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QUATERNION APPLICATIONS FOR ROBOT

Z. Ehrenberger[1], T Březina[2], P. Houška[3]

***Abstract**: The control of motion a robot manipulator gripper and a mobile (locomotion) robot behaviour requires the solution of the direct and inverse kinematics and dynamics. This paper describes a method for computing such models with the aid of the Denavit-Hartenberg matrix representation for manipulator linkages, the hypercomplex numbers witch take full advantage of limited number of arithmetic operations and substantial reduction in real-time computation. The purpose of this paper is to show how can be used quaternion in robotics and to find out the merits of such application.*

Keywords: robot, quaternion, manipulation, locomotion, modelling, coordinate transformation

1. Introduction

This paper presents Rodrigues-Hamilton parameters method for obtaining of robot kinematics and dynamics models and to ensure real-time control of such systems. The hypercomplex formalism (quaternion) starts to be used by robotic specialists. An oriented extension to the application of quaternion technique is presented in this paper. In the first part we deal in review of the main elements of quaternion method with an accent to the computation of the only direct kinematics and differential models. In the next parts we present model of manipulator and locomotion robot computation.

2. Quaternion method

Hypercomplex numbers called quaternion satisfy the general relationship:

$$Q = q_0 + i \cdot q_1 + j \cdot q_2 + k \cdot q_3 = s + v \tag{1}$$

where q_0, q_1, q_2, q_3 are real numbers, $q_0 = s$ is the scalar part, $v = i \cdot q_1 + j \cdot q_2 + k \cdot q_3$ is the vector part (pure quaternion). In Equation (1) i, j, k are basic elements satisfying the relations $i^2 = j^2 = k^2 = -1$, $(i \cdot j)z = i(j \cdot k) = -1$, $i \cdot j = -j \cdot i = k$, $j \cdot k = -k \cdot j = i$, $k \cdot i = -i \cdot k = j$. The addition of two quaternions $p = s_1 + v_1$, $r = s_2 + v_2$ is written as:

$$p + r = (s_1 + s_2) + (v_1 + v_2)\,. \tag{2}$$

The multiplication of p and r is associative and non-commutative:

$$p \cdot r = s_1 \cdot s_2 - v_1 \cdot v_2 + s_2 \cdot v_1 + s_1 \cdot v_2 - v_1 \times v_2 \tag{3}$$

with inner and cross product of the respective vector parts.

[1] Zdeněk Ehrenberger, Prof. Ing. DrS., Mechatronic Center of IT, SA, Technická 2, 61669 Brno, e-mail:ehrenberger@uvssr.fme.vutbr.cz

[2] Tomáš Březina, Ing. RNDr. Csc., VUT v Brně,FSI ÚAI, Technická 2, 61669 Brno, e-mail: brezina@uai.fme.vutbr.cz

[3] Pavel Houška, Ing., VUT v Brně, FSI ÚMT, Technická 2, 61669 Brno, e-mail: houska@umt.fme.vutbr.cz

The conjugate quaternion is defined as $Q^{*} = s - v$, the quaternion norm is $\|Q\| = \left(Q \cdot Q^{*}\right)^{1/2} = N(q)$ and the norm of unit quaternion $\left\|\mathring{Q}\right\| = \mathring{Q} \cdot \mathring{Q} = 1$.

By applying the symbolic relation (3) there is

$$Q = p \cdot r = q_0 + i \cdot q_1 + j \cdot q_2 + k \cdot q_3 ,$$

where $q_0 = p_0 \cdot r_0 - p_1 \cdot r_1 - p_2 \cdot r_2 - p_3 \cdot r_3$,

$$q_1 = r_0 \cdot p_1 + p_0 \cdot r_1 + p_2 \cdot r_3 - p_3 \cdot r_2 ,$$

$$q_2 = r_0 \cdot p_2 + p_0 \cdot r_2 + p_3 \cdot r_1 - p_1 \cdot r_3 ,$$

$$q_3 = r_0 \cdot p_3 + p_0 \cdot r_3 + p_1 \cdot r_2 - p_2 \cdot r_1 ,$$

and also the attitude quaternion

$$|Q| = \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} p_0 & -p_1 & -p_2 & -p_3 \\ p_1 & p_0 & -p_3 & p_2 \\ p_2 & p_3 & p_0 & -p_1 \\ p_3 & -p_2 & p_1 & p_0 \end{bmatrix} \begin{bmatrix} r_0 \\ r_1 \\ r_2 \\ r_3 \end{bmatrix} = P \cdot \bar{r} . \tag{4}$$

A quaternion, which has only the vector part can represent:

i) A position of a point in a 3-dimensional space. The position quaternion is written as

$$\vec{r} = \vec{i} \cdot x + \vec{j} \cdot y + \vec{k} \cdot z . \tag{5}$$

The position quaternion and Cartesian vector are then synonymous.

ii) A translation of position vector $\vec{r}$ to another position vector $\vec{r}_0$. Therefore translation quaternion

$$Q_T = \vec{i} \cdot q_1 + \vec{j} \cdot q_2 + \vec{k} \cdot q_3 \tag{6}$$

and translation is represented as

$$\vec{r}_0 = \vec{i} \cdot x_0 + \vec{j} \cdot y_0 + \vec{k} \cdot z_0 = \vec{r} + Q_T = \vec{i} \cdot (x + q_1) + \vec{j} \cdot (y + q_2) + \vec{k} \cdot (z + q_3) \tag{7}$$

A rotation of a position vector is expressed by the rotation quaternion which is unit quaternion

$$Q_R = \cos\frac{q}{2} + \vec{n} \cdot \sin\frac{q}{2} \tag{8}$$

where unit vector $\vec{n}^0$ represents the direction of the axis about which a vector rotates, q represents the angle rotated. If a position vector expressed by (5) realizes a rotation defined by rotation quaternion Q_r , and $\vec{r}$ transform to a new position vector $\vec{r}_0$, the relationship between $\vec{r}_0$ and $\vec{r}$ is

$$\vec{r}_0 = \vec{i} \cdot x_0 + \vec{j} \cdot y_0 + \vec{k} \cdot z_0 = Q_R \cdot \vec{r} \cdot Q_R^{*} \tag{9}$$

A direction of unit vector is determined by direction cosines $l = \cos a$, $m = \cos b$, $n = \cos g$, where a, b, g are Euler angles and $l^2 + m^2 + n^2 = 1$, one obtains polar representation of a quaternion

$$Q = q_0 + i \cdot q_1 + j \cdot q_2 + k \cdot q_3 = \sqrt{N(q)} \cdot \left[\cos\frac{q}{2} + \sin\frac{q}{2} \cdot \left(i \cdot \cos a + j \cdot \cos b + k \cdot \cos g\right)\right] = \sqrt{N(q)} \cdot Q_R$$

Any elementary motion of a position vector is the combination of its rotation around an axis carrying a unit vector $\vec{n}$ of an angle q with translation along this axis. When the vector $\vec{r}$ realizes a rotation Q_R and a translation Q_T , the new vector $\vec{r}^0$ is

$$\vec{r}^0 = Q_R \cdot \vec{r} \cdot Q_R^{*} + Q_T .$$

On the base of quantities transcription of the last equation, the applications can be the the following:

a) The manipulator motion is specified by a sequence of the base transformations giving the location and orientation of the center of the end effector with respect to the coordinate system of the workstation. Each such base consists of rotation followed by translation and therefore the location of an A object point with respect to the workstation is given by

$$\vec{p}_w = R_A \cdot \vec{p}_f + \vec{p}_A, \tag{10}$$

where R_A is the rotation quaternion operator, $\vec{p}_f$ is the location vector of a point with respect to the origin of A, $\vec{p}_A$ is translation vector at a displacement of an A object.

b) The 3-dimensional motion of a rigid body is expressed as

$$\vec{p}' = R \cdot \vec{p} + \vec{L},$$

where $\vec{p}'$ and $\vec{p}$ are position vectors of an object, for instance in times t, t'; R is a rotation quaternion operator, $\vec{L}$ is an object translation vector.

We frequently express a rotation as a right-hand twist by an angle q about an axis $\vec{n}$, therefore the quaternion operator (orthogonal matrix)

$$R = Rot(\vec{n}, q) \tag{11}$$

where $R^{-1} = Rot(\vec{n}, -q) = Rot(-\vec{n}, q) = R$ and by means of equation (8), the "null" rotation would be given by

$$Rot(\vec{n}, 0) = (1 + \vec{n}, 0) = (1 + 0).$$

Notice that if a rotation R is represented by a quaternion Q_R, then the quaternion corresponding to R^{-1} can be obtained by negating the vector part of Q_R. If rotations R_1 and R_2 are represented by quaternion Q_{R1} and Q_{R2}, respectively, then the resultant rotation $R_1 R_2$ will be represented by the quaternion product $Q_{R1} \cdot Q_{R2}$.

3. Quaternion kinematic model for robot manipulation

In this section we derive forward recursive symbolic relations for evaluating the robot manipulator open chain structure. Each component of the chain is taken to a supposedly undeformable solid body marked with subscript i varying form 0 to n. Body C_i is linked to body C_{i-1} by means of a revolute or prismatic joint (i-1) marked with subscripts. A body C_i is associated with body C_{i+1} by means of another revolute or prismatic joint.

We associate the local coordinate system (base) $F_i(O_i; x_i, y_i, z_i)$ with body C_i in accordance with Denavit-Hartenberg's notation [4]. When the joint is prismatic, only translation parameters are variables and identified with the generalized coordinates and when the joint is revolute only revolute parameters are variables identified with generalized coordinates.

Taking into account the theory of homogenous transformation matrices, the elementary enlarged homogenous matrix (where $\sin q, a = sq, a$, $\cos q, a = cq, a$) relation the i^{th} coordinate system to the $(i-1)^{th}$ one

- for a rotary joint

$$T_{i-1}^{i}=\left[\begin{array}{cccc}cq_i & -sq_i\cdot ca_i & sq_i\cdot sa_i & a_i\cdot cq_i\\ sq_i & cq_i\cdot ca_i & -cq_i\cdot sa_i & a_i\cdot sq_i\\ 0 & sa_i & ca_i & a_i\cdot d_i\\ \hline 0 & 0 & 0 & 1\end{array}\right] \qquad (12)$$

where q_i -is the joint angle, a_i -is the twist angle, d_i - the distance along joint axis z_{i-1} between the origin of system $(i-1)$ and the intersection joint with the common normal.

- for a prismatic joint the elements $t_{1,4}, t_{2,4}$ are omitted. Therefore the base transformations are commonly represented by enlarged (4x4) matrices, created by 4 submatrices for rotation (orientation), translation (column vector of the origin position), perspective transformation and a scale. The rotation submatrix R_{3x3} has the following property: $R^{-1}=R^{T}$, (orthogonal submatrix).

The transformation matrix between the coordinate frame of the last $(n-th)$ line and the base reference coordinate system is

$$T_n^0=T_1^0\cdot T_2^1 ... T_{n-1}^{n-2}\cdot T_n^{n-1}, \qquad (13)$$

where multiplication may be executed either by applying backward recursive relations

$$T_n^{i-1}=T_i^{i-1}\cdot T_n^i,\ i=n-1,..,1$$

or by forward recursive relations

$$T_i^0=T_{i-1}^0\cdot T_i^{i-1},\ i=2,..,n.$$

The employment of recursive relation type depends on the purpose of matrix elements using and plays an important role in reducing the computational complexity.

The direct kinematic model is relationship defining the situation of the manipulator end effector as a function of its configuration. Any elementary motion can be also expressed by a quaternion operator [1]

$$Q=\left(1+i\cdot\frac{Q_T}{2}\right)\cdot Q_R, \qquad (14)$$

where Q_T, Q_R are given by equations (6), (8), $\vec{n}=\vec{j}\cdot l+\vec{k}\cdot m+\vec{s}\cdot n$, and $\vec{j},\vec{k},\vec{s}$ -unit vectors. This operator characterizes the screwing process along an axis carrying vector $\vec{n}_0$.

Let as define a required orientation such that a rotation of angle $q_{0,n}$ around an axis carrying the unit vector $\vec{n}^{0}$ orients a base F_0 parallel (or identifies) to a base F_n. Parameters of the rotation quaternion are then defines as:

$$C=\frac{\cos q_{0,n}}{2},\ L=l\cdot\sin\frac{q_{o,n}}{2},\ M=m\cdot\sin\frac{q_{0,n}}{2},\ N=n\cdot\sin\frac{q_{0,n}}{2}$$

and the respective rotation quaternion is expressed as:

$$Q_{R_0,n}=C+\vec{j}\cdot L+\vec{k}\cdot M+\vec{s}\cdot N. \qquad (15)$$

The position is defined by the coordinates of the origin of the base F_n in the reference base F_0, equation (7).

The manipulator configuration is defined by so called generalized coordinates $\left(q_{0,1},..,q_{n-1,n}\right)$ to which correspond the elementary quaternion operators

$\left(Q_{0,1},..,Q_{n-1,n}\right)$ with the index n – number of a joint. Then situation of the end effector and computation of the direct kinematic model is deduced from equation (14) with the product of n-elementary quaternion operators, as

$$Q_{0,n}=\left(1+\vec{i}\cdot\frac{Q_{T_0,n}}{2}\right)\cdot Q_{R_0,n}=\prod_{i=1}^{n}q_{i-1,i} \tag{16}$$

which leads to put equal primary and secondary parts (denote P_r, S_e respectively) of both sides of equation (16)

$$Q_{R_0,n}=P_r\left(\prod_{i=1}^{n}q_{i-1,i}\right),\ Q_{T_o,n}=2\cdot S_e\left(\prod_{i=1}^{n}q_{i-1,i}\right)\cdot Q^{*}_{R_0,n} \tag{17}$$

The primary part allows obtain the parameters C, L, M, N and the secondary part the position coordinates (5).

This representation is rather more efficient than the matrix representation. Storage requirements are reduced, and calculations involving rotations can be done with fewer primitive operations (adds and multiplies) than are required if matrices are used.

4. Locomotion robot model by quaternion

In this section a brief survey of quaternion use angular coordinates for locomotion robot is presented. Then the differential equations of elements of a quaternion are developed [2].

Quaternion expressed like four-dimensional vectors are convenient to describe rotation and translation of three-dimensional space of coordinate bases and they are useful for description of rigid body orientation. Let x is a pure quaternion (i.e. a vector) described by:

$$\vec{x}=\vec{i}\cdot x_1+\vec{j}\cdot x_2+\vec{k}\cdot x_3\,. \tag{18}$$

Let l be a unit quaternion (that is, $l\cdot l^{*}=1$) described by

$$l=l_0+i\cdot l_1+j\cdot l_2+k\cdot l_3\,. \tag{19}$$

According to equation (9) now consider the quaternion y defined by the triple product defining rotation of $\vec{x}$

$$y=l\cdot\vec{x}\cdot l^{*} \tag{20}$$

By performing the indicated multiplications, or writing (20) with the aid of (4)

$$h=l\cdot\vec{x} \tag{21}$$

$$y=\left(l\cdot\vec{x}\right)\cdot l^{*}=h\cdot l^{*} \tag{22}$$

and substituting (21) into (22) upon returning from four to three dimensions for y one obtains the equation

$$y=T\cdot\vec{x}$$

where

$$T=\begin{bmatrix}2\cdot\left(l_0^2+l_1^2\right)-1 & 2\cdot\left(l_1\cdot l_2-l_0\cdot l_3\right) & 2\cdot\left(l_1\cdot l_3+l_0\cdot l_2\right)\\ 2\cdot\left(l_1\cdot l_2+l_0\cdot l_3\right) & 2\cdot\left(l_0^2+l_2^2\right)-1 & 2\cdot\left(l_2\cdot l_3+l_0\cdot l_1\right)\\ 2\cdot\left(l_1\cdot l_3-l_0\cdot l_2\right) & 2\cdot\left(l_2\cdot l_3+l_0\cdot l_2\right) & 2\cdot\left(l_0^2+l_3^2\right)-1\end{bmatrix} \tag{24}$$

It can be shown that (20) preserves the norm, that is

$$\vec{x}\cdot\vec{x}^{*}=y\cdot y^{*}\,. \tag{25}$$

This demonstrates that quaternions can be used as angular coordinates.

A development of differential equations of the elements of a quaternion is convenient for robot modelling. Let $\vec{n}$ and $\vec{h}$ be fixed vectors attached to local earth-fixed base and rotating body base respectively. Consider transformation defined by (20)

$$h = l \cdot \vec{n} \cdot l^* . \tag{26}$$

Rewriting (26) in the body base a:

$$h_b = l \cdot \vec{n}_b \cdot l^* . \tag{27}$$

where the subscript b implies the corresponding vectors expressed in body base (x, y, z). Let us further assume that two vector systems coincide with $l = 1$. That is

$$\vec{h}_b = \vec{n}_e \tag{28}$$

where the subscript e implies the corresponding vector expressed in the local earth-fixed base. Since $\vec{h}$ moves with body frame, (28) holds for all values of l . By means of (27) we obtain

$$n_e = l \cdot \vec{n}_b \cdot l^* \text{ or } n_b = l^* \cdot \vec{n}_e \cdot l . \tag{29}$$

This relates in quaternion notation the components of the vector $\vec{n}$ in the two coordinate bases. In matrix notation it becomes

$$Y = \Lambda^T \cdot Y_e \cdot \Lambda , \tag{30}$$

where Y is the matrix representation of quaternion n . Differentiating this and noting that Y_e is a fixed quaternion matrix yields

$$\dot{Y} = \dot{\Lambda}^T \cdot Y_e \cdot \Lambda + \Lambda^T \cdot Y_e \cdot \dot{\Lambda} , \tag{31}$$

where from (30)

$$Y_e = \Lambda \cdot Y \cdot \Lambda^T . \tag{32}$$

Substituting (32) into (31) and simplifying yields

$$\dot{Y} = \dot{\Lambda}^T \cdot \Lambda \cdot Y + Y \cdot \Lambda^T \cdot \dot{\Lambda} .$$

As y is a pure quaternion so that [see equation (4), matrix P], $Y^T = -Y$ result in

$$\dot{Y} = \dot{\Lambda}^T \cdot \Lambda \cdot Y - \left(\dot{\Lambda}^T \cdot \Lambda \cdot Y\right)^T \tag{33}$$

If n_e an n are component representation of an arbitrary fixed vector in the earth-fixed and body bases respectively, then one can write for position vector of body point from a gravity center

$$n = E \cdot n_e \tag{34}$$

where E is an orthogonal transformation (3x3) matrix ($E^T = E^{-1} = \Omega$, equation (24)) with goniometric Euler angle functions [1]. By differentiating w.r.t. time and noting $\frac{dn_e}{dt} = 0$ yields

$$\dot{n} = \dot{E} \cdot n_e \tag{35}$$

and using (34) too

$$\dot{n} = E \cdot E^T \cdot n . \tag{36}$$

On the other hand, the rotating base fixed vector time derivative $\dot{\vec{n}}$ w.r.t. rotating base is a velocity of point relative to earth-fixed base given by

$$\frac{d\vec{n}}{dt} = \frac{d\tilde{n}}{dt} + \vec{w} \times \vec{n} = 0 , \; \dot{\vec{n}} = -\left(\vec{w} \times \vec{n}\right) , \tag{37}$$

where

$$\frac{d\vec{n}}{dt} = \vec{i} \cdot \frac{dn_x}{dt} + \vec{j} \cdot \frac{dn_y}{dt} + \vec{k} \cdot \frac{dn_z}{dt}$$

which is a velocity of center of mass relative to earth-fixed base and $\vec{w}$ is an angular velocity vector of the body relative to earth-fixed base.

In matrix notation (37) will be

$$\dot{\vec{n}} = -\Omega \cdot \vec{n}\ , \tag{38}$$

where Ω is the quaternion matrix of the angular velocity vector of the rotating base presented in the rotation base, therefore

$$\Omega = \begin{bmatrix} 0 & -w_z & w_y \\ w_z & 0 & -w_x \\ -w_y & w_x & 0 \end{bmatrix}$$

and $w_{x,y,z}$ are angular velocity components. Comparing (36) and (38) yields

$$\dot{E} \cdot E^T = -\Omega \ \text{ or } \ \dot{E} = -\Omega \cdot E\ . \tag{39}$$

As was proved the matrix quaternion representation is also

$$\dot{Y} = -\frac{1}{2} \cdot \left[\tilde{\Omega} \cdot Y - \left(\tilde{\Omega} \cdot Y \right)^T \right]. \tag{40}$$

Comparing (40) and (33) gives

$$\dot{\Lambda}^T \cdot \Lambda = -\frac{1}{2} \cdot \tilde{\Omega},\ \dot{\Lambda}^T = -\frac{1}{2} \cdot \tilde{\Omega} \cdot \Lambda^T\ . \tag{41}$$

In this case the quaternion matrix $\tilde{\Omega}$ is enlarged matrix Ω with respect to four-dimensional space (equation 4). Therefore (41) may be expressed

$$\begin{bmatrix} \dot{l}_0 \\ \dot{l}_1 \\ \dot{l}_2 \\ \dot{l}_3 \end{bmatrix} = -\frac{1}{2} \cdot \begin{bmatrix} 0 & w_x & w_y & w_z \\ -w_x & 0 & -w_z & w_y \\ -w_y & w_z & 0 & -w_x \\ -w_z & -w_y & w_x & 0 \end{bmatrix} \begin{bmatrix} l_0 \\ l_1 \\ l_2 \\ l_3 \end{bmatrix} = \frac{1}{2} \cdot \begin{bmatrix} -l_1 & -l_2 & -l_3 \\ l_0 & -l_3 & l_2 \\ l_3 & l_0 & -l_1 \\ -l_2 & l_1 & l_0 \end{bmatrix} \begin{bmatrix} w_x \\ w_y \\ w_z \end{bmatrix}. \tag{42}$$

An initial value of l must be specified for integrating this set of equations. The method of obtaining this value may be the following: elements of complete transformation E can be reached by writing the matrix as the triple product of the separate rotations for E_1, E_2, E_3 transformations describing the rotations by means of Euler angles a, b, g respectively. The product matrix $E = E_3 \cdot E_2 \cdot E_1$ then follows as

$$E = \left[\begin{array}{c:c:c} \cos b \cdot \cos a & \cos b \cdot \sin a & -\sin b \\ -\cos g \cdot \sin a + \sin g \cdot \sin b \cdot \cos a & \cos g \cdot \cos a + \sin g \cdot \sin b \cdot \sin a & \sin g \cdot \cos b \\ \sin g \cdot \sin a + \cos g \cdot \sin b \cdot \cos a & -\sin g \cdot \cos a + \cos g \cdot \sin b \cdot \sin a & \cos g \cdot \cos b \end{array} \right] \tag{43}$$

The complete transformation from robot body-to-local earth system is given by

$$\begin{bmatrix} x_e \\ y_e \\ 0 \end{bmatrix} = E^T \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} \ \text{ and } \ \begin{bmatrix} \dot{x}_e \\ \dot{y}_e \\ 0 \end{bmatrix} = E^T \cdot \begin{bmatrix} v_x \\ v_y \\ 0 \end{bmatrix} \tag{44}$$

with initial conditions $x_e(0), y_e(0)$.

As $\Omega = E^T$, by comparing corresponding elements of matrices (43) and (24), the components of the quaternion can be related to Euler angles for initial conditions as follows

$$l_0(0) = c\frac{a_0}{2} \cdot c\frac{b_0}{2} \cdot c\frac{g_0}{2} + s\frac{a_0}{2} \cdot s\frac{b_0}{2} \cdot s\frac{g_0}{2}\ ,$$

$$l_1(0) = c\frac{a_0}{2} \cdot c\frac{b_0}{2} \cdot s\frac{g_0}{2} - s\frac{a_0}{2} \cdot s\frac{b_0}{2} \cdot c\frac{g_0}{2}\ ,$$

$$l_2(0) = c\frac{a_0}{2}\cdot s\frac{b_0}{2}\cdot c\frac{g_0}{2} + s\frac{a_0}{2}\cdot c\frac{b_0}{2}\cdot s\frac{g_0}{2}, \tag{45}$$

$$l_3(0) = -c\frac{a_0}{2}\cdot s\frac{b_0}{2}\cdot s\frac{g_0}{2} + s\frac{a_0}{2}\cdot c\frac{b_0}{2}\cdot c\frac{g_0}{2}$$

where $\mathrm{c}\,a, b, g = \cos a, b, g$ and $s a, b, g = \sin a, b, g$.

The current values of Euler angles are reached by matrix (24) elements where $a = \tan^{-1}\left(\frac{t_{21}}{t_{11}}\right)$, $b = -\sin^{-1}(t_{31})$, $g = \tan^{-1}\left(\frac{t_{32}}{t_{33}}\right)$.

In development of a locomotion robot model the equations of motion are described at first, the influencing forces are taken to be through the mass center and the moments are about the body axes. The accelerations on the robot are computed by means of forces from the drivers. Differential equations (42) have to be solved with initial conditions (45) and local earth to body transformation matrix (24). Robot motion trajectories are described by (44) for its mass center in the local earth-fixed coordinates.

5. Conclusion

This paper has investigated the usefulness of quaternion representation mainly of kinematics for robotics applications. Quaternions has been meationed as an attractive mathematical tool also for robot dynamic and combined manipulator/locomotion behaviour representation. There are two kinematics problems in the command and control of a robot manipulator and locomotion. In the case of a robot manipulator one is the direct problem and the other is inverse problem. The direct problem only solved in this paper is the determination of the position and orientation of the end effector of the manipulator given the joint angles and arm lengths of the manipulator. The inverse problem is to find a set of joint angles and arm lengths for a prescribed position and orientation of the end effector. Traditionally both problems are handled using the homogenous matrix representation [4]. In the case of a locomotion robot there is the problem to determine and solve equations of motion. This paper handles these problems using quaternions and advances a navigation approach to track the position and orientation of the locomotion robot. Quaternion method offers the best computation efficiency for these applications.

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