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THE EFFECTS OF MASS TRANSFER IN RAYLEIGH-PLESSET BUBBLE DYNAMICS AND CAVITATION MODELING IN A CONVERGENT-DIVERGENT NOZZLE

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Summary: *The paper studies the effects of the mass transfer term in the previously presented modified Rayleigh-Plesset equation which includes mass flow across the boundary of a spherical bubble. Some interesting conclusions are drawn regarding the relative importance of the term during the unbounded bubble growth modeled numerically. In addition, the paper proposes an experimental cavitation modeling design based on a convergent-divergent nozzle, as the focus of the experimental investigation of cavitation/bubble nucleation in a gas contaminant water mixture has progressively turned from the nucleation pulse technique applied at a shock-tube to depressurization in a flowing liquid.*

1. INTRODUCTION

In the previous works of the authors, a modified form of the Rayleigh-Plesset equation has been introduced in order to account for mass transfer effects occurring at the interface between the bubble content and the surrounding liquid. Such mass transfer can be caused by phase transition (i.e. evaporation or condensation) and/or gas diffusion (dissolving of the contaminant gas). The modified Rayleigh-Plesset equation replaces the traditional Rayleigh-Plesset equation (which neglects evaporation, condensation and diffusion) and has the following form:

$$\frac{p_B - p_\infty}{\rho_L} = R\ddot{R} + \frac{3}{2}\dot{R}^2 + \left(4\frac{\mu_L}{\rho_L} + \frac{2Rj_B}{\rho_B}\right)\frac{\dot{R}}{R} + \frac{R\dot{j}_B}{\rho_L} + \frac{2\sigma}{\rho_L R} \tag{1}$$

The newly added terms are the 4th and 5th terms on the right-hand side of the equation, where j_B is mass flux across the bubble boundary in the radial outward direction (see Fig. 1) defined as follows:

$$j_B(t,r) = \rho_B\left(v_B - \dot{R}\right) = \rho_L\left(v_L - \dot{R}\right) \tag{2}$$

Nomenclature:

B properties on the inner side of the bubble interface (bubble contents)

L properties on the outer side of the interface (liquid ambience)

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∞	properties in infinity
R	bubble interface radius [m]
r	distance from bubble center [m]
v	velocity [m/s]
ρ	density [$\mathrm{kg/m^3}$]
p	pressure [Pa]
μ	dynamic viscosity [Pa.s]
σ	surface tension [N/m]

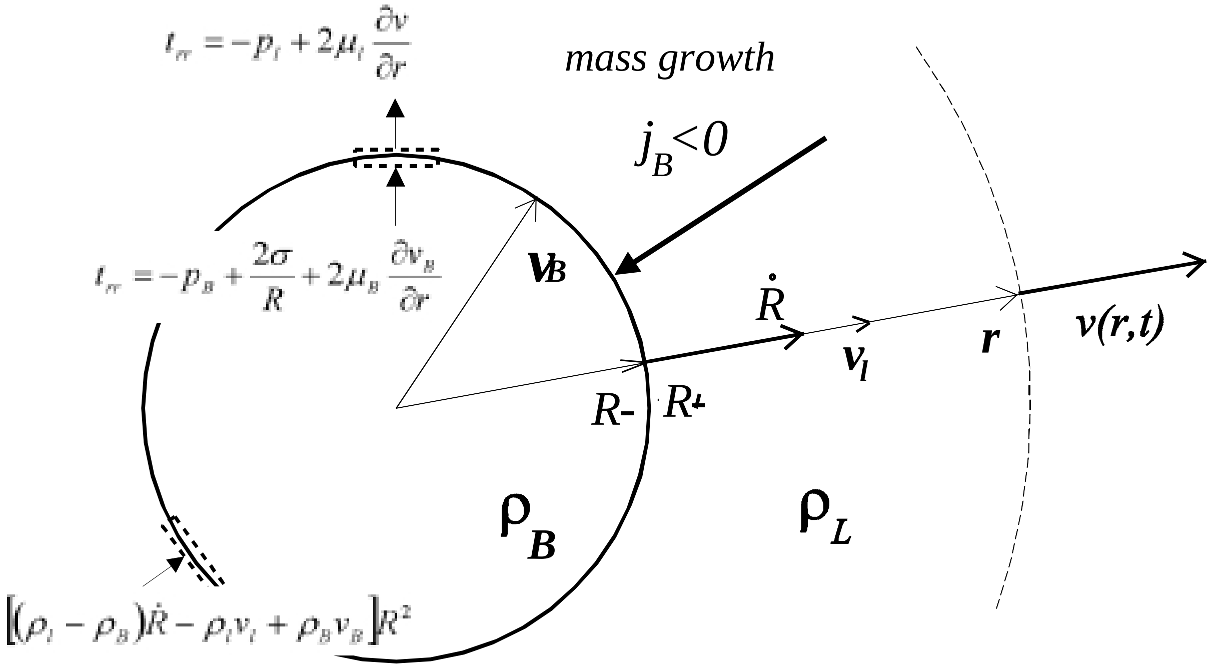

Fig. 1 Balance of mass and momentum at the interface of a spherical bubble in liquid

In the current work, the modified Rayleigh-Plesset equation has been subject to theoretical investigation to determine the magnitude of importance of the mass transfer term relative to the other terms in the equation. In addition, the paper presents a design sketch of convergent-divergent nozzle for experimental investigation of cavitation phenomena in water-gas solution.

2. THE EFFECT OF MASS TRANSFER IN BUBBLE DYNAMICS

In Eq. 1 and Fig. 1, the mass flux is oriented in the radial outward direction. For $j_B > 0$ mass flows out of the bubble and supports closing of the bubble, for $j_B < 0$ mass goes into the bubble and supports opening of the bubble and for $j_B = 0 = const$ mass transfer across the boundary is zero – the modified equation degenerates into the standard Rayleigh-Plesset equation.

For a spherical bubble boundary and multi-component liquid mixture, one can define the mass flux using concentrations as a sum of partial mass fluxes of multiple components/phases:

$$j_B = \sum_i j_{Bi}\ ,\ \ j_{Bi} = -\frac{\rho_L D_{Li}\left(c_{Li\infty} - c_{LiB}\right)}{R\sum_j \left(c_{vj\infty} - c_{vjB}\right)} \tag{3}$$

Here, D is diffusion coefficient, index v denotes vapor and c is mass fraction defined as follows:

$$c_B = \rho_B/\rho\ ,\ c_v = \rho_v/\rho \tag{4}$$

In order to evaluate the importance of the mass transfer term on the solution of the modified Rayleigh-Plesset equation, one has to define a dimensionless quantity that could capture the addition of mass transfer phenomena. Such a quantity can be the Schmidt number defined as the ratio of viscous and diffusive forces.

$$Sc = \mu_L/\rho_L D \tag{5}$$

To complete the mathematics, we need to express the viscosity term in the modified Rayleigh-Plesset equation as equal to

$$\left(4\frac{\mu_L}{\rho_L} + \frac{2Rj_B}{\rho_B}\right) = 4\frac{\mu_L}{\rho_L}\left(1 + \frac{j_B R}{2\mu_L}\frac{\rho_L}{\rho_B}\right), \tag{6}$$

while the mass flux is proportional to

$$j_B \approx -D\frac{\partial \rho_B}{\partial r}\bigg|_R \approx -\frac{\rho_L D\left(c_{lB_\infty} - c_{lB}\right)}{R_0}. \tag{7}$$

Here, R_0 is the initial bubble radius (equilibrium radius) constituting the initial condition for the numerical solution.

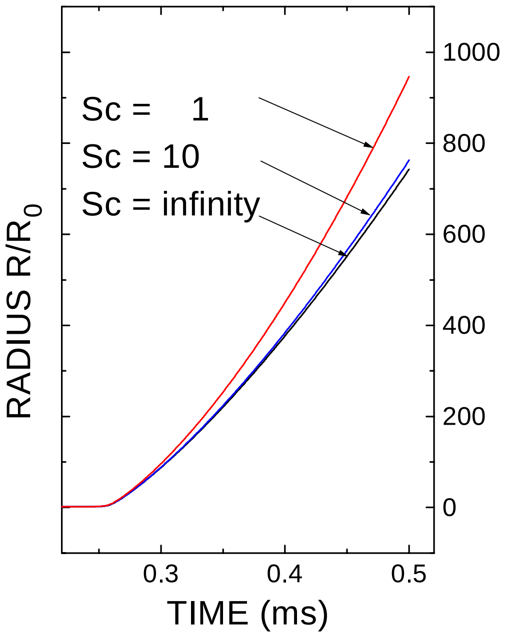

Fig. 2 Response of a single spherical bubble to a sudden pressure decrease ($R_0 = 2.10^{-6} m$)

Now, we can analyze the effects of diffusive phenomena on the numerical growth rate of a spherical bubble. The following equilibrium condition is applied:

$$p_V + \sum_i p_{G0i} - p_\infty(0) = \frac{2\sigma}{R_0}, \quad (8)$$

where G denotes properties of contaminant gas(es).

Fig. 2 clearly demonstrates that the rate of growth indeed depends on the Schmidt number (the presented result was calculated numerically for the case of an unbounded stable bubble growth. For the traditional Rayleigh-Plesset equation, we obtain the reference curve with no mass transfer and $Sc \to \infty$. For air at normal temperature, we have $Sc = 500$ and slightly faster growth rate. The difference between these two cases does not appear significant, however, one has take into account the time scale used to plot the figure. For the sake of lucidity, the time scale was reduced to 0.5 ms. Typically the time of growth can be longer, for example in the nozzle presented later in this article the time to study bubbles in their growth phase may reach 2-4 ms. Finally, at $Sc = 1$ (near phase transition) the curve declines from the reference curve notably, showing that evaporation may play a significant role in bubble dynamics, when suitable conditions are set.

3. APPLICATION OF CONVERGENT-DIVERGENT NOZZLE IN CAVITATION EXPERIMENTS

In the previous publications the authors focused their experimental preparations on the principle of nucleation pulse, hoping to use the experience gained during the experimental measurement of droplet growth and nucleation (condensation). However, after thoroughly analyzing the specific requirements of the cavitation experiments, the authors have decided to follow the more conventional path for studying bubble creation and growth, i.e. depressurization of liquid in a convergent-divergent nozzle.

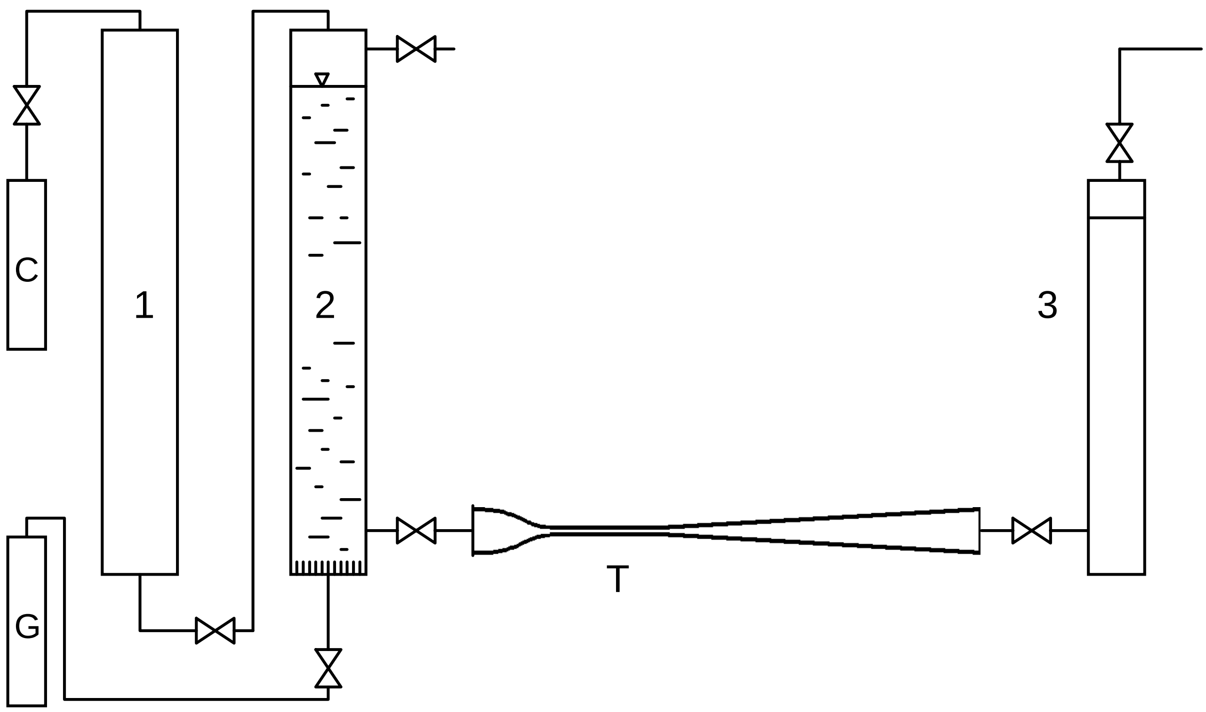

Fig. 3 Scheme of experimental setup for studying cavitation in convergent-divergent nozzle.

Fig. 3 shows the design of the experimental configuration being designed in the Institute of Thermomechanics. Vessel 2 is filled with the test liquid (tap water). The contaminant gas G is added at the bottom of vessel 2 until the required gas concentration in the liquid is achieved. The liquid in vessel 2 is pressurized using a carrier gas C (vessel 1) and released into the convergent-divergent nozzle. In the test section of the nozzle (the section with the lowest nozzle diameter) large negative pressures are attained and cavitation is observed and measured (using optical methods and pressure measurement).

4. CONCLUSION

The mass transfer term in the modified Rayleigh-Plesset equation has proven to be of indispensable importance in describing the dynamics of spherical bubbles in water. The now used equation provides a good base for theoretical and numerical completion of the experimental study of cavitation phenomena, which will bear on the more promising principle of depressurization in convergent-divergent nozzles.

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