention to numerical simulation, which allows them to analyse more general systems. (Choy and Padovan, 1987) performed extensive numerical simulations of rotor/stator rub

[1] Pavel Hynek, Loughborough University, Wolfson School of Mechanical and Manufacturing Engineering, Mechatronic Research Group, Holywell Way LE11 3TU, UK
email: P.Hynek@lboro.ac.uk

interactions in order to develop a more detailed understanding of rotor/stator interactions. (Muszynska et al. 1990) performed numerical simulations of rotor/stator partial rubbing and compared the simulation and experimental results. (Al-Bedoor, 2000) developed a model for coupled torsional and lateral vibration of unbalanced rotor which takes into account rotor/stator rubbing. (Choi and Noah, ; Chu and Zhang, 1998; Goldman and Muszynska, 1994) studied chaotic behaviour of rotor/stator systems with rubs. However, numerical simulation gives little insight into behaviour of the dynamic system. For instance, a number of simulations are required in order to find out relation between system parameters (e.g. friction coefficient) and system response. Thus the time needed for numerical integration of system's equations of motion becomes crucial.

This paper describes three approaches to modelling of rub related phenomenon and compares them from numerical simulation point of view. First approach uses piecewise non-linear spring connected between contacting linear surfaces. Second approach employs two sets of equations, one for no contact regime and one for contact regime. The last approach solves a set of non-linear equations in order to find out the relevant contact force which ensures the boundary conditions i.e. rotor must remain within stator. These approaches to rotor/stator rub modelling are tested on a simple Jeffcott rotor, depicted in Figure 1, and the numerical results are compared.

## 2 The dynamic model

Dynamic models of a rotor system, in which contact between rotor and stator can occur, has generally two parts. The first part is a dynamic model of rotor and stator comprised of equations of motion for uncoupled rotor and stator. As has been mentioned in section 1, a simple Jeffcott rotor with additional bearing shown in Figure 1 has been chosen for this first part for all three modelling techniques.

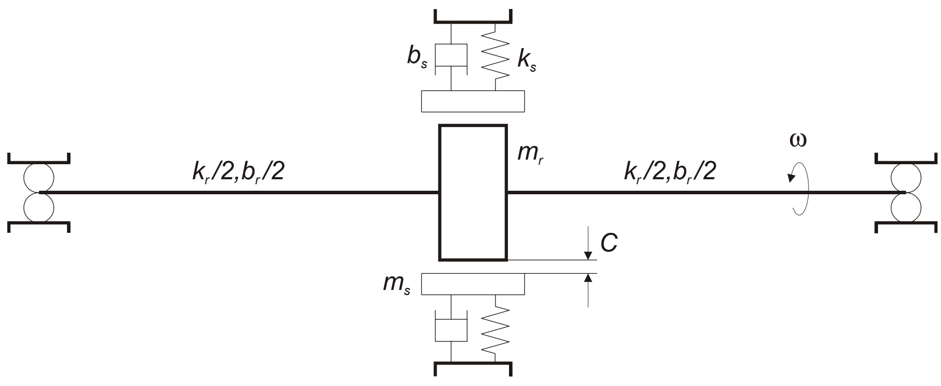

Figure 1 Jeffcott rotor

Jeffcott rotor equations of motion can be written in matrix form of (2.1) where $\boldsymbol{q}$ is a system displacement vector, $\boldsymbol{M}$ is mass matrix, $\boldsymbol{B}$ is damping matrix, $\boldsymbol{K}$ is stiffness matrix and $\boldsymbol{F}$ is vector of external forces which can include contact forces etc.. Unbalance forces $\boldsymbol{F}_{un}$ are considered in form of (2.2), where $\omega$ is rotor speed, $A$ is rotor angular acceleration $e$ is distance between rotors geometrical and mass centres, $m_r$ is rotor mass and $\boldsymbol{C}_{13}$ is a transformation matrix form coordinate system $x_3y_3$ to $x_1y_1$.

$$\begin{aligned} &\mathbf{M}\cdot\ddot{\mathbf{q}}+\mathbf{B}\cdot\dot{\mathbf{q}}+\mathbf{K}\cdot\mathbf{q}=\mathbf{F}_{un}+\mathbf{F} \\ &\mathbf{q}=\begin{bmatrix} x_r & y_r & x_s & y_s \end{bmatrix} \end{aligned} \tag{2.1}$$

$$\mathbf{F}_{un}=\mathbf{C}_{13}\cdot\begin{bmatrix} m_r\cdot e\cdot\varpi^2 \\ m_r\cdot e\cdot A \end{bmatrix} \tag{2.2}$$

The second part models the interaction between rotor and stator. This part is different for each modelling method. A detailed description for each method follows.

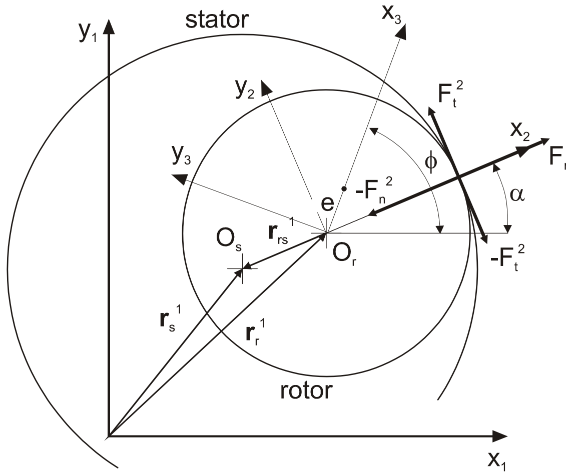


Figure 2 Coordinate systems

## 2.1 METHOD I - Piecewise non-linear spring characteristic

This method uses a piecewise non-linear spring characteristics connected between stator and rotor surface as shown in Figure 3. An example of the spring characteristic is shown in Figure 4. The spring stiffness depends on the distance between rotor and stator surfaces i.e. distance $O_rO_S$. The stiffness is zero, if rotor and stator are not in contact and it raises to a high value $k_c$ e.g. $10^9$ $N/m$ when rotor touches stator. This can be expressed as follows

$$F_n = \begin{cases} 0 \text{ if } \delta < C \\ k_c \text{ if } \delta \geq C \end{cases} \tag{2.3}$$

$$\delta = \sqrt{(x_r - x_s)^2 + (y_r - y_s)^2} \tag{2.4}$$

where $F_n$ is normal contact force, $k_c$ spring stiffness, $C$ rotor stator clearance and $\delta$ is distance $O_rO_s$. This contact model needs to be embedded into rotor/stator model. Boundary condition is ensured by contact force $F_n$ thus this force can be included into the vector of external forces $\boldsymbol{F}$ at right side of the equations of motion (2.1).

$$\mathbf{M}\cdot\ddot{\mathbf{q}}+\mathbf{B}\cdot\dot{\mathbf{q}}+\mathbf{K}\cdot\mathbf{q}=\mathbf{F}_{un}+\begin{bmatrix}\mathbf{C}_{12} & 0\\ 0 & \mathbf{C}_{12}\end{bmatrix}\cdot\begin{bmatrix}-F_n\\ -F_n\cdot f\\ F_n\\ F_n\cdot f\end{bmatrix} \tag{2.5}$$

Where $\boldsymbol{C_{12}}$ is transformation matrix from coordinate system $x_2y_2$ to $x_1y_1$ (see Figure 2), $\boldsymbol{F_{un}}$ is unbalance force vector and $f$ is friction coefficient. Equations (2.3),(2.4) and (2.5) form complete model of rotor/stator system.

As can be seen, there is always interference between rotor and stator surface when rotor is in contact with stator. The size of the interference depends on the spring stiffness $k_c$, so that the higher stiffness the smaller interference. Thus, this contact model does not ensure the boundary condition between rotor and stator exactly. However, if the spring stiffness is high enough e.g. $10^9$ then the interference is negligible in comparison with clearance $C$. This contact model is relatively simple and it is not difficult to be implemented even with more complicated rotor/stator models, for example obtained using FEM (Hynek, 1998). The disadvantage of this model is a sudden change in system stiffness. This leads to some numerical problems (discussed in section 3). However, they can be mitigated by employing a linear change of spring stiffness (Hynek, 1997).

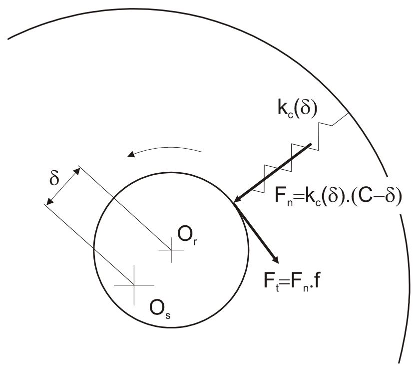


Figure 3 Rotor/stator contact model

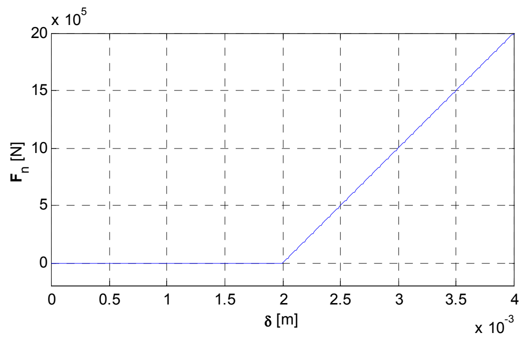


Figure 4 Non-linear spring characteristic

## 2.2 METHOD II – Two sets of equations of motion

This method uses two sets of equations of motion one for the regime when rotor is not in contact with stator and the other one for case when rotor is in contact with stator. Transition from one regime to the other is governed by a set of rules incorporating values of the normal force $F_n$ and the gap $\delta$. The transition from one regime to the other includes two steps. Firstly, an iterative procedure is invoked in order to find the out precise time when rotor leaves stator or when rotor comes into contact with stator. Secondly, initial conditions for next regime are worked out from the last state of previous regime. The flowchart of the algorithm is shown in Figure 5. Equations of motion for non-contact regime can be written as follows

$$\mathbf{M}\cdot\ddot{\mathbf{q}}+\mathbf{B}\cdot\dot{\mathbf{q}}+\mathbf{K}\cdot\mathbf{q}=\mathbf{F}_{un} \tag{2.6}$$

Equations of motion for contact regime are same as equations (2.5) but accompanied by boundary condition (2.7) which ensures that rotor remains within stator.

$$\begin{bmatrix} x_s \\ y_s \end{bmatrix}=\begin{bmatrix} x_r \\ y_r \end{bmatrix}+\mathbf{C}_{12}\cdot\begin{bmatrix} -C \\ 0 \end{bmatrix} \tag{2.7}$$

An advantage of this approach is an exact implementation of boundary condition (2.7), which is accomplished by use of two different sets of equations of motion, each for different motion regime, and switching between them according to the system state. However, the switching is a disadvantage of this algorithm, because neither rule (i.e.

$\delta$<$C$, $F_n$>0) can be met due to numerical inaccuracies. Thus the algorithm does enter either non-contact regime or contact regime and computation effectively stalls. This can be partially solved by forcing algorithm to enter next regime at least once when it leaves previous regime.

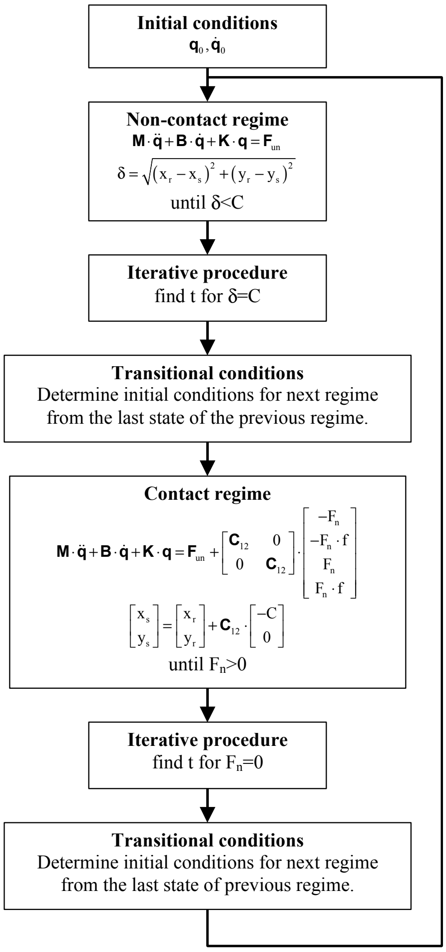


Figure 5 Method II - Algorithm flowchart

### 2.3 METHOD III – Finding appropriate contact force

This method uses one set of equations of motion (2.5) and computes appropriate the contact force $F_n$ to ensure rotor/stator boundary condition, which is considered in form of (2.4). The procedure of numerical simulation is that, at every integral step the value of $\delta$ is calculated according to the system state obtained. If the value is lower than clearance *C*, the simulation will go on as contact free system case i.e. the contact force $F_n$ is set to zero. If the value of $\delta$ is higher than clearance *C* then an iterative procedure is invoked to find out the value of $F_n$ that forces the gap $\delta$ be equal to clearance *C*. On the other hand, if a negative value of $F_n$ is obtained, it means that the rotor and the stator will not touch each other at the next integral step and $F_n$ is set to zero. The flowchart of the algorithm is shown in Figure 6.

Exactness of the boundary condition (2.7) implementation depends largely on precision of the iterative procedure. Clearly, the more precise the iterative procedure is, the more exact implementation of the boundary condition is achieved. However, high precision leads to longer simulation times, because the iteration takes place in every integration step in case when rotor is in contact with stator.

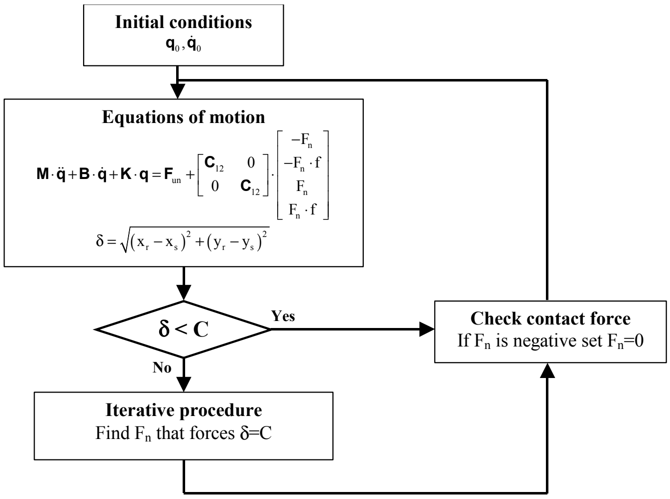


Figure 6 METHOD III – Algorithm flowchart

## 3 Simulation results and comparison

A number of simulation tests have been performed in order to evaluate usability of methods described in section 2 for rotor/stator rub phenomenon simulation. The methods has been implemented in program system MATLAB© employing standard numerical procedures supplied with MATLAB©. The integration method used to solve the equations of motion was Runge-Kutta method with automatic integration step control. The tolerances were set as follows; relative tolerance 1e-3 and absolute tolerance 1e-6. Numerical method for solving one non-linear equation was used in iterative procedures of methods II and III. It is a combination of bisection, secant, and inverse quadratic interpolation methods. As has been motioned above, Jeffcott rotor has been chosen as a representative of rotor systems for its simplicity. One simulation of rotor's run-up with constant angular acceleration has been performed for each method. The speed range has been chosen so that rotor goes through its critical speed during its run-up. Model and simulation parameters are stated in Table 1.

| Parameter | Value | Unit | Description |
|---|---|---|---|
| $m_r$ | 0.418 | kg | rotor mass |
| $k_r$ | 2493 | N/m | rotor stiffness |
| $b_r$ | 1.3 | N·s/m | rotor damping coefficient |
| $m_s$ | 0.86 | kg | stator mass |
| $k_s$ | 38166 | N/m | stator stiffness |
| $b_s$ | 3.61 | N·s/m | stator damping coefficient |
| C | 2 | mm | rotor/stator clearance |
| f | 0.1 | - | Coulomb friction coefficient |
| $k_c$ | $10^9$ | N/m | contact stiffness (used only in METHOD I) |
| $R_r$ | 10 | mm | rotor radius in location of contact (needed for relative speed calculation) |
| e | 0.4 | mm | distance between rotor's mass and geometrical centres |
| A | 70 | rad/s$^2$ | rotor's angular acceleration |
| T | 4 | sec | simulation time |
| rtol | $10^{-3}$ | - | relative tolerance |
| atol | $10^{-6}$ | - | absolute tolerance |

Table 1 Simulation parameters

Rotor goes through several regimes during its run-up. First regime is a rub free regime; rotor does not touch the stator. Rotor comes into contact with stator around *t=1.14 sec* and maintains contact until *t=2.6 sec*, where conditions for maintaining contact are no longer fulfilled. At this time rotor leaves contact with stator and enters again rub free regime. Rotor vibration $x_r$ is depicted in Figure 7. Figure 8 and Figure 9 show stator and rotor vibration respectively in vicinity of contact beginning. Displacements of rotor and stator centres computed by the methods are in good agreement as can be seen from Figure 7, Figure 8, and Figure 9. Interestingly, there are discrepancies of a few orders between normal contact forces as can be seen in Figure 10, because each method uses different approach for computing contact forces. For instance METHOD I uses a contact spring of which stiffness is very high (i.e. $10^9$), thus small inaccuracies in rotor and stator displacement causes large fluctuations in contact force. This is a disadvantage if magnitude of the contact force is in interest.

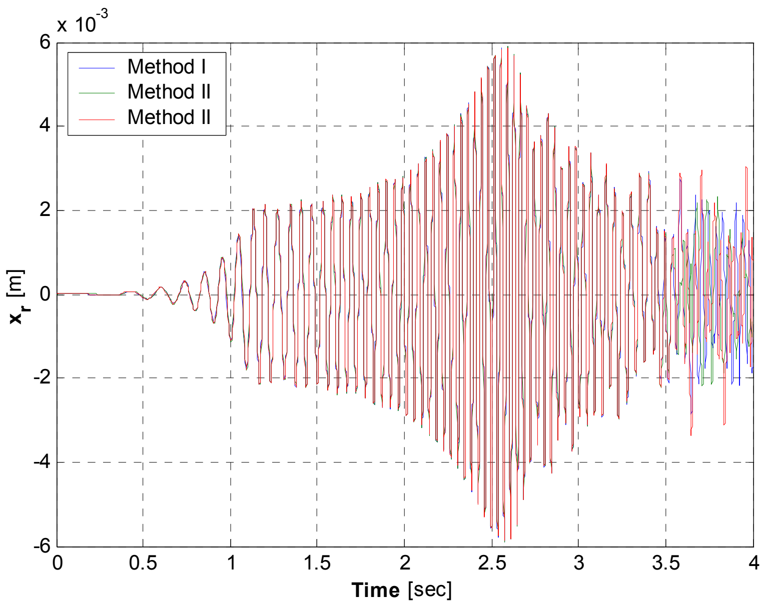


Figure 7 Rotor vibration

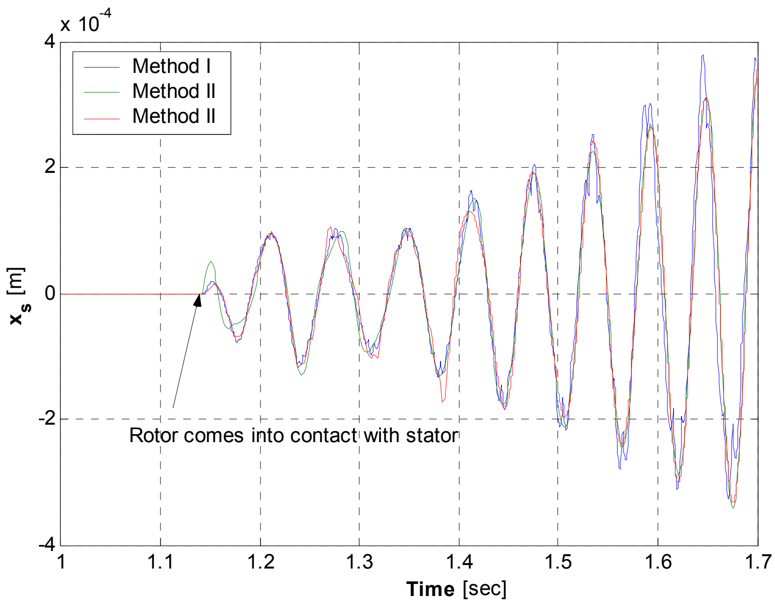


Figure 8 Stator vibration in vicinity of contact beginning

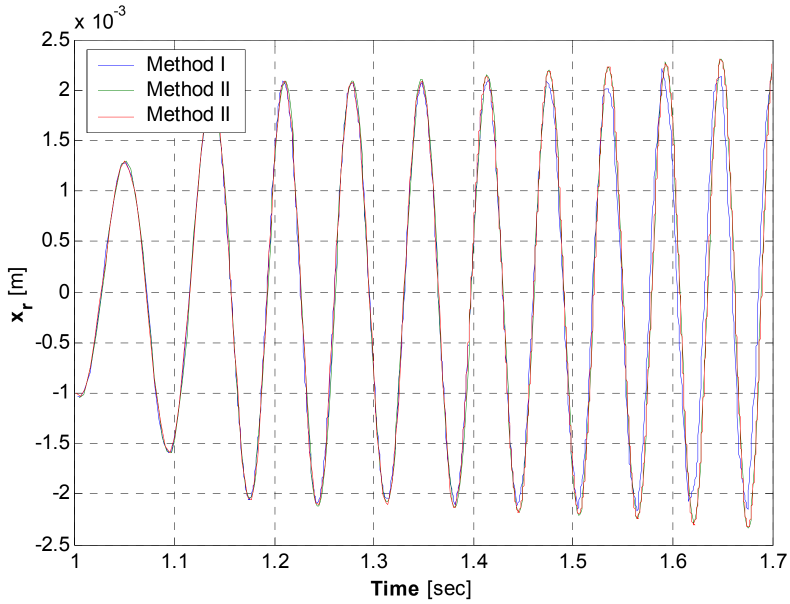


Figure 9 Rotor vibration in vicinity of contact beginning

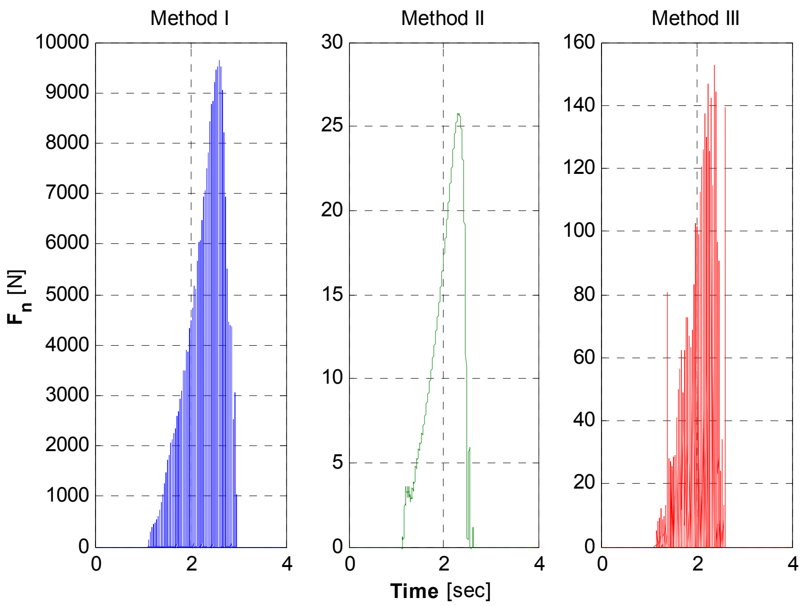


Figure 10 Normal contact force

Table 2 shows comparison between times needed to perform simulation. The fastest method is METHOD II and it also requires lowest number of integration steps needed to complete the simulation. The slowest method is METHOD I, which also requires the highest number of integration steps. This is caused by introducing the non-linear contact stiffness into dynamic model and it can be explain as follows. Ability and successfulness of integration methods are greatly influenced by matrix spectral condition number $\sigma(A)$ which is defined as follows

$$\sigma(\mathbf{A}) = \frac{|\lambda_i|_{max}}{|\lambda_i|_{min}} \tag{2.8}$$

where $\boldsymbol{A}$ is the system matrix from state space interpretation of the model or system Jacobian in case of non-linear systems and $\lambda_i$ are its eigenvalues. The higher $\sigma(A)$ the more difficult is to perform the simulation. If $\sigma(A)$ is within range $1 < \sigma(A) < 10^2$ then the simulation has positive preconditions. Conversely, if $\sigma(A)$ exceeds $10^2$ then the system is said to be "stiff" and its simulation is more difficult. The matrix condition number $\sigma(A)$ for system model by METHOD I changes through the simulation depending whether rotor/stator contact occurs as can be seen from Figure 11. The value of $\sigma(A)$ well exceeds $10^2$ which leads to shorter integration steps and consequently to longer simulation times. Simulation duration for system modelled by METHOD III is roughly equal to the time for simulation by METHOD I. However METHOD III requires significantly less integration steps than METHOD I. This is advantage when a complicated rotor system, of which first derivative evaluation is a time-consuming operation, is simulated. The use of METHOD III can reduce the simulation duration, because less evaluation of first derivatives is needed.

| Task | Method I | Method II | Method III |
|---|---|---|---|
| Overall simulation duration | 1123 sec | 45 sec | 1050 sec |
| Time for evaluation of first derivatives | 62% | 64% | 19% |
| Time for iteration | 0% | 1% | 65% |
| Number of integration steps | 90459 | 1857 | 8040 |

Table 2 Simulation duration comparison

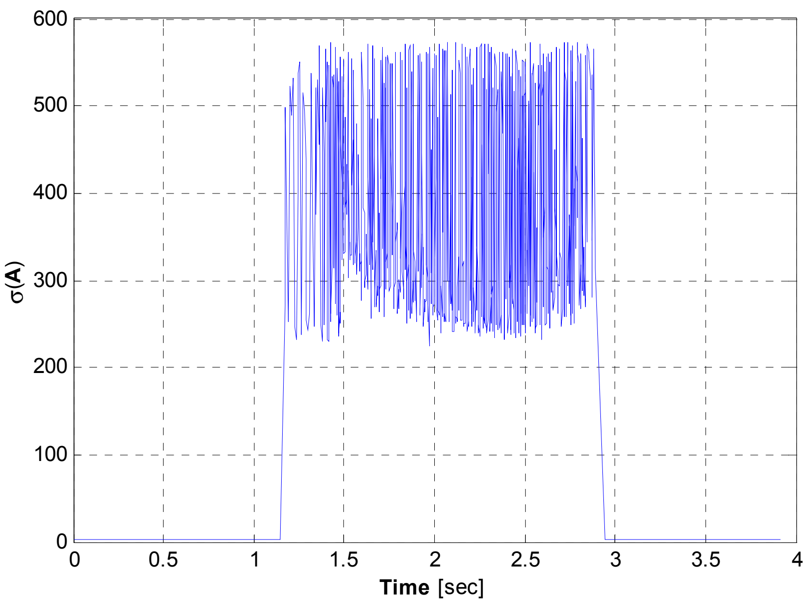


Figure 11 Matrix spectral condition number

## 4 Conclusion

Three modelling techniques have been implemented and compared in terms of their suitability for numerical simulation rather than their precision and accuracy to describe the phenomenon. The numerical experiments that have been performed are not comprehensive. More experiments are needed in order to make a thorough comparison of the method described in this paper. However some basic conclusions can be made.

Performance of METHOD I is greatly influenced by the contact stiffness $k_{c,}$ the higher contact stiffness the lower rotor/stator interference and the better phenomenon description. However, the higher contact stiffness the longer simulation times and the higher susceptibility to numerical instability. Nonetheless, METHOD I is very easy to implement even for complicated systems.

METHOD II is the fastest method, but it needs two different sets of equations of motion, which is disadvantage when used with more complicated dynamic models (e.g. models having a few tens of DOF). Moreover if contact can occur at more than one place within the rotor system, then a number of different sets of equations of motion is needed, one for each combination of contact places.

Performance of METHOD III largely depends on the performance of the iterative algorithm employed to find appropriate value of the contact force $F_n$ as can be seen from Table 2 (65% of overall duration was spent in the iteration procedure). This method is suitable for complicated systems where because it reduces number of integration steps required and thus can significantly reduce simulation duration.

The methods have been implemented in program system MATLAB© using standard numerical procedures supplied with the program system. Thus some space for speed improvement of the method is in the implementation itself. For instance, coding the algorithms in a high level programming language (e.g. C/C++) can significantly reduce simulation duration.

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## References

Al-Bedoor, B.O. (2000) Transient torsional and lateral vibrations of unbalanced rotors with rotor-to-stator rubbing. Journal of Sound and Vibration, v 229, n 3, p 627-645

Billett, R.A. (1965) Shaft Whril Induced by Friction. The engineer 713-714.

Choi, S.-K. and Noah, S.T. Mode-locking and chaos in a modified Jeffcott rotor with a bearing clearance. American Society of Mechanical Engineers, Design Engineering Division (Publication) DE, v 50, (1992), p 21-28

Choy, F.K. and Padovan, J. (1987) NON-LINEAR TRANSIENT ANALYSIS OF ROTOR-CASING RUB EVENTS. Journal of Sound and Vibration 113, 529-545.

Chu, F. and Zhang, Z. (1998) Bifurcation and chaos in a rub-impact Jeffcott rotor system. Journal of Sound and Vibration, v 210, n 1, (Feb 12 1998), p 1-18

Ehrich, F.F. (1969) The Dynamic Stability of Rotor/Stator Radial Rubs in Rotating Machinery. Journal of Engineering for Industry ASME 10251028

Goldman, P. and Muszynska, A. (1994) Chaotic behavior of rotor/stator systems with rubs. Journal of Engineering for Gas Turbines and Power, Transactions of the ASME, v 116, n 3, p 692-701

Hynek, P. (1997) Analýza pohybu rotoru vsamomazných ložiskách. Diplomová práce, Ústav mechaniky těles FS VUT Brno

Hynek, P. (1998) Modelování rotoru se samomaznými ložisky pomocí MKP. Kolokvium Dynamika strojů 99', ÚT AV ČR, Praha

Johnson, D.C. (1962) SYNCHRONOUS WHRIL OF A VERTICAL SHAFT HAVING CLEARENCE IN ONE BEARING. Journal Mechanical Engineering Science 4, 85-93.

Muszynska, A., Wesley D. Franklin and Robert D. Hayashida (1990) Rotor -to-stator partial rubbing and its effects on rotor dynamic response.