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MODELLING OF MEASURING SYSTEM FOR TESTING OF STEPPING MOTORS

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Summary: *At present worked out is new measuring system for evaluation of dynamic properties of stepping motors. There is prepared mathematical model of the measuring system and stepping motor under test. Authors present the way of modelling and some effects of computer simulation.*

1. INTRODUCTION

Designers of drives with stepper motors are interested in characteristics illustrating *dependency of the torque* developed at the motor shaft *on its average rotational speed* or *the frequency of the controlling pulses* (being proportional to the speed). These characteristics should be obtained for a given controlling method and a given kind of load. While determining these characteristics, the basic measuring problems are: choice of a configuration of the test station (having a stationary torque meter for measuring reaction of the motor stator, or rotary torque meter for measuring the torque "at the shaft"), selection of design features of a torque meter, and accepting appropriate measuring procedures - i.e. algorithms of controlling the test station, changing the loads and data acquisition.

Problems of experimental determining dynamic properties of stepper motors (as well as other types of brushless motors with electronic commutation) are complicated because of a character of operation of these micro-machines that introduces pulsatory input functions into an flexible electromechanical structure (created by attaching the torque transducer). Interactions within the structure of the test station not only influence quality of the measuring process (accuracy of transformation of the electromagnetic torque) but also disturb operation of the motor by generating resonance phenomena.

In the *Institute of Micromechanics and Photonics at the Warsaw University of Technology* (*IMiF PW*), one designed a test station enabling to carry out comparative experiments related to the methodology of determining these characteristics [1]. Members of the mechanical structure are shown in Fig. 1, and view of the test station is shown in Fig. 2.

One acknowledged that an effective method of improving the measuring procedures of the test station as well as evaluating various methods of determining the characteristics would be simulation studies. At the first stage, one created a mathematical model of the electromechanical system, created by the stepper motor and torque transducers.

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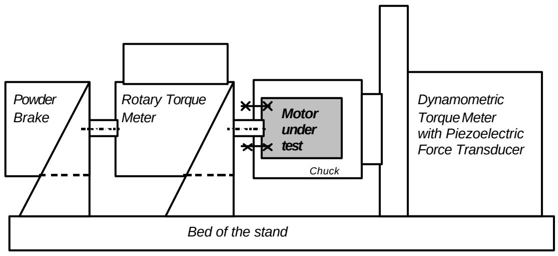

Fig. 1. Mechanical structure of the test stand

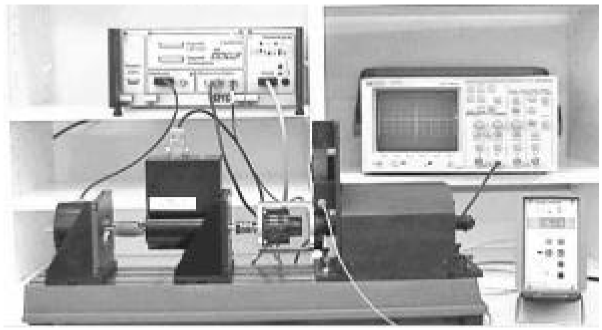

Fig. 2. View of the stand

2. GENERAL FORM OF THE MODEL OF ELECTROMECHANICAL STRUCTURE OF THE TEST STAND

While modelling a two-phase motor, one accepted the following equation of electric circuits (for a k circuit):

$$u_k = Ri_k + \frac{d\,\mathbf{Y}_k}{dt},\ k \in \{1,2\}, \tag{1}$$

Independent variables of the process of energy conversion are phase-currents i_k and angular position of the rotor g. Exciting forces are supply voltages u_k (the load are reduced moments of inertia T_f and T_L, occurring in the equations of the electromechanical system). Conversion of energy is expressed by fluxes associated with the phases - $\mathbf{Y}_k$ and electromagnetic torque T_e. General functions $\mathbf{Y}_k$ and T_e are nonlinear functions of angular position and phase-currents [4]:

$$\Psi_k = \Psi_k(g, i_1, i_2), \tag{2}$$

$$T_e = T_e(g, i_1, i_2), \tag{3}$$

where, $i_k = i_k(t)$ and $g = g(t)$.

Mechanical structure, created by attaching torque meters and a braking system to the motor, is expressed by a model with concentrated-parameters, in which torsional rigidities of the mechanical transducer within the torque meter (if need be, also torsional rigidities of the couplings) were accepted for flexible elements [2,3]. General model of such a system is expressed by the following equation:

$$J\ddot{g} + D\dot{g} + Kg = T \tag{4}$$

in which the following are taken into consideration: moments of inertia of undeformable solids (matrix J), damping (e.g. within the couplings - matrix D), torsional rigidities (of couplings and mechanical transducers of the torque meters) - matrix K, matrix of generalised coordinates (angular positions of the solids) and matrix of torque's T (whose elements are electromagnetic torque of the motor and torque related to all the loads - both active and reactive).

3. MODEL OF THE TEST STAND

In order to model a configuration with two torque meters, which allows one to carry out simultaneous comparative experiments related to measurement of the torque "at the shaft" or "at the stator" of the motor, one accepted a three-mass system for further considerations - Fig. 3. Such system simulates also a test station with one stationary torque meter and a brake attached by means of a flexible coupling.

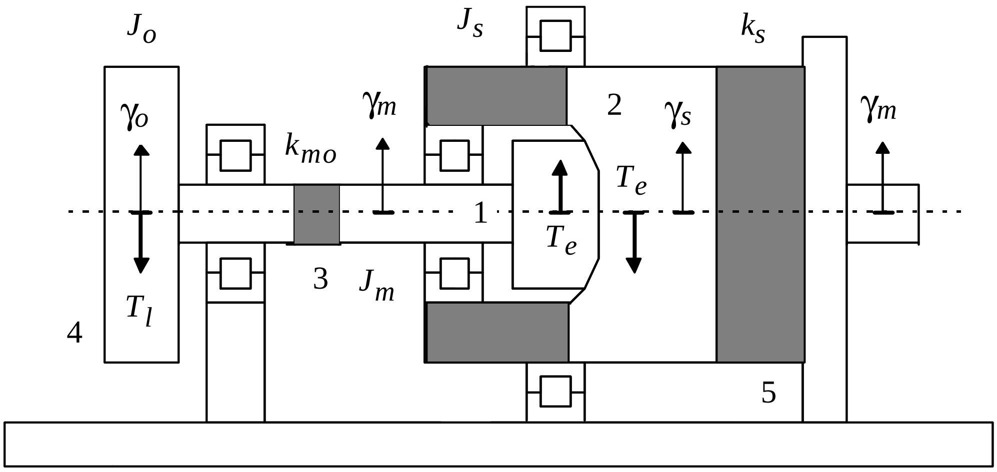

Fig. 3. Model of the mechanical unit of the test station containing a motor and two torque meters – presented as a three-mass system

1- rotor, 2 – stator, 3 – mechanical (flexible) transducer of the rotary torque meter, 4 – load, 5 – mechanical (flexible) transducer of the stationary torque meter (other designation in the text below).

Mathematical model of the system consists of three differential equations:

- of the rotor:

$$J_m \frac{d^2\gamma_m}{dt^2} + D_{ms}\left[\frac{d\gamma_m}{dt} - \frac{d\gamma_s}{dt}\right] + k_{mo}(\gamma_m - \gamma_o) + T_{ms}\, sgn\left[\frac{d\gamma_m}{dt} - \frac{d\gamma_s}{dt}\right] = T_e \tag{5}$$

- of the stator:

$$J_s \frac{d^2\gamma_s}{dt^2} + D_s \frac{d\gamma_s}{dt} + D_{ms}\left[\frac{d\gamma_s}{dt} - \frac{d\gamma_m}{dt}\right] + k_s\gamma_s + T_s\, sgn\left[\frac{d\gamma_s}{dt}\right] + T_{ms}\, sgn\left[\frac{d\gamma_s}{dt} - \frac{d\gamma_m}{dt}\right] = -T_e \tag{6}$$

- of the load:

$$J_o \frac{d^2\gamma_o}{dt^2} + D_o \frac{d\gamma_o}{dt} + k_{mo}(\gamma_o - \gamma_m) + T_o\, sign\left[\frac{d\gamma_o}{dt}\right] + T_l = 0 \tag{7}$$

under initial conditions of angular positions and velocities specified as follows:

$$\gamma_m(0) = \gamma_{m0},\ \gamma_s(0) = \gamma_{s0},\ \gamma_o(0) = \gamma_{o0};\ \left.\frac{d\gamma_m(t)}{dt}\right|_{t=0} = \omega_{m0};\ \left.\frac{d\gamma_s(t)}{dt}\right|_{t=0} = \omega_{s0};\ \left.\frac{d\gamma_o(t)}{dt}\right|_{t=0} = \omega_{o0}.$$

One used the following designations in the system of equations (5-7):

$\gamma_{m(s)(o)}$ - rotation angle of the rotor (stator) (of the load) in relation to a permanent axis;

$J_{m(s)(o)}$ - moment of inertia of rotating masses of the rotor (stator) (of the load);

$D_{s(o)}$ - coefficient of viscous friction between the stator (the load) and the surroundings;

$D_{m\ s}$ - coefficient of viscous friction between the stator and the rotor;

k_s - coefficient of elasticity of the stator fixing;

$k_{m\ o}$ - coefficient of elasticity of the coupling between the rotor and the load;

T_o - torque of dry friction related to the load, or torque of a reactive-type of load; i.e. having the sense contrary in relation to the sense of angular velocity of the load;

T_S - torque of dry friction related to the stator;

T_{ms}- torque of dry friction between the stator and the rotor;

T_l - torque of an active-type of load; i.e. having the sense independent on the angular velocity;

T_e - electromagnetic torque (in this case – exciting torque).

The presented system of equations is nonlinear because of two reasons: appearance of the signum function (*sgn*), and nonlinear character of the dependency of the driving torque T_e on the state variables; i.e. on the angle of position of the rotor in relation to the stator ($g_m - g_s$) and on the phase-currents. This means that there is no general, analytic solution of such system of equations. Nevertheless, it is possible to obtain an approximate solution while applying numerical methods, provided there are known values of the parameters, initial conditions, and the form of the input function.

The presented equations allow us to solve an opposite problem of dynamics, i.e. to determine parameters of motion under a given exciting torque. In the case of stepper motors, the torque is a function of angular position and angular velocity as well as of time. In order to determine unequivocally motion of a system whose parameters are known, one should also specify the initial conditions of position and angular velocity.

Mechanical system has specified dynamic properties, which according to the circumstances can significantly influence parameters of the exciting torque. In the case of the real stepper motors, the exciting torque T_e introduces a nonlinearity into the equation of motion since even while the most far-reaching simplifying assumptions accepted - what means e.g. taking into account fundamental harmonic only - it is a sinusoidal function of the angular position:

- for a permasyn motor with p pairs of poles:

$$T_e = -T_{max} \sin(p\boldsymbol{d}) \tag{8}$$

- for a reluctance motor with Z_r number of rotor teeth:

$$T_e = -T_{max} \sin(Z_r \boldsymbol{d}), \tag{9}$$

where: $\boldsymbol{d}$ denotes a discrepancy angle of position of the rotor axis in relation to the central electric axis of the stator (i.e. in relation to the point of stable equilibrium for unloaded motor).

Studies of response of a stepper motor to a step input function of the position can be carried out by applying an initial position of the rotor g_{m0} (different from the point of stable equilibrium), or by applying the point of stable equilibrium g_u (under zero initial conditions of the rotor position). In the case of a three-mass system with a stepper motor, the following formula is valid:

$$d = (g_m - g_s + g_{m0} + g_{s0} - g_u), \tag{10}$$

whereas the fundamental harmonic of the electromagnetic torque, e.g. of a permasyn motor, will be:

$$T_e = -T_{max}\, sin\,(p d). \tag{11}$$

While studying a single nonstationary state of the system around a position of stable equilibrium, there has been applied a step-wise change of the initial position of the rotor g_{m0}, and thus the equation (10) is changed into the following form:

$$d = (g_m - g_s + g_{m0}). \tag{12}$$

However, while studying resonance phenomena of the system, there has been applied a step-wise change of the position of stable equilibrium $g_u(t)$ with a certain frequency. Then, the discrepancy angle d is defined as:

$$d = (g_m - g_s - g_u(t)). \tag{13}$$

Change of the position of stable equilibrium is accepted most often as:

$$g_u(t) = g_{u0} E(1 + w \cdot t), \tag{14}$$

where: g_{u0} - value of basic step; w - pulsation of commutation; E - *entier* function.

4. SIMULATION TESTS – WORK OF THE MOTOR

Simulation model was elaborated with application of *MATLAB/SIMULINK* package. In Fig. 4, there is presented response, of a system whose parameters are as follows:

$J_m = J_s = J_o = 2 \cdot 10^{-6}\ \mathrm{kg \cdot m^2}$; $D_{ms} = D_s = D_o = 0$; $k_{mo} = k_s = 1\mathrm{N \cdot m/rad}$; $T_{\max} = 1\mathrm{N \cdot m}$,

to a step input function of the initial position of the rotor $g_{m0} = -p/2$. On the basis of the course of angular path, one can evaluate vibration frequency of particular masses of the system. E.g. in the course of rotor vibration (Fig. 4a), two distinct frequencies can be distinguished: 187.5Hz and 43.3Hz. The course presented in Fig. 4d illustrates changes of the discrepancy angle of the rotor position in relation to the stator (the electromagnetic torque varies in function of that angle).

In Fig. 5, there are presented courses of angular path of the rotor of stepper motor operating in a three-mass system whose parameters are the same as in the case of the experiment introduced in Fig. 4. However, the input function of changing the position of stable equilibrium $g_u(t)$ is of a frequency as indicated in the figure, called a commutation frequency. Value of the basic step was $g_{u0} = p/2$. Analysis of the course of angular path of the rotor is a basis for studying resonance phenomena in a system with stepper motor. Comparison of the courses in Fig. 5a and 5b allows us to evaluate an influence of the rigidity of a stationary torque meter on values of the resonance frequencies of the rotor.

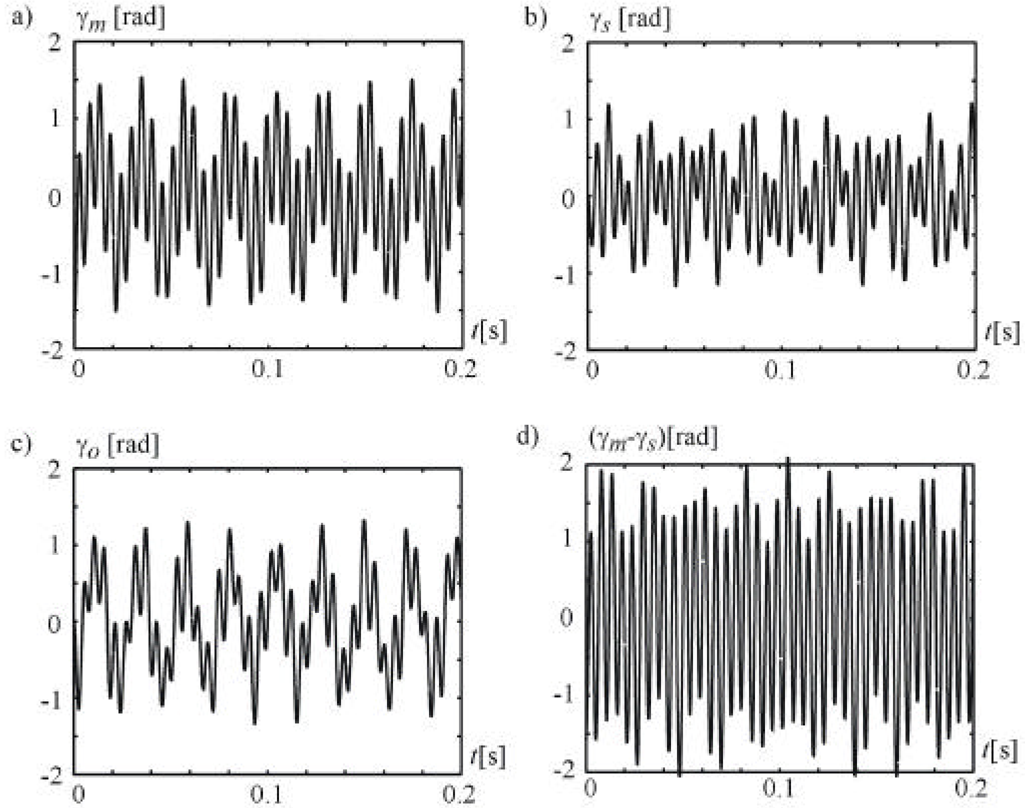

Fig. 4. Angular position of the rotor (a), stator (b), load (c) and the discrepancy angle of the rotor position in relation to the stator (d) during response of the three-mass system of the test station to a step input function of the rotor position $g_m(0) = -\pi/2$ (description in the text).

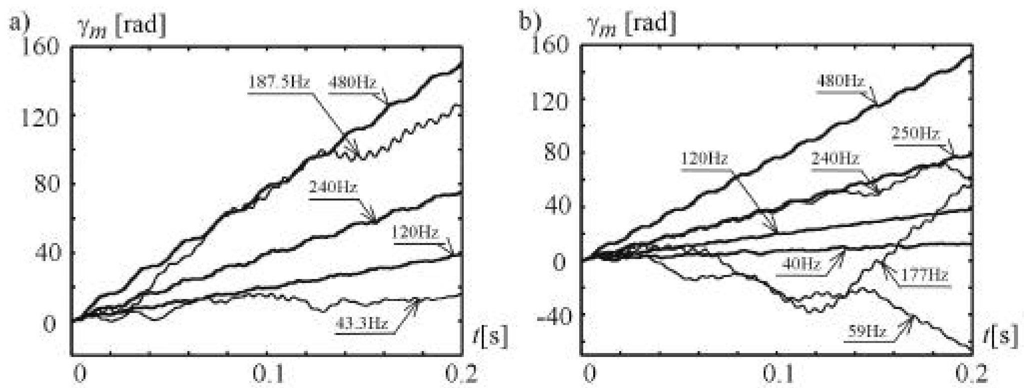

Fig. 5. Course of angular path of the rotor of stepper motor operating in a test station by full-step commutation of a given frequency, for two values of the coefficient of elasticity of the stator fixing: a) k_S=1N·m/rad; b) k_S=100N·m/rad (the other parameters same as in Fig. 4).

5. Simulation tests – measuring process

The presented model makes it possible to foresee resonance phenomena in the test station and to limit them, e.g. by selecting appropriate design features of the torque meters. It allows us also to compare the electromagnetic torque of the motor with torque signals in the mechanical transducers of the torque meters. While determining the electromagnetic torque of the motor with application of the method based on measuring the stator reactions (when the force transducer used in a piezo-electric stationary dynamometric torque meter is characterised by linearity and high rigidity, and diameter of the stator is small) the torque of reaction will be proportional to the product of coefficient of elasticity of the sensor (being equal to the coefficient of elasticity of the stator fixing) and angular path of the stator:

$$T_{rs} = -k_s \cdot g_s . \qquad (15)$$

On the other hand, in the case of a rotary torque meter, the torque transmitted by a flexible element, will equal the product of angle difference ($g_o - g_m$) and rigidity k_{mo}:

$$T_{mo} = -k_{mo} \cdot (g_o - g_m) . \qquad (16)$$

In Fig. 6a, there are presented two simulation courses – electromagnetic torque T_e in comparison with signal of stationary torque meter (T_{rs}). These courses were obtained during testing of response on $g_{u0} = p/2$ angular step of the rotor position. The parameters are as follows:

$J_m = J_s = J_o = 2 \cdot 10^{-6}\ kg \cdot m^2;\ D_{ms} = D_s = D_o = 0;\ k_{mo} = 1\ N \cdot m/rad\ ;$
$k_s = 100\ N \cdot m/rad\ ,\ T_{max} = 1N \cdot m,$

However, in the Fig 6b, there are presented courses of basic harmonic of torque signal generated by of the stationary torque meter against a background of the electromagnetic torque T_e. This comparison indicated the accuracy of measurement.

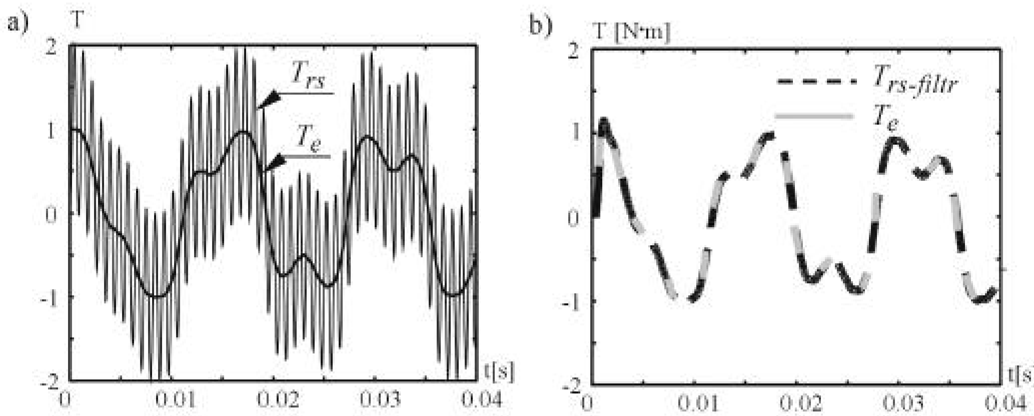

Fig. 6. An example of the measurement of torque and signal processing

a) signal of stationary torque meter without filtering, b) result of the filtering operation

T_e – electromagnetic torque, T_e– measuring signal, $T_{rs\text{-}filtr}$ – torque signal after filtering

6. CLOSING REMARKS, ACKNOWLEDGEMENT

Presented results of the simulation tests show that the parameters of the mechanical structure of the test stand have the determined influence on the measurement of the motor torque. Model of the electromechanical structure is a basis for a complex modelling of the test station - where the following are also taken into consideration: dynamics of the system for applying load (an electromagnetic brake with a power supply unit) and processing of the signals in torque meters. This complex model makes it possible to optimize the measuring process during determination the frequency characteristics of the stepping motors.

ACKNOWLEDGEMENTS

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