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VOLD-KALMAN ORDER TRACKING FILTRATION AS A TOOL FOR MACHINE DIAGNOSTICS

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***Summary:** The main topic of the paper deals with diagnostics of rotating and reciprocating machines based on RPM, noise and vibration measurements. Key words are the Vold-Kalman order tracking filtering. An example of vibration measurements is taken from the diagnostics of the machining center.*

1. INTRODUCTION

The dynamic test of rotating machines is based on the noise and vibration measurements. The base frequency of all of these exciting forces is related to the machine rotation frequency. An extensive vibration is excited when the base frequency or its harmonics meet the structural resonance frequency of machines. The machine is tested during steady-state rotation or run up / coast down. Clear information about the origin of the extensive vibration cannot be given by a single frequency spectrum but by a multispectrum recorded during variation of the machine RPM. It should be mentioned that any driven unit does not rotate at a purely constant speed but its speed slowly varies around an average value. Spectrum components of the diagnostic signal originate from simultaneously amplitude and phase modulation of so called carrying harmonic components that correspond to the excitation at a purely steady-state rotation. The amplitude modulation of harmonic signals arises from the non-uniform periodic load while a phase modulation is there due to the non-uniform rotational speed. Rotation speed variations at the fixed signal sampling frequency cause the smearing of the dominating components in the frequency spectra.

An analysis of signals from machines running in cyclic fashion is preferred in terms of order spectra rather than frequency spectra. The order spectra are evaluated using time records that are measured in dimensionless revolutions rather than seconds and the corresponding FFT spectra are measured in dimensionless orders rather than frequency. This technique is called order analysis or tracking analysis, as the rotation frequency is being tracked and used for analysis. The resolution of the order spectrum is equal to the reciprocal value of the revolution number per a record corresponding to input data for the Fast Fourier Transform (FFT).

A lot of practical mechanical systems contain multiple shafts that may run coherently through fixed transmissions, or partially related through belt slippage and control loops, or independently, as for instance a cooling fan in an engine compartment. For coherently running shafts it is possible to use the above mentioned order analysis technique. On the other hand, non-coherently running systems with multiple orders decoupling close and crossing order can be extracted by the Vold-Kalman order tracking. The standard methods based on FT enables only speed limited order tracking while the Vold-Kalman order tracking filtering is without slew rate limitation.

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The fundamentals given in this paper are based on the theory described by [1,3]. The evaluation of the frequency responses is an original contribution to the Vold order techniques. An example of employing of the Vold-Kalman order tracking filtration deals with an analysis of the horizontal machining center.

2. PRINCIPLES OF VOLD-KALMAN FILTERING – ALGORITHM OF THE FIRST GENERATION

A harmonic signal with continuous time, $x(t) = A\cos(\omega t)$, is a solution of the second order differential equation in which the first derivative of the time signal is missing. The harmonic oscillations are neither amplified nor damped. If the time signal is sampled, $t_n = n\Delta t, n = 0, 1, 2,...$ where Δt is the sampling time increment, the harmonic oscillations are a solution of the second order difference equation with a characteristic equation having two complex conjugate roots which equal to the following values of $z_1 = \exp(j\omega\Delta t)$ and $z_2 = \exp(-j\omega\Delta t)$, where ω is an angular velocity. The solution of the mentioned equation takes the form $x(n) = C(z_1^n + z_2^n)$. A characteristic polynomial corresponding to the characteristic equation can be written in the form $(z - z_1)(z - z_2)$ that gives the original difference equation

$$x(n) - 2\cos(\omega\Delta t)x(n-1) + x(n-2) = 0\,, \qquad (1)$$

where the coefficient of the delayed sample $x(n-1)$ can be designated by $c(n) = 2\cos(\omega\Delta t)$. The equation (1) is a linear, frequency dependent constrain equation on the sine wave, and it is called the structural equation of the Kalman filtration [1]. The solution of the equation (1) is based on the first two samples, $x(1)$ and $x(2)$, and the angular velocity (angular frequency), ω.

Noise or vibration signal generated by a rotating machine consists of the sinusoids differing in their frequencies and the signal is contaminated with background noise. The sinusoid frequency is an integer or fractional multiple of the machine shaft rotational frequency that is called a fundamental frequency. The sine wave can slightly change its amplitude and frequency over the time samples involved in the equation (1). In order to express deviations from the true stationary sine wave, the unknown non-homogeneity term, $\varepsilon(n)$, is incorporated on the right side of the mentioned equation

$$x(n) - c(n)x(n-1) + x(n-2) = \varepsilon(n). \qquad (2)$$

Note that the number of samples in the equation (2) is equal to three, hence $x(n-2)$, $x(n-1)$ and $x(n)$. The system of the structural equations containing all the samples $x(1),..., x(N)$ takes the following form

$$\begin{bmatrix} 1 & -c & 1 & 0 & ... & 0 & 0 & 0 \\ 0 & 1 & -c & 1 & ... & 0 & 0 & 0 \\ ... & ... & ... & ... & ... & ... & ... & ... \\ 0 & 0 & 0 & 0 & ... & 1 & -c & 1 \end{bmatrix} \begin{bmatrix} x(1) \\ x(2) \\ ... \\ x(N) \end{bmatrix} = \begin{bmatrix} \varepsilon(3) \\ \varepsilon(4) \\ ... \\ \varepsilon(N) \end{bmatrix} \qquad (3)$$

which can be rewritten in the matrix and vector form

$$\mathbf{A\,x} = \boldsymbol{\varepsilon} \qquad (4)$$

The sum of the squares of all the unknown non-homogeneity terms can be expressed as a scalar product

$$\boldsymbol{\varepsilon}^T\boldsymbol{\varepsilon} = \mathbf{x}\mathbf{A}^T\mathbf{A\,x}\,, \qquad (5)$$

where a row vector $\boldsymbol{\varepsilon}^T$ is a transpose of the column vector $\boldsymbol{\varepsilon}$.

The value of the instantaneous angular frequency, ω, which is a multiple of the machine fundamental frequency, cannot be usually measured at each recorded sample. Tacho pulses, generated once in a rotation of the shaft, give a reduced information on the instantaneous fundamental frequency. Thus the Vold-Kalman order tracking filtration needs a very accurate estimation of the instantaneous fundamental frequency. The methodology that has been chosen for this filtration in PULSE, the Brüel & Kjær signal analyzer is based on fitting cubic splines in a squares sense.

Instead of observing the sampled sinusoidal signal $x(n)$, samples $y(n)$ are recorded. The signal $y(n)$ is combined from both the signals satisfying the structural equation (2) as well as random noise and other sinusoidal components differing in the frequency with the sinusoidal signal $x(n)$. The random noise and other sinusoidal are combined into the signal $\eta(n)$. Formally, it can be written as

$$y(n) = x(n) + \eta(n). \tag{6}$$

The system of the data equations containing all the samples $x(1),\ldots,x(N)$ similar to the system of the structural equations takes the following form

$$\begin{bmatrix} y(1) \\ y(2) \\ \ldots \\ y(N) \end{bmatrix} - \begin{bmatrix} x(1) \\ x(2) \\ \ldots \\ x(N) \end{bmatrix} = \begin{bmatrix} \eta(1) \\ \eta(2) \\ \ldots \\ \eta(N) \end{bmatrix}. \tag{7}$$

The system (7) can be expressed in the vector form

$$\mathbf{y} - \mathbf{x} = \boldsymbol{\eta}. \tag{8}$$

The sum of the squares of the signal $\eta(n)$ can be written in the form

$$\boldsymbol{\eta}^T \boldsymbol{\eta} = \left(\mathbf{y}^T - \mathbf{x}^T\right)\left(\mathbf{y} - \mathbf{x}\right). \tag{9}$$

The equations (3) and (7) give a system of overdetermined equations for the unknown waveform $x(n)$ which can be solved using standard least square techniques such as normal equations allowing very fast solutions algorithm. Both of the sums consisting of the square samples, corresponding to the non-homogeneity term $\varepsilon(n)$ and other sinusoidal and background random noise $\eta(n)$, must be minimized. To control relationship between the standard deviation of the non-homogeneity term $\varepsilon(n)$ and the standard deviation of the mentioned signal $\eta(t)$, the loss function is combined into the form

$$J = r^2 \boldsymbol{\varepsilon}^T \boldsymbol{\varepsilon} + \boldsymbol{\eta}^T \boldsymbol{\eta}. \tag{9}$$

The choice of a large value for a weighting factor r leads to the highly selective filtration in frequency domain that takes a long time to converge in amplitude. In contrast, fast convergence with low frequency resolution is achieved by choosing r small.

The first derivative of the loss function (9) with respect to the vector $\mathbf{x}$ gives a condition for the minimum of this function called a normal equation.

$$\frac{\partial J}{\partial \mathbf{x}} = 2 r^2 \mathbf{A}^T \mathbf{A}\,\mathbf{x} + 2(\mathbf{x} - \mathbf{y}) = 0. \tag{10}$$

The solution of the equation (10) is given by the formula for the unknown waveform $x(n)$

$$\mathbf{x} = \left(r^2 \mathbf{A}^T \mathbf{A} + \mathbf{E}\right)^{-1} \mathbf{y}. \tag{11}$$

The banded square positive definite matrix $r^2\mathbf{A}^T\mathbf{A}+\mathbf{E}$ consists of non-zero elements that are arranged into 5 diagonals. Therefore, it is easy to invert it.

To test selectivity of the Vold-Kalman order filter, the 100-sample and 500-sample waveforms with sampling frequency of 1 Hz are employed. The center frequency of both the filters is chosen $0.1f_s$. The frequency response is inspected for the frequency ranging from $0.05f_s$ to $0.15f_s$. The weighting factor r ranges from 1 to 1000. The frequency responses of the Vold Kalman order filter for both the waveforms are shown in figure 1. The limit value of the weighting factor is equal to 100 for the 100-sample waveform. One can conclude that the largest number of the sample the better selectivity of the Vold-Kalman order filter.

A chirp signal and the result of the Vold-Kalman order filtration in the form of the difference between $y(n)$ and $x(n)$ is shown in figure 2.

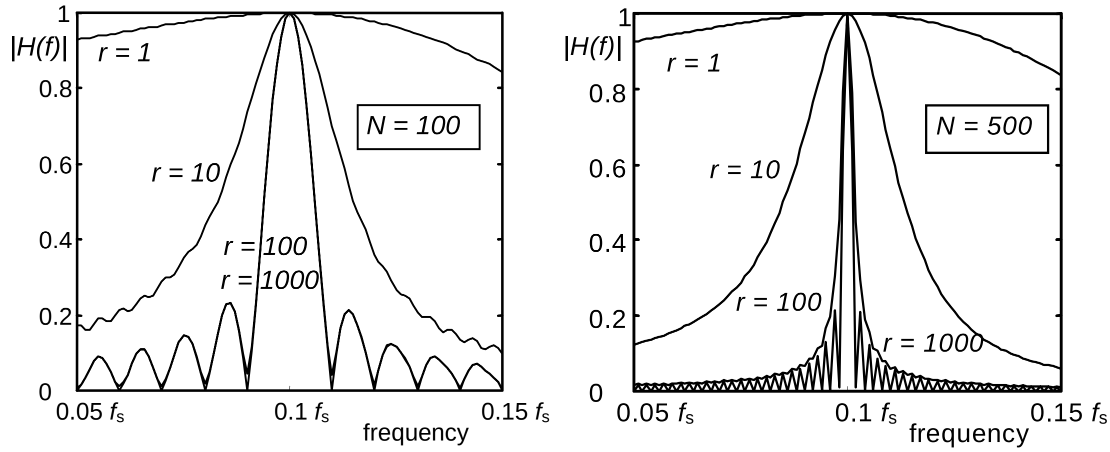

Fig. 1. Frequency response of the Vold-Kalman order tracking filter

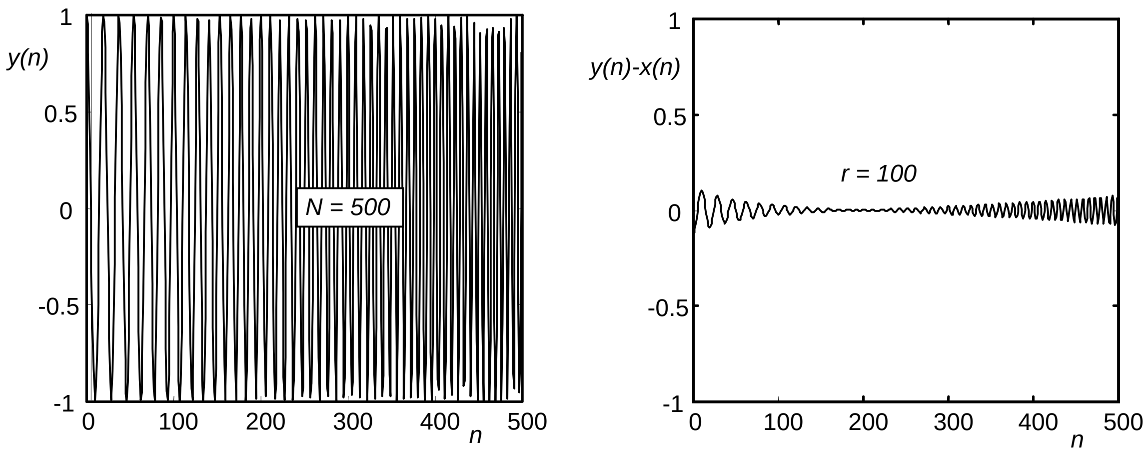

Fig. 2. 500-sample chirp signal and result of the Vold-Kalman order tracking filter

3. ALGORITHM OF THE SECOND GENERATION

The algorithm of the first generation gives the time signal $x(t)=A\cos(\omega t)$ as a component of the recorded signal $y(t)$. The real time signal can be written as the sum of the products $A_k(t)\Theta_k(t)$, where *k* runs over all of the positive and negative multiples of the fundamental frequency

$$x(t) = \sum_{k=-\infty}^{+\infty} A_k(t)\Theta_k(t). \quad (12)$$

The term $A_k(t)$ is a complex envelope and the term $\Theta_k(t)$ is a function corresponding to the rotating unit vector, called a phasor, in the complex plane. The phasor is lying on the unit circle and it is defined by the following formula

$$\Theta_k(t) = \exp\left(jk\int_0^t \omega(\tau)d\tau \right), \quad (13)$$

where the integral of frequency gives the angle traveled by the axis up to the current time. Note that the terms $A_k(t)$ for the positive index $+k$ and negative index $-k$ are complex conjugate quantities, $A_{-k}(t) = A_k^*(t)$.

The second-generation algorithm [3] results in the time series $A_k(n)$. The complex envelope $A_k(t)$ is the low frequency modulation of the carrier wave $\Theta_k(t)$. Low frequency modulation causes envelope smoothness. In other words envelope is locally approximated by a low order polynomial. This condition can be expressed by the structural equation with the non-homogeneity term $\varepsilon(n)$

$$\nabla^s A_k(n) = \varepsilon(n). \quad (14)$$

Note that the difference operator of a given order s annihilates all polynomial of one order less. The order of the difference equation (14) equals one, at the least. For instance, the value of s equaling to one gives the following structural equation $A_k(n) - A_k(n-1) = \varepsilon(n)$. The order of the difference operator designates the order of the filter that is equal to the number of poles in the filter.

Data equation is given by the following formula

$$y(n) - \sum_{k \in K} A_k(n)\Theta_k(n) = \eta(n), \quad (15)$$

where the summation is for a desired subset of orders.

4. EXAMPLE OF EMPLOYING THE VOLD-KALMAN ORDER TRACKING FILTRATION

Employing the Vold-Kalman tracking filtering is demonstrated on analysis of the acceleration signal that is measured on the spindle head of the horizontal-machining center (see figure 3). The analytical instrumentation used for the Vold-Kalman tracking filtering was of the Brűel & Kjæer origin and comprised the PULSE signal analyzer.

An optical trigger attached to the spindle gives the tacho pulses. Setting up and the zoom of time signal are shown in Figure 4. The RPM value is evaluated from the time interval between two consecutive impulses. The trigger level, hysteresis and slope are set up in the tacho setting property page.

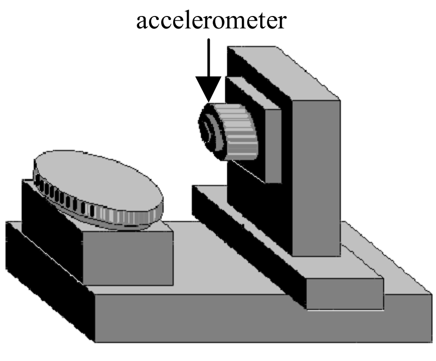

Fig. 3. Horizontal machining center

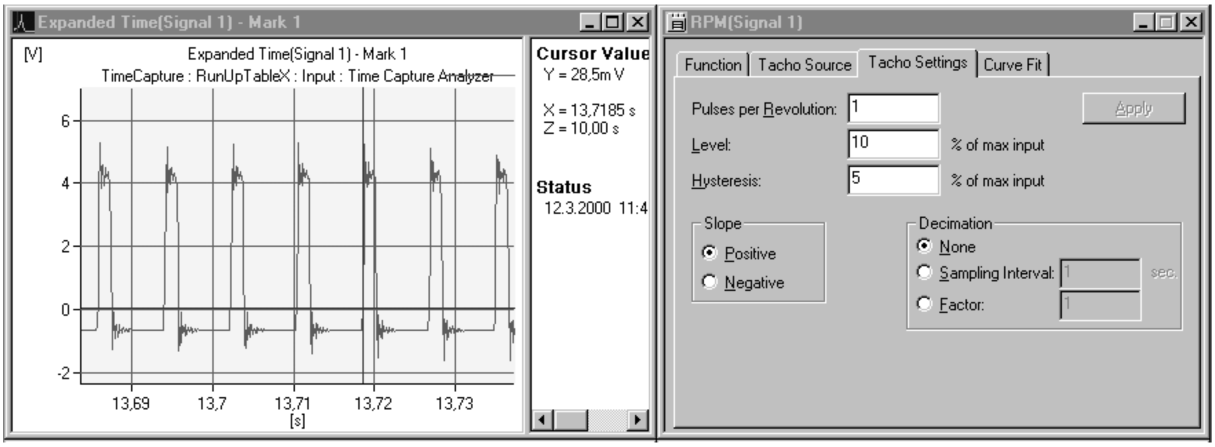

Fig. 4. Tacho pulses and tacho setting property page

Vibration is excited by the natural unbalance of the spindle. The RPM run-up ranges from 50 to 10000 RPM and it takes 18s. The sampling frequency of both the tacho and vibration signals is chosen 8192 Hz that corresponds to the frequency span of 3200 Hz. Results are shown in figure 5, 6 and 7. Vibration time signal of the run-up and Vold-Kalman filter property page are shown in figure 5.

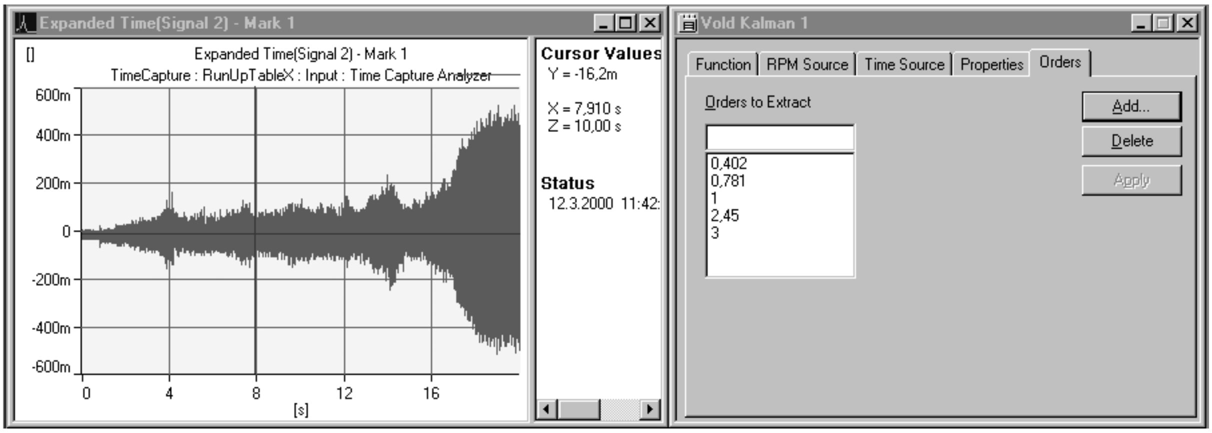

Fig. 5. Vibration time signal of the run-up and order setting property page

A short time Fourier transform of the acceleration signal is shown in figure 6. A time slice at 18.68s shows that dominating orders of the spindle rotational speed are equal to the 0.402th, 0.781th, 1st, 2.450th and 3rd order of the rotational speed of the spindle. The quantities are entered as the orders to be extracted on the order setting property page.

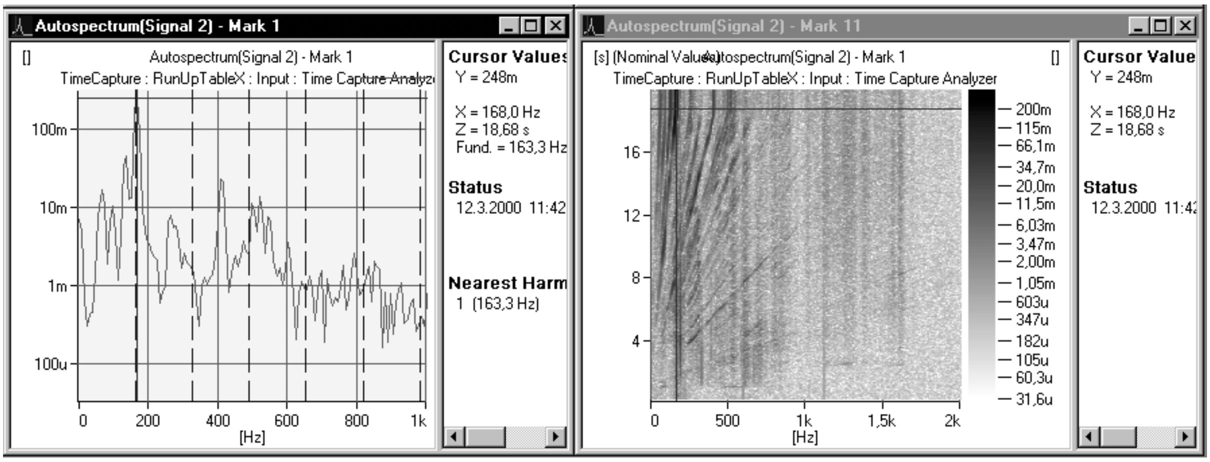

Fig. 6. A short time Fourier transform of the acceleration signal

The unbalance of a driving motor excites the 0.781th order. The unbalance or misalignment of the spindle excites the 1st and 3rd orders. Faults in rolling-bearings probably excite the 0.402th and 2.450th orders.

Overlapped waveforms (spindle acceleration signal in Z direction) of the 0.402th, 0.781th, 1st, 2.450th and 3rd order of the rotational speed of the spindle extracted using a two-pole Vold-Kalman filter with 10% bandwidth are shown in figure 7.

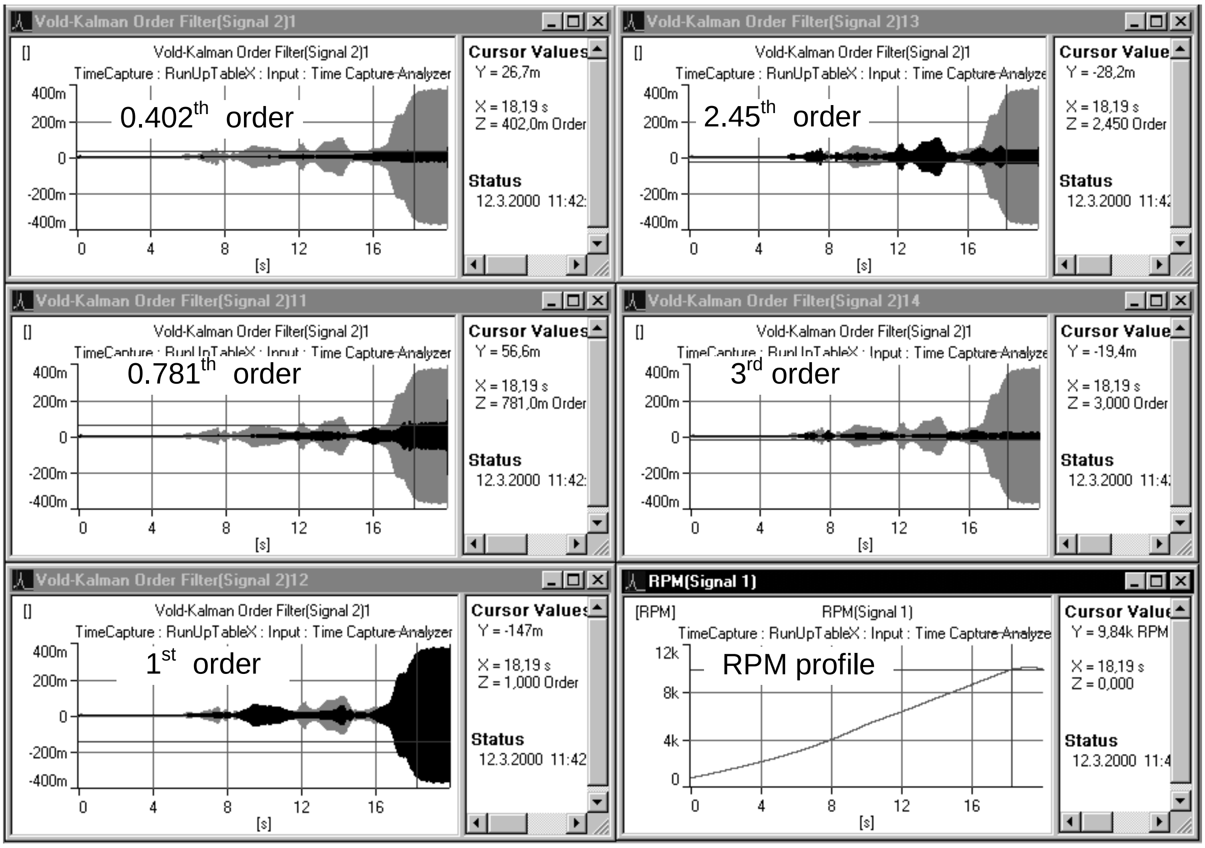

Fig. 7. Overlapped waveforms of the 0.402th, 0.781th, 1st, 2.450th and 3rd order of the rotational speed of the spindle extracted using a two-pole Vold-Kalman filter

5. Conclusion (Closing Remarks, Acknowledgement)

The paper gives an overview of Vold-Kalman order tracking filtration as an analytical tool for the diagnostics of rotating and reciprocating machines. The main advantage of this analytical technique consists in ability to track an order without slew rate limitations. The only speed limitation is due to the filter response. Stepwise changes of the RPM and tacho signal drop-outs can be handled. Decoupling of close and crossing orders is possible.

The only disadvantages are not real time processing, longer calculation time and some prior knowledge of the signal required.

6. References

[1] Vold, H. & Leuridan, J. Order Tracking at Extreme Slew Rates, Using Kalman Tracking Filters. SAE Paper Number 931288.

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[3] Gade, S. & Herlufsen, H. Konstantin-Hansen, H. & Vold, H. Characteristics of the Vold-Kalman Order Tracking Filter. Brüel & Kjær Technical Review, No.1 - 1999.

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