National Conference with International Participation
ENGINEERING MECHANICS 2001
Svratka, Czech Republic, May 14 - 17, 2001

METHODS OF MEASUREMENT OF VIBRO-ACOUSTIC TRANSFER PROPERTIES OF RESILIENT ELEMENTS

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Summary: *Passive vibration isolators of various kinds are used to reduce the transmission of vibrations. The paper describe the theoretical principles of the measurement for the determination of the most important quantities which govern the transmission of vibration through linear isolators. This paper shows the limitations of the methods which are used for application in practice. The results of the methods are useful for isolators which are used to prevent low-frequency vibration problems and attenuate structure-borne sound.*

1. Introduction

Isolation and structural damping constitute the two most widely applicable means for the control of vibration or structure-borne sound, particularly in the audio frequency range. Vibration isolation in essence involves use of a resilient connection between a source of vibration and an item to be protected, so that this item vibrates less than it would if a rigid connection were used. In some typical situation the source consists of a vibration machine or structure and the item to be protected is a human being, environment, instruments, machines and so on. Many salient features of vibration isolation can be analysed in terms of a simple model consisting of a rigid mass that is connected to a support via a isolator and that is constrained to translate along a single axis. More complex models are needed to address situations where the magnitude of excitation depends of the motions, where an additional isolator-mass (spring-mass) system is inserted between the primary one and the support, and/or at comparatively high frequencies where the isolator mass plays a significant role or where the isolated items do not behave as rigid masses. Other complications arise because of non-uniaxial motions and nonlinearities.

2. Dynamic transfer stiffness matrix of vibration isolators

The dynamic transfer stiffness is determined by elastic, inertia and damping properties of the isolators. The reason for choosing a presentation of test results in terms of a stiffness is the practical consideration that it complies with date of static and/or low-frequency dynamic stiffness which are commonly used. The additional importance of inertial forces makes the dynamic transfer stiffness at high frequencies more complex than at low frequencies. Because at low frequencies only elastic and damping forces are important, the low-frequency dynamic stiffness is only weakly dependent on frequency due to material properties. In principle the dynamic transfer stiffness of vibro-acoustic isolators is dependent on static preload and temperature.

A familiar approach to the analysis of complex vibratory systems is the use of stiffness – compliance – or transmission matrix concepts. The matrix elements are basically special forms of frequency-response function; they describe linear properties of mechanical and acoustical systems. On the basis

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of knowledge of the individual subsysten properties, corresponding properties of assemblies of subsystem can be calculated. The general conceptual framework for the proposed isolator characterisation is shown in figure 1.

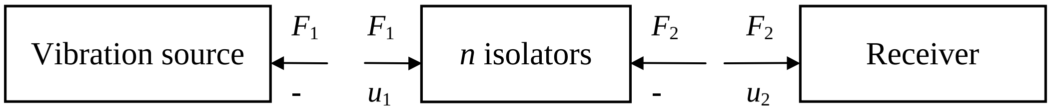

Figure 1. – Block diagram of source/isolators/receiver system

A point contact is assumed at each connection between source and isolator and between isolator and receiver. To each connection point a force vector $\boldsymbol{F}$ containing three orthogonal forces and three orthogonal moments and a displacement vector $\boldsymbol{u}$ containing three orthogonal translational and three orthogonal rotational components are assigned.

In figure 1 just one component of each of the vector F_1, u_1, F_2 and u_2 is shown. To show that the *blocked transfer stiffness* is suitable for isolator characterisation in many practical cases, the discussion will proceed from the simplest case of unidirectional vibration to the multidirectional case for single isolator as follows

$$F_1 = k_{1,1}u_1 + k_{1,2}u_2 \tag{1}$$

$$F_2 = k_{2,1}u_1 + k_{2,2}u_2 \tag{2}$$

where $k_{11}$and k_{22} are driving point stiffnesses when the isolator is blocked at the opposite side.

k_{12} and k_{21} are blocked transfer stiffnesses.

The matrix form of equations (1) and (2) is

$$\boldsymbol{F} = [k]\, \boldsymbol{u} \tag{3}$$

with the dynamics stiffness matrix

$$[k] = \begin{bmatrix} k_{1,1} & k_{1,2} \\ k_{2,1} & k_{2,2} \end{bmatrix} \tag{4}$$

For excitation of receiving structure via the isolators

$$k_{\mathrm{r}} = -\frac{F_2}{u_2} \tag{5}$$

where k_τ denotes the dynamics driving point stiffness of the receiver.

From equation (2) and (5) it follows that

$$F_2 = \frac{k_{2,1}}{1 + \dfrac{k_{2,2}}{k_1}} u_1 \tag{6}$$

If $\left|k_{2,2}\right| \leq 0{,}1\left|k_\tau\right|$, then F_2 approximates the so-called blocking forces to within 10 %, i.e.

$$F_2 = F_{2,\mathrm{blocking}} = k_{2,1}u_1 \tag{7}$$

Because vibration isolators are only effective between structures of relatively large dynamic stiffness on both sides of the isolator, equation (7) represents the intended situation at the receive side. Measurement of the blocked transfer stiffness for an isolator under static preload is easier than measurement of the complete stiffness matrix. Moreover it forms the representative isolator characteristic under the intended circumstances.

If forces and motions at each interface can be characterised by six orthogonal components, the isolator may be described as a 12-port [3]. If the receiver has relatively large driving point dynamic stiffnesses compared to the isolator, the forces exerted on the receiver approximate the blocking forces given by equation (7). Therefore, the blocked transfer stiffnesses are appropriate quantities to characterise vibro-acoustic transfer properties of isolators, and also in the case of multidirectional vibration transmission.

3. Energy dissipation and loss factor

The *damping capacity* ψ of a vibrating system is defined as $\psi = D/W$, where D represents the energy that is removed from the system per cycle and W denotes the vibrational energy stored in the system. The *loss factor* η, defined by $\eta = \psi/2\pi$, represents the ratio of the energy removed per radian to the stored vibrational energy. In most practical situations the loss factors less than 0,2 and one may take the stored energy to be equal to the total vibrational kinetic and potential energy. In general, the various measures of damping are related to each other by

$$\eta = \frac{\psi}{2\pi} = \mathrm{tg}\phi = \frac{k_i}{k_0} \tag{8}$$

or at resonance

$$\eta = 2\varsigma = \frac{1}{Q} \tag{9}$$

where ζ is *damping ratio* (viscous damping coefficient/critical damping coefficient)

Q *quality factor*.

The phase angle ϕ by which the displacement of the mass of the isolator lags the excitation force may also be used to characterise the system's damping. For example, at radian frequencies ω much below the system's natural frequency $\mathrm{tg}\phi = k_i/k_0$. Where k_i is the imaginary part of complex stiffness

$$k = k_0 + \mathrm{j}k_i. \tag{10}$$

This complex stiffness, which is equal to the ratio of the phasor of the applied force to that resulting displacement, characterizes both, the energy storage and energy dissipation of the system.

For the purpose of the discussion it is sufficient to consider a single isolator with a single vibration direction. Because only measurements with a block output side are considered, the phasor equation (1) and (2) are reduced to

$$F_1 = k_{1,1} u_1 \tag{11}$$

$$F_2 = k_{2,1} u_1 \tag{12}$$

At low frequencies, where inertial forces play no role, there is a simple relationship between the phase angle of the dynamic transfer stiffness and the damping properties of the resilient element and the frequency-dependent stiffness can be approximated by $k \approx k_{1,1} \approx k_{2,1}$.

This complex low-frequency dynamic stiffness can be written from equation (8) and (10) as

$$k = k_0(1 + \mathrm{j}\eta) \quad (13)$$

The loss factor of a resilient element can be estimated according to

$$\eta \approx \mathrm{tg}\phi_{2,1} \quad (14)$$

where $\phi_{2,1}$ is the phase angle of the dynamic transfer stiffness $k_{2,1}$.

4. Conclusion

The model shown in figure 1 and equation (1) to (7) is correct under the assumption that the isolators forms the only transfer path between the vibration source and receiver. In practice there may be mechanical or acoustical parallel transmission paths which cause flanking transmission. For any measurement method of isolator properties, the possible interference of such flanking with proper measurements has to be minimised.

The measurement of small loss factors using equation (14), is extremely sensitive to phase measurement errors. For higher frequencies, where the approximations $k \approx k_{1,1} \approx k_{2,1}$ are no longer valid, it is no longer correct to use equation (14) as a characterisation of the damping properties of the resilient elements.

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