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GLOBAL VIEW ON DYNAMICS OF IMPACT OSCILLATOR

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Summary: *Impact oscillator is the simplest mechanical system with one degree of freedom, the periodically excited mass of which can impact on the stop. The aim of this paper is to explain the dynamics of the system, when the stiffness of the stop changes from zero to infinity. It corresponds to the transition from the linear system into strongly nonlinear system with rigid impacts. The Kelvin-Voigt and piecewise linear model of soft impact was chosen for the study. New phenomena in dynamics of motion with soft impacts in comparison with known dynamics of motion with rigid impacts are introduced in this paper.*

1. INTRODUCTION

The one-degree-of-freedom impact oscillator is one of the simplest strongly non-linear mechanical systems. It consists of an elastically suspended and periodically excited mass *m,* which can impact against a rigid stop (Fig. 1(c)). Its dynamics has been thoroughly investigated theoretically, experimentally and using simulation methods (see references in (Peterka and Vacík 1992) and (Peterka 1981)).

The analytical and simulation solution of the impact oscillator motion uses mathematical model of the impactless motion (Fig. 1(a)) and a certain model of impact. Strong nonlinearity is caused by an additional stiffness and damping during the contact of mass with stop. The stiffness coefficient should lie in the interval $(0, \infty)$. The left boundary value corresponds to impactless motion (Fig. 1(a)) and right boundary represents motion with absolutely rigid impacts (Fig. 1(c)). The behavior of the system motion is well known in both boundary cases. Motion of oscillator with rigid impacts is most complex, but theoretical analysis of periodic motions and their stability, using Newton model of impact with restitution coefficient *R*, is simplest. The Kelvin-Voigt (Fig. 2(a)) and piecewise linear (Fig. 2(b)) model of 'soft' impacts (Fig. 1(b)) was chosen for the explanation of development of nonlinear phenomena inside the mentioned stiffness interval. The notion 'soft' means that model of impact does

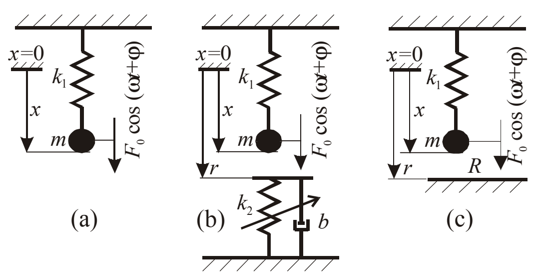

Figure 1. Scheme of transition from linear motion **(a)** through motion with soft impacts **(b)** into motion with rigid impacts **(c)** of one degree of freedom oscillator

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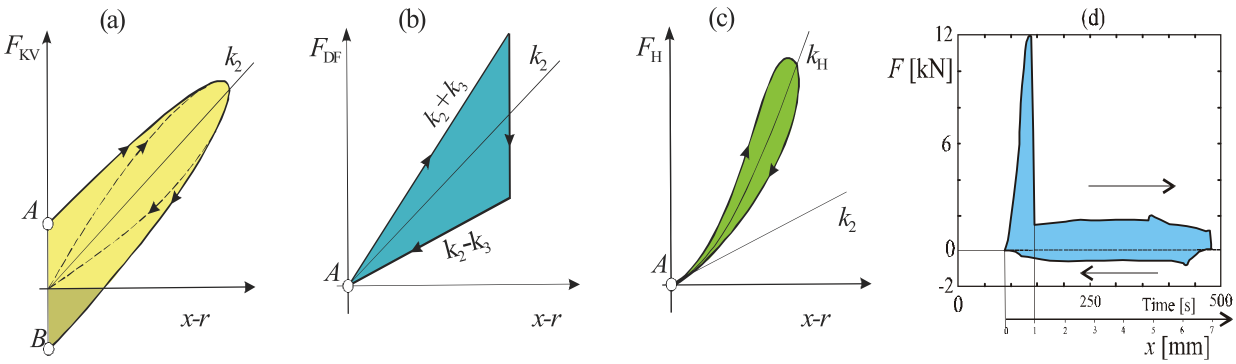

Figure 2. Examples of force interactions during soft impacts

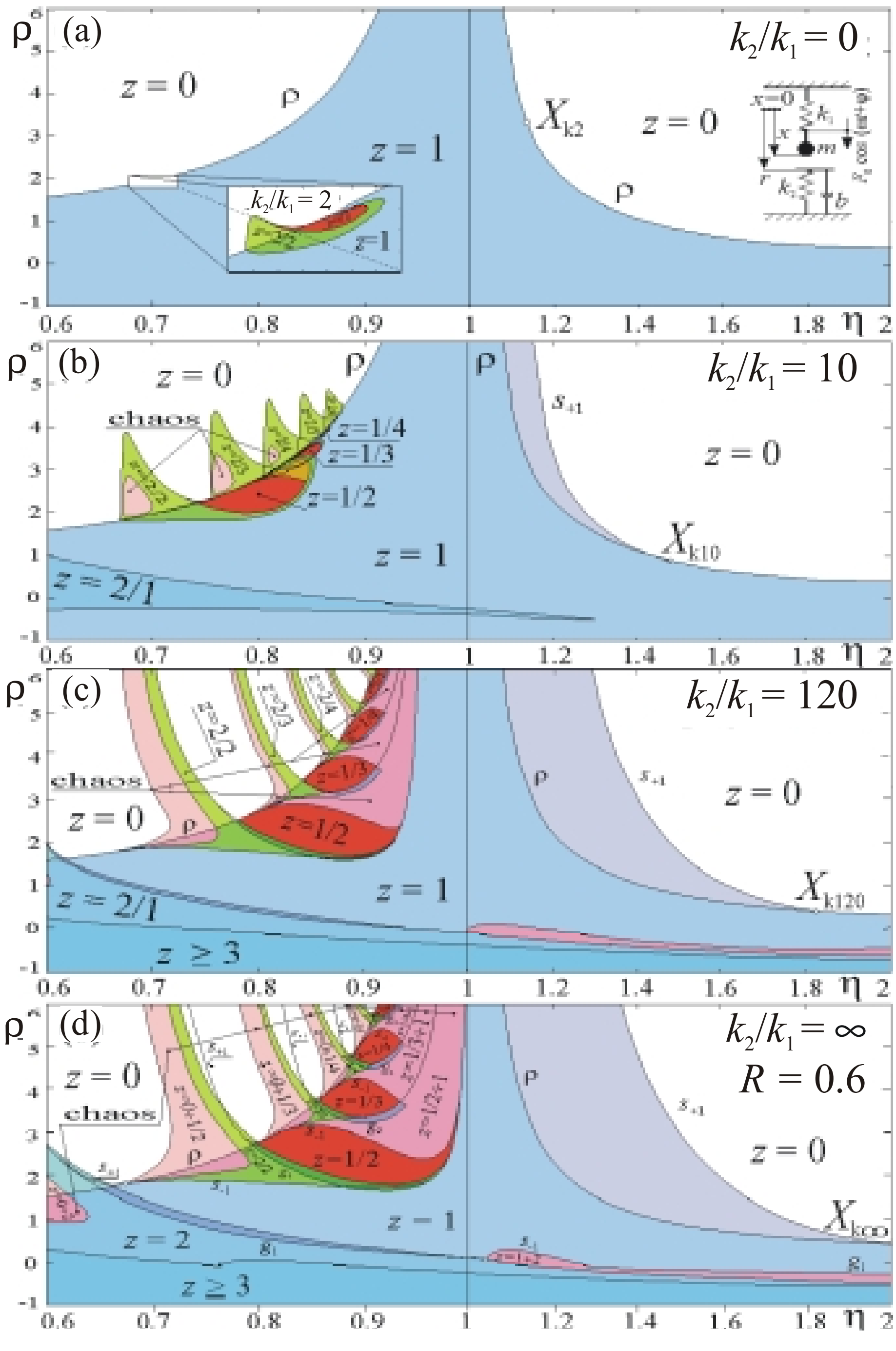

Figure 3. Regions of motions with the Kelvin-Voigt model of soft impacts **(a)**-**(c)** and rigid impacts according to Newton's model of impact **(d)**

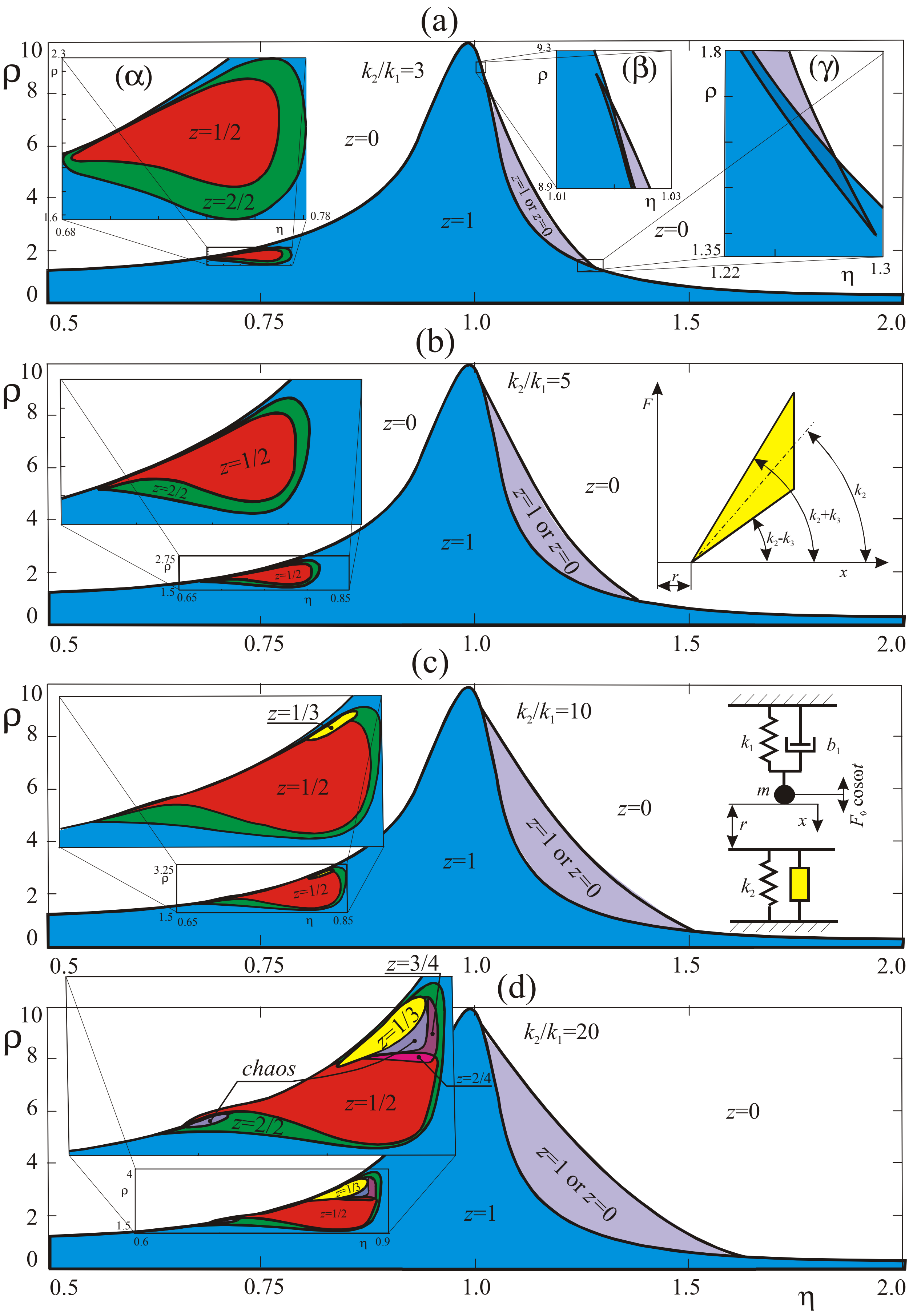

Figure 4. Regions of motions of oscillator with peicewise linear model of soft impact

not neglect the impact duration and simulates a force interaction of impacting bodies during impact.

Dynamics of oscillator with Hertz's quasistatic model (Fig. 2(c)) was also explored using numerical simulation (Půst and Peterka 2001). Figure 2(d) shows diagram of elastic, plastic and dry friction forces measured during the piercing of the hole in a chain link. This diagram is more complicated and will be used for the simulation of motion and an optimisation of system parameters of the real machine.

2. REGIONS OF DIFFERENT IMPACT MOTIONS

The motion with rigid impacts has very diverse response, depending on the system parameters, especially on dimensionless static clearance $\rho = rk/F_0$ between mass m and the stop and on dimensionless excitation force frequency $\eta = \omega/\Omega$, where $\Omega = (k/m)^{1/2}$. Special structure of periodic and chaotic impact motion regions was found and four ways from periodic into chaotic impact motions were explained (Peterka and Vacík 1992), (Peterka 1999).

Periodic motions are characterized by quantity $z=p/n$, where p is number of impacts and n is number of excitation periods $T=2\pi/\omega$ in one period of motion. This quantity characterizes also chaotic impact motions and corresponds to mean value of impacts in excitation period T.

There exists fundamental series $z=0,1,2,3,\ldots$. Motions repeat with period T ($n=1$) and differ by number $p=0,1,2,..$ of impacts. Impactless motion ($z=0$) belongs also into this series. Another series correspond to periodic subharmonic impact motions. Their period is integer multiple of T ($n \geq 2$). One of series is typical by $p=1$. These motions and fundamental impact motion $z=1$ ($p=1$, $n=1$) were analyzed analytically. There exist also other series of more complicated periodic impact motions, e.g. $p=1,3,5,7,\ldots$ impacts with period $2T$ ($n=2$) or $p=2$ impacts with period $n=2,3,4,5,\ldots$.

Every periodic impact motion has a region of existence and stability in space of system parameters. Regions of different motions are usually illustrated in plane ($\eta \times \rho$), because they considerably influence the system response. Examples of regions are shown in Fig.3 for the Kelvin-Voigt model and increasing relation k_2/k_1, which express the hardening of the stop elasticity. Similar regions are introduced in Fig.4 for piecewise linear model of soft impact and for viscous damping of impactless motion (constant $b_1/(2\sqrt{km})=0.1$).

Results of the hardening of impacts can be expressed as it follows:

1) Nonlinear phenomena birth on grazing boundary ρ where impacts appear in impactless motion. No hysteresis regions exist for small values of stiffness of spring k_2.

2) Regions of existence and stability of subharmonic and chaotic impact motions and their hysteresis regions into the region of impactless motion ($z=0$) become wider and develop from grazing boundary ρ. Similarly the hyseteresis region of the fundamental motion ($z=1$) increases, but its region under boundary ρ becomes narrower due to the expansion of regions of subharmonic impact motions.

3) Transition cross grazing boundary ρ from impactless into one-impact motion is continuous for motion with soft impacts and very narrow region of one-impact motion with weak impacts exists along the boundary ρ. Nevertheless hysteresis regions of impact motions into impactless motion region exist. It is caused by the existence of saddle-node stability boundaries where regimes with weak impacts jumps into the same regime with strong impacts and not till these motions exhibit hysteresis phenomena. It will be explained in next chapter.

4) Comparison of Figs.3(a),(b) and Fig.4 shows that the structure of fundamental and subharmonic impact motions does not depend considerably on the model of soft impact.

5) Regions of existence and stability of periodic subharmonic impact motions, beginning from a certain order, lie only over the grazing bifurcation boundary ρ. Their appearance is conditioned by a special selection of initial conditions of the system motion and basins of attraction of all possible motions for certain combination of parameters should be ascertained.

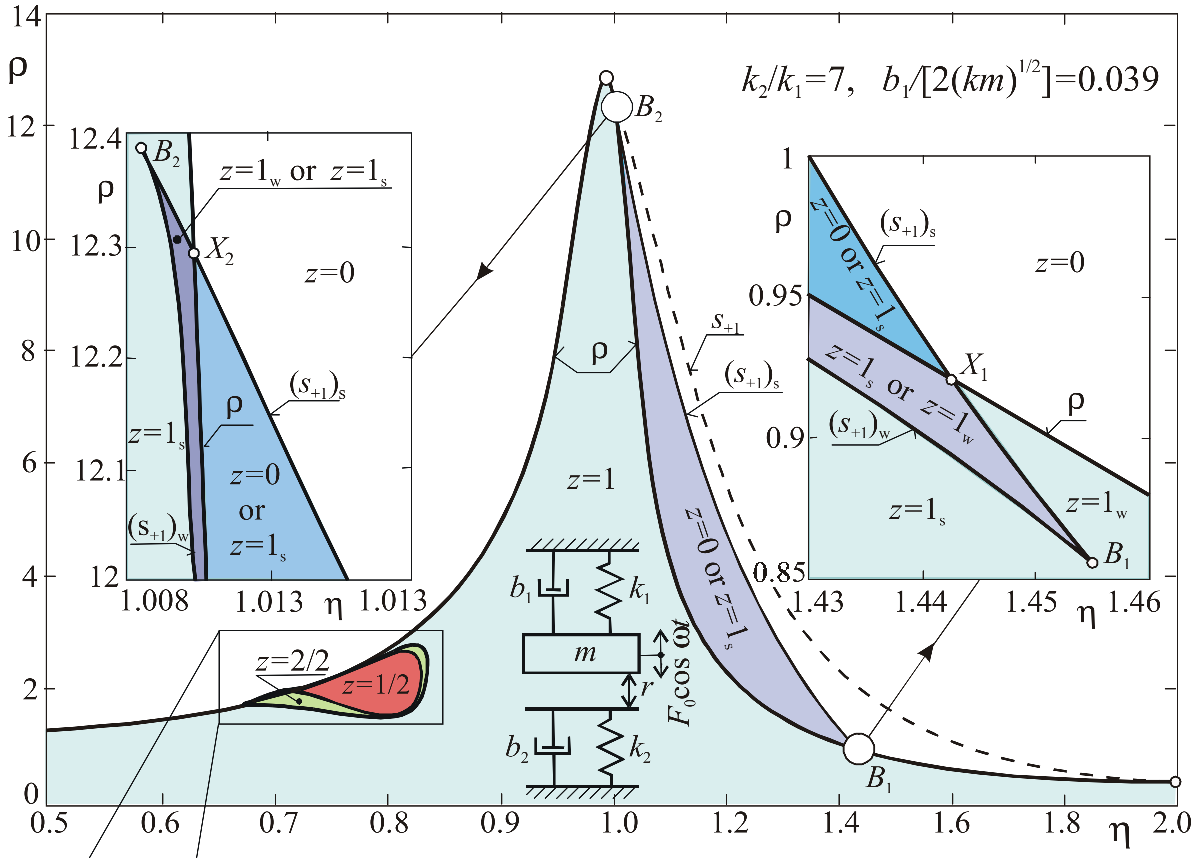

Figure 5. Regions of impact motions for oscillator with soft impacts

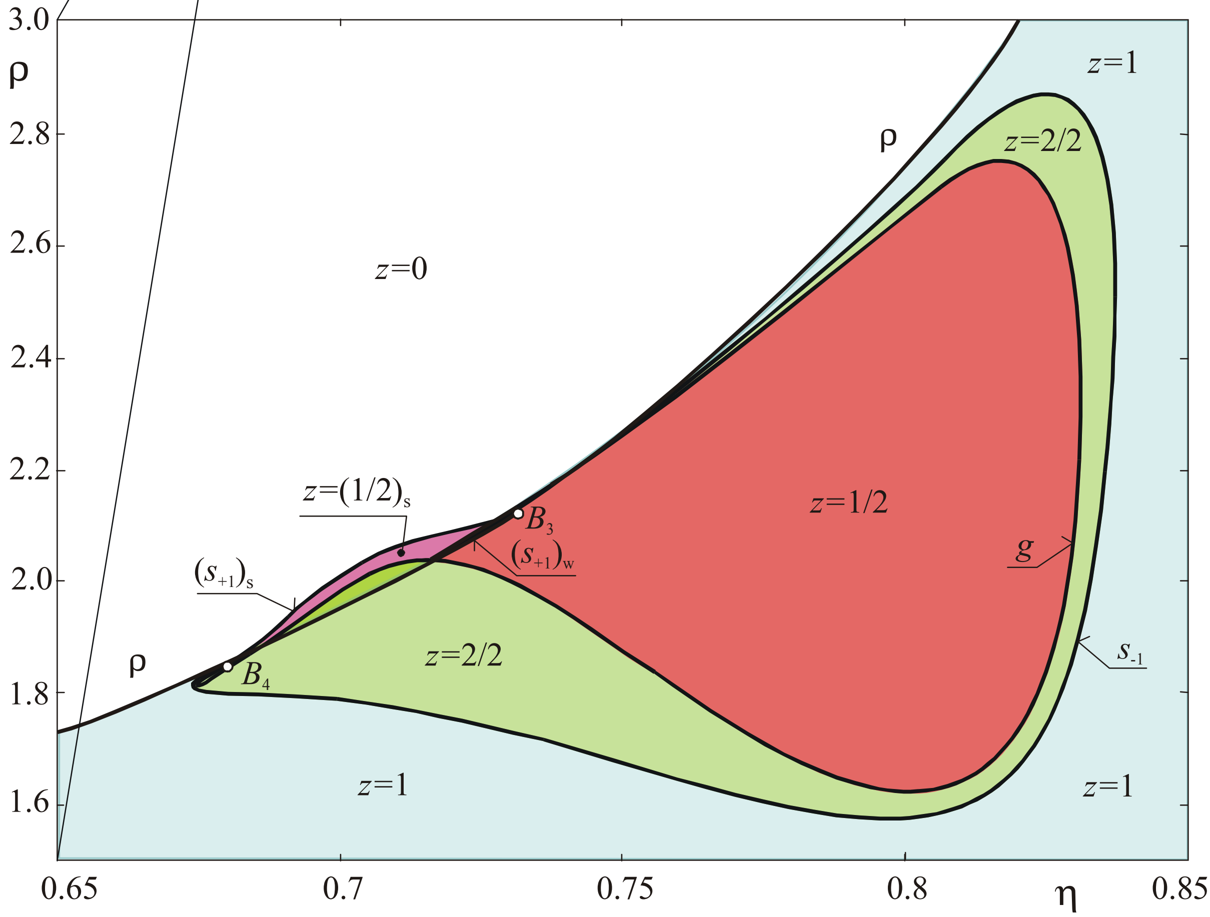

Figure 6. Enlarged regions of subharmonic impact motions

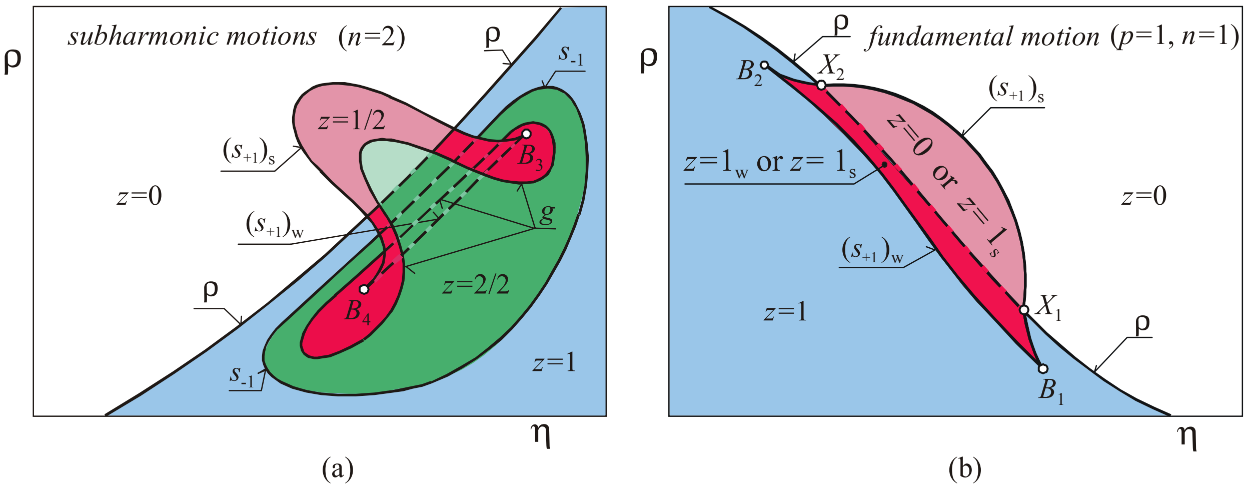

Figure 7. Schematic explanation of hysteresis phenomena in motion with soft impacts

3. HYSTERESIS PHENOMENA OF IMPACT MOTION

Typical feature of the impact motion with rigid impacts ($k_2/k_1=\infty$) is the non-continuous transition from the impactless motion into impact motion along grazing bifurcation boundary ρ (see boundary ρ in Fig. 3 (d)). The motion with repeated touch of moving body with the stop (grazing) is never stable and a jump into impact motion with strong impacts appears. $z=1$ impact motion with strong impacts stabilizes suddenly on segment ρ in frequency interval ($1<\eta<2$). This impact motion has hysteresis up to saddle-node stability boundary s_{+1}, where it jumps back into impactless motion. Both neighboring $z=0$ and $z=1$ motions can exist in hysteresis region between boundaries ρ and s_{+1}.

Transition from impactless motion into one-impact motion cross grazing boundary ρ is continuous for motion with soft impacts. It will be explained using the system with the Kelvin-Voigt model of soft impact and $k_2/k_1=7$ (see Fig.5). $z=1_w$ impact motion with weak impacts arises on grazing boundary ρ and is stable (before-impact velocity on boundary ρ is zero). This motion losses stability on saddle-node bifurcation boundary $(s_{+1})_w$ between points B_1 and B_2 under boundary ρ (see Fig.5) and changes by jump into $z=1_s$ impact motion with stronger impacts. This motion exhibits hysteresis up to saddle-node stability boundary $(s_{+1})_s$ where it transfers again by jump either into motion $z=1_w$ (under boundary ρ on segments between points B_1, X_1 and B_2, X_2) or into impactles motion (over ρ on segment between points X_1, X_2). Stability boundary $(s_{+1})_w$ is near grazing boundary ρ and approaches it with increasing relation k_2/k_1 (hardening of the stop). This legality is schematically expressed in Fig.7(b).

Similar but more complex structure exists for explanation of hysteresis phenomena of subharmonic impact motions, which appear under resonance of linear system in frequency interval $2/3<\eta<1$. Fundamental motion with rigid impacts losses its stability on period-doubling bifurcation boundary s_{-1} (see Fig.3(d)). Similar situation exists also for motion with soft impacts (see small regions of subharmonic motions $z=2/2$ and $z=1/2$, which are enlarged in Fig.6). Nevertheless, stability boundary s_{-1} is too nearby to grazing boundary ρ, especially near the incidence of hysteresis phenomena and it is impossible to express graphically the results of very accurate numerical simulations despite of considerable enlargement. Therefore the schematic Fig.7(a) will be used for the explanation of hysteresis phenomena.

Impactless motion transits again into fundamental $z=1$ motion at grazing boundary ρ and later losses its stability at boundary s_{-1}, where subharmonic motion $z=2/2$ continuously arises. Periodic impact splits on two very near impacts - weaker and stronger. Every impact repeats after two excitation periods and period of motion doubles. There exist again grazing bifurcation boundary g inside region of $z=2/2$ motion (similarly as boundary ρ between $z=1$ and impactless motion regions), where the splitting of motion is so large that weaker impact vanishes and subharmonic impact motion

z=1/2 appears. This boundary is also reversible, so transition between motions z=2/2 and z=1/2 is continuous and without hysteresis. As lately as inside region of z=1/2 the saddle-node stability boundary $(s_{+1})_w$ exists (see dashed line between points B3 and B4 in Fig.7(a)), where regime $z=(1/2)_w$ with weak impact jumps into the same regime with stronger impact ($z=(1/2)_s$). Not until this motion exhibits hysteresis up to its saddle-node stability boundary $(s_{+1})_s$ (see full line between points B3 and B4 in Fig.7(a)). As this boundary crosses different regions of periodic motions (Fig.7(a)), periodic $z=(1/2)_s$ motion jumps into $z=(1/2)_w$ or z=2/2 or impactles (z=0) motion.

It is necessary to specify next three features of explained scheme:

1) hysteresis regions of impact motions express many of possible subharmonic and ultra-subharmonic resonances of the system motion (Peterka 1981), where considerable part of natural vibration exists in the motion besides a part of excited vibration,

2) region of z=2/2 motion exhibits also the hysteresis into regions of $z=(1/2)_w$, z=2/2 and (z=0) motions, but it does not transit into these motions, because it continuously changes into $z=(1/2)_s$ motion. It follows from described situation that z=2/2 motion should be distinguished on two regimes $z=(2/2)_w$ and $z=(2/2)_s$ too,

3) $z=(1/2)_w$ impact motion transits into $z=(1/2)_s$ motion only along short segments of saddle-node stability boundary $(s_{+1})_w$ near points B3 and B4 (in region of z=1/2 motion in Fig.7(a)). The transition of system motion along remaining part of stability boundary $(s_{+1})_w$ (in region of z=2/2 motion) is characterized by the tendency of stabilization of $z=(1/2)_s$ motion too, but during the transition the second impact in motion period appears due to a larger intensity of motion and $z=(2/2)_s$ stabilizes.

4. CONSLUSION

New phenomena of motion of impact oscillator with soft impacts were obtained. The development of nonlinear characteristics of system motion when the hardening of the stop changes from zero to infinity was explained. It corresponds to the transition from the linear oscillator motion into the motion of oscillator with rigid impacts.

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