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DYNAMICS OF OSCILLATOR WITH PIECEWISE LINEAR MODEL OF SOFT IMPACTS

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Summary: *The aim of this contribution is to present a more detail explanation of different types of motion of the oscillator with soft impacts using regions of existence and stability, phase trajectories and time series of impact motions. The explanation is extended into lower and negative values of static clearance. Negative clearance corresponds to a static prestress of vibrating mass to the stop.*

1. INTRODUCTION

This contribution enlarges paper [1] dealing with the analysis of the impact oscillator (Fig. 1). Mass *m*, which is excited by force $F_0 \cos \omega t$ and damped by viscous damping b_1, can impact against a soft stop situated in distance *r* from the mass equilibrium position. The stop is characterised by spring k_2 and damping expressed by term $k_3(x-r)\sin \dot{x}$ (Fig. 2).

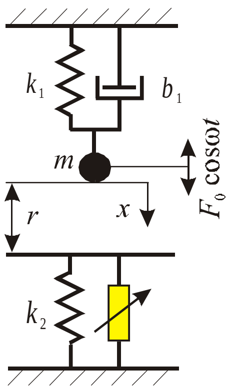

Figure 1. Scheme of impact oscillator

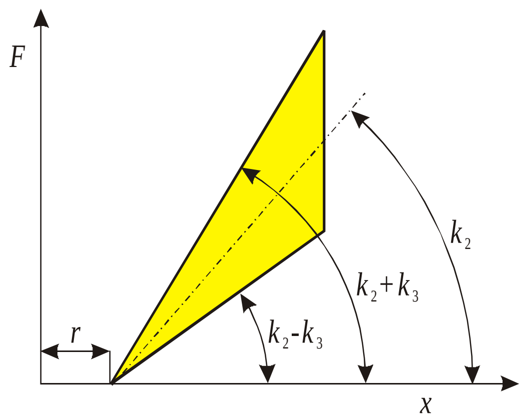

Figure 2. Piecewise model of soft impact

Denoting dimensionless deflection $X=x/x_{st}$ and time $\tau = \Omega t$, where $x_{st}=F_0/k_1$ and $\Omega = \sqrt{k_1/m}$, the impact oscillator motion is described by equation

$$X'' + 2\beta X' + X + F = \cos \eta\tau ,$$

where $X' = \mathrm{d}X/\mathrm{d}\tau$, $2\beta = b_1/(k_1 m)^{1/2}$, $\eta = \omega/\Omega$, F=0 for $X<\rho$, $\rho = r/x_{st}$,

$$F = (X-\rho)\frac{k_2}{k_1}\left(1+\frac{k_3}{k_2}\operatorname{sign} X'\right) \quad \text{for } X>\rho .$$

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2. RESULTS OF NUMERICAL SIMULATION

Following parameters 2β =0.1, k_2/k_1 =20 and k_3/k_2 =0.2 are common for all presented results. Regions of different motions are marked by values of quantity $z=p/n$ in plane (ρ, η) (Fig. 3). p and n denote number of impacts and number of excitation periods T, respectively, in the motion period.

The impactless motion (z=0, see Fig. 4, F0) exists for higher clearances ρ and is bounded by grazing boundary

$$g_0 = \left((1-\eta^2)^2 + (2\beta\eta)^2\right)^{-1/2},$$

where mass m begins touch the stop. This boundary is the amplitude-frequency characteristics of the impactless motion.

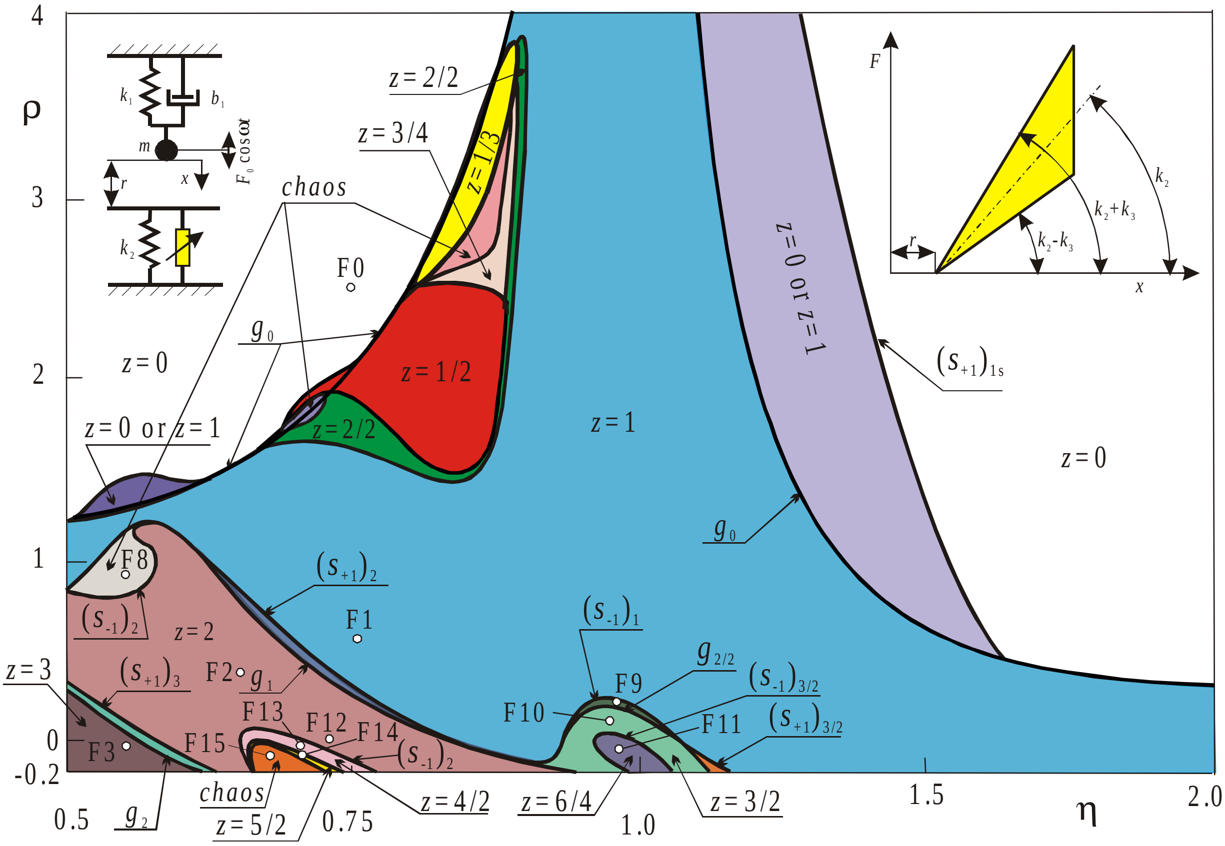

Figure 3. Regions of impact oscillator motion

The quasistationary transition from impactless (z=0) motion into the fundamental z=1 impact motion (Fig. 4, F1) cross boundary g_1 is always continuous and stable for systems with soft impacts. This impact motion is unique and stable for very soft impacts ($k_2/k_1 \to 0$), but nonlinear phenomena as instabilities and ambiguities appear with increasing k_2/k_1 in the region of z=1 impact motion.

There are two typical instabilities of impact motions:

a) *saddle–node* characterised by a transition into another motion and accompanied with quantitative changes – jumps,

b) *period-doubling* characterised by a transition into very similar motion without quantitative changes but with the doubling of motion period. Phase trajectories split and this qualitative change is described by the change of quantity z from value $z=p/n$ into value $z=2p/2n$.

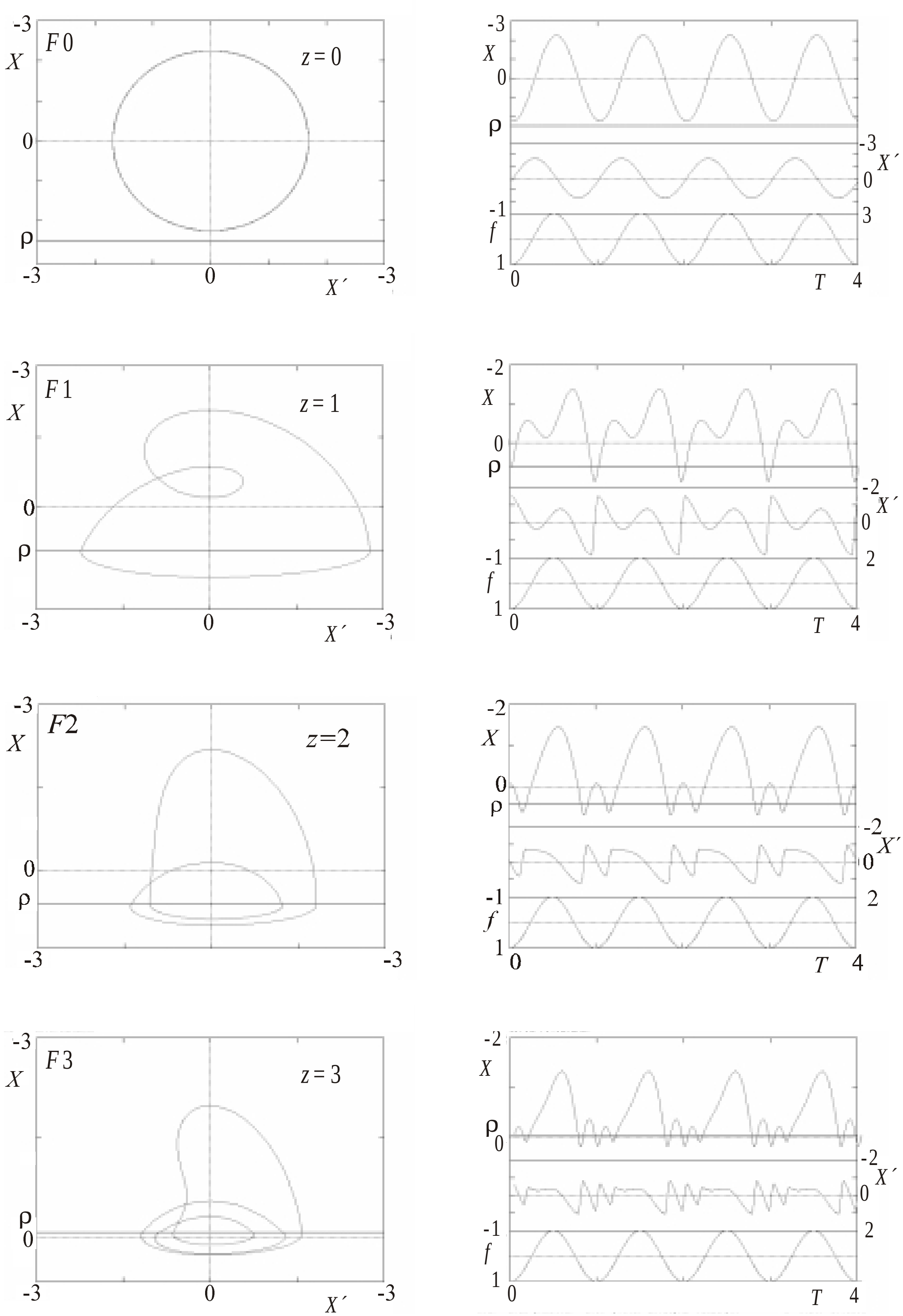

Figure 4. Phase trajectories and time series of fundamental periodic motions of oscillator with soft impacts in points F(η,ρ) of Fig. 3, F0(0.75,2.5), F1(0.75,0.6), F2(0.65,0.4), F3(0.5,-0.05)

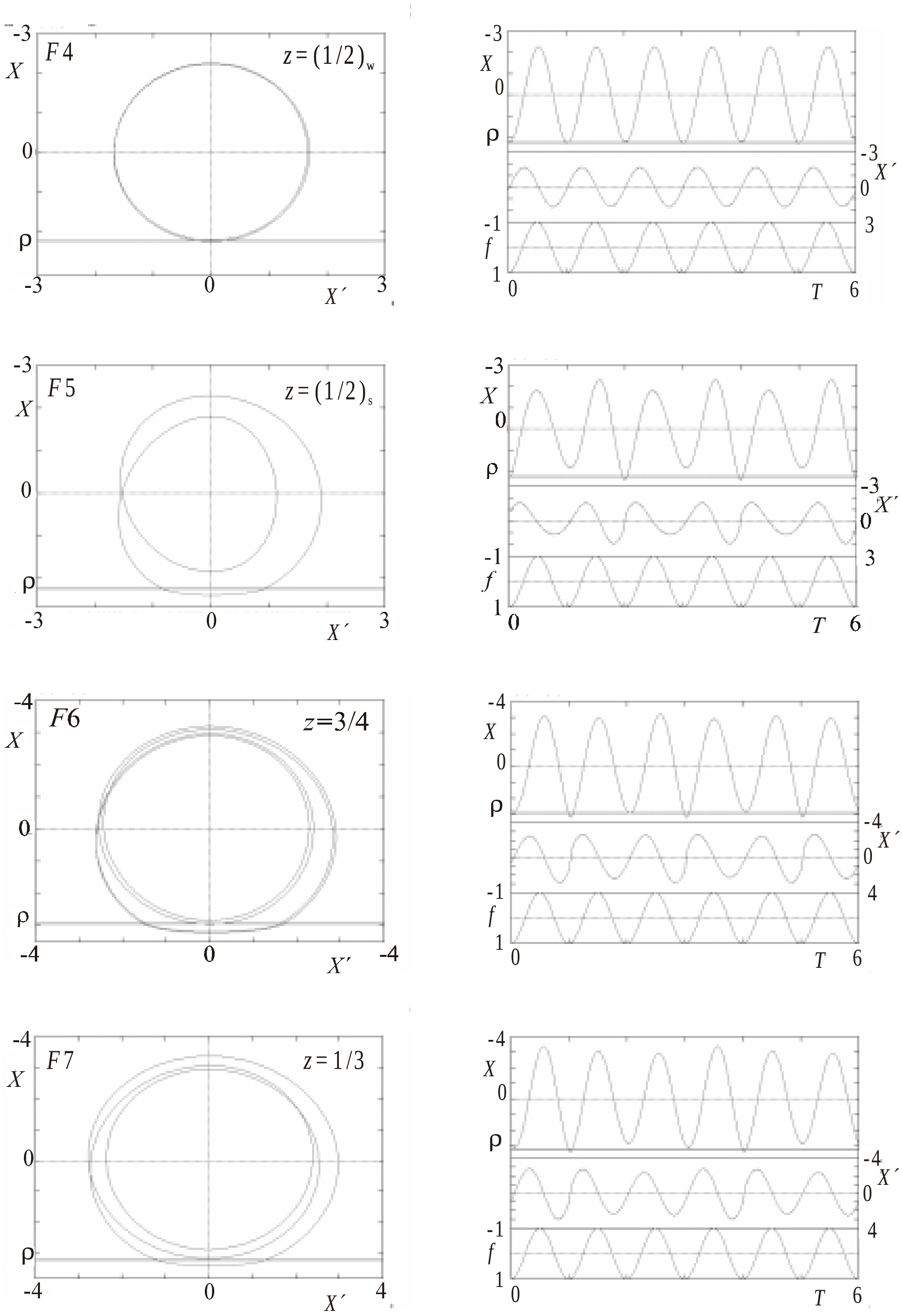

Figure 5. Phase trajectories and time series of subharmonic periodic motions of oscillator with soft impacts in points *F*(η,ρ) of Fig. 3, *F*4(0.75,2.2488), *F*5(0.75,2.2486), *F*6(0.8735,2.93), *F*7(0.85,3.212)

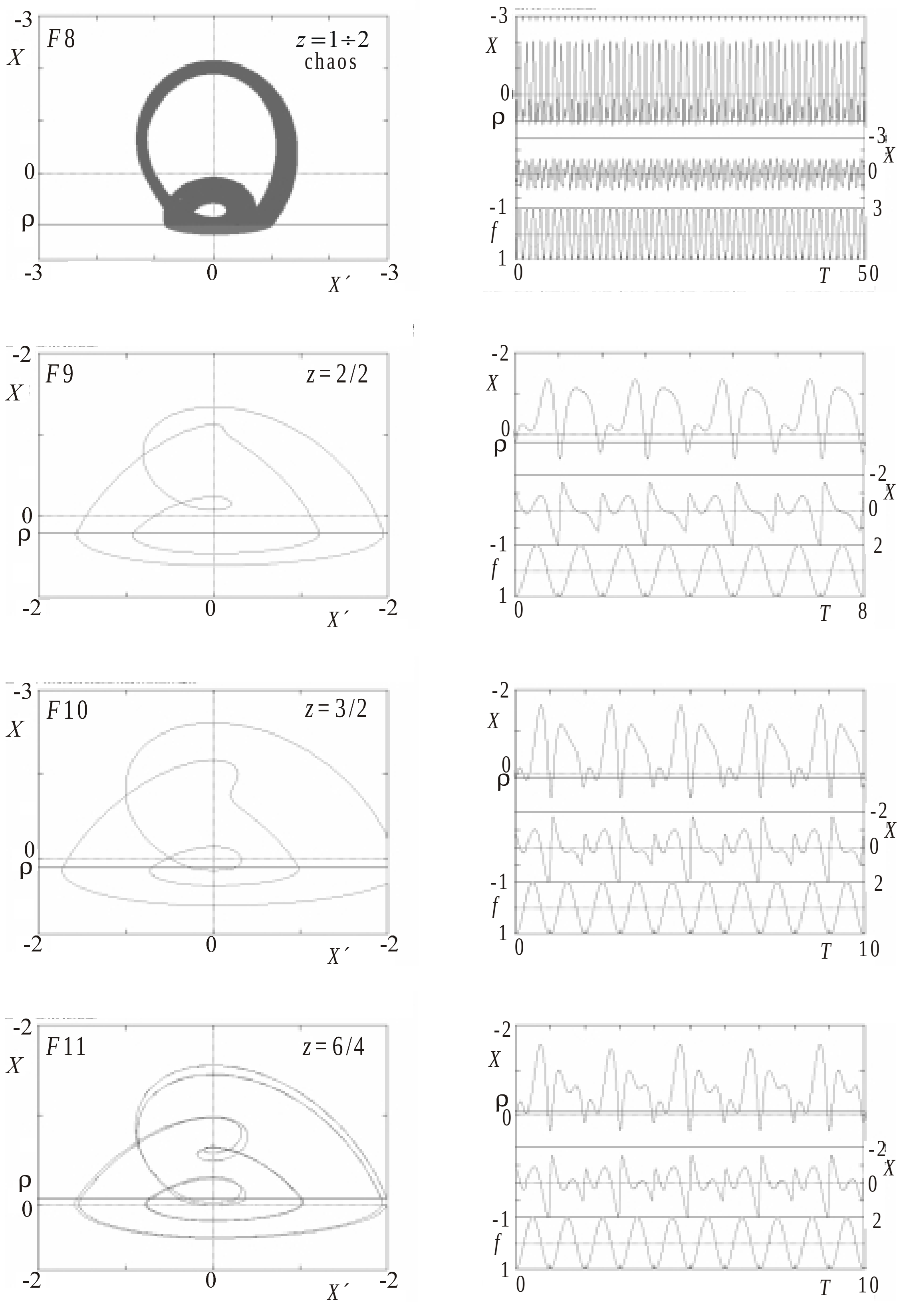

Figure6. Phase trajectories and time series of chaotic and periodic motions of oscillator with soft impacts in points *F*(η,ρ) of Fig. 3, *F*8(0.55,1), *F*9(0.975,0.2), *F*10(0.97,0.1), *F*11 (0.975,-0.07)

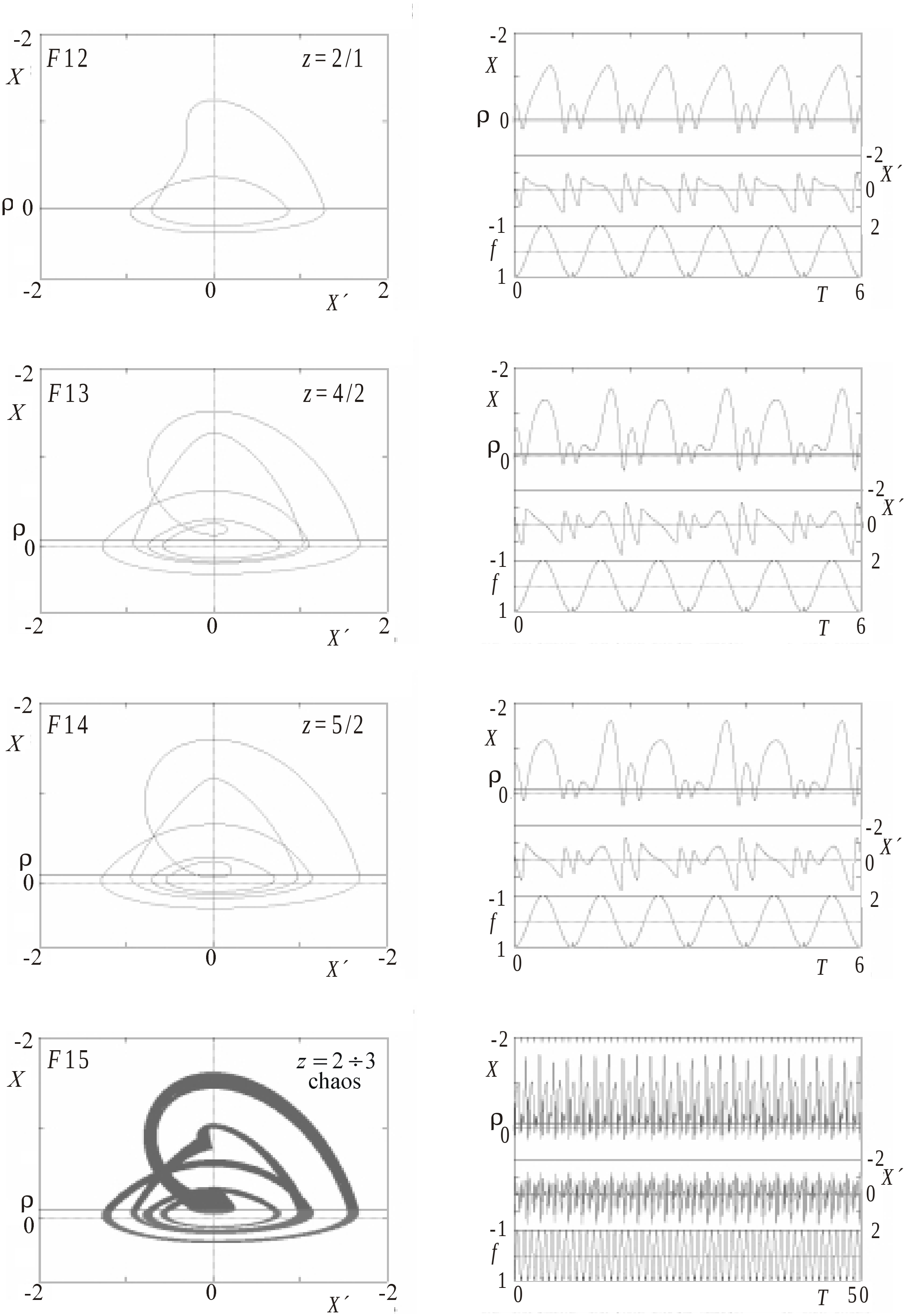

Figure7. Phase trajectories and time series of periodic and chaotic motions of oscillator with soft impacts in points *F*(η,ρ) of Fig. 3, *F*12(0.725,0), *F*13(0.7,-0.07), *F*14(0.7,-0.1), *F*15 (0.675,-0.1)

Threre exist narrow band of $z=(1/1)_w$ impact motion along grazing boundary g_0 where before-impact velocity is weak. It is zero just on boundary g_0 and increases with the change of parameters ρ, η towards the inside of $z=1$ motion region. The second boundary of this band is saddle-node stability boundary $(s_{+1})_w$, where $z=(1/1)_w$ motion jumps into $z=(1/1)_s$ motion with stronger impacts. Region of this motion is bounded by saddle-node stability boundary $(s_{+1})_s$ which goes through the region $z=0$ and creates hysteresis region of $z=(1/1)_s$ impact motion into region of impactless motion. Such hysteresis regions exist in frequency intervals $0.5<\eta<0.65$ and $1<\eta<1.5$ (they are marked ($z=0$ or $z=1$) in Fig. 3).

More complex behaviour of system motion exists in intermediate interval $0.65<\eta<1$ where $z=(1/1)_w$ motion splits on period-doubling stability boundary s_{-1} and motion $z=(2/2)_w$ appears. Nevertheless the Feigenbaum cascade of period doublings does not realise and with increasing splitting the weaker impact in the motion period disappears and $z=(2/2)_w$ motion transits into $z(1/2)_w$ impact motion. This transition is reversible, so the boundary between regions $z=2/2$ and $z=1/2$ in Fig.3 can be assumed as grazing boundary $g_{1/2}$ of $z=1/2$ impact motion. Only inside $z=1/2$ region the saddle-node stability boundary exists where $z=(1/2)_w$ motion (Fig. 5, F4) with weak impact jumps into $z=(1/2)_s$ motion (Fig. 5, F5) with stronger impact. This motion is stable up to boundary $(s_{+1})_{1/2}$ which goes through the impactless motion region and also creates the hysteresis region of subharmonic impact motion into impactless motion region.

The sequence of $z=(1/1)$ motion splitting continues in upper part of $z=2/2$ motion region to appearance of $z=4/4$ motion, which later loses the weakest impact in the motion period and new periodic regime $z=3/4$ stabilises (Fig. 5, F6). The Feigenbaum cascade of period doublings of $z=3/4$ motion ($z=6/8$, 12/16…) terminates in the chaotic motion (Fig. 3).

There exist also other subharmonic impact motions, which cannot be obtained by quasistationary changes of parameters η, ρ, but only by specific motion initial conditions. For example the $z=1/3$ impact motion is shown in Fig. 5, F7 and its region $z=1/3$ exists around parameters $\eta=0.85$ and $\rho=3.5$ in Fig. 3.

More detail description is introduced in [2].

It follows from Fig. 3 that hysteresis regions of fundamental motion $z=1$ into region of impactless motion ($z=0$) along grazing boundary g_0 alternate with regions of subharmonic motions $z=2/2$, $z=1/2$, which also can exhibit hysteresis regions. Similar alternation of stability boundaries s_{+1} and s_{-1} of fundamental impact motion $z=1$ and alternation of hysteresis and beat motion regions was theoretically derived and using the simulation verified for the oscillator with rigid impacts [3].

3. IMPACT MOTIONS FOR SMALL AND NEGATIVE CLEARANCES ρ

Similar alternation exists between regions of neighbour motion $z=1$ and $z=2$, $z=2$ and $z=3$ …., which was found also for oscillator with soft impacts (Fig. 3). Grazing boundaries g_1 and g_2 of $z=1$and $z=2$ motions create together with stability boundaries $(s_{+1})_2$ and $(s_{+1})_3$ hysteresis regions, where both neighbour motions $z=1$ and $z=2$, and $z=2$, $z=3$ can exist. These hysteresis regions alternate with beat motion regions, where neighbour motions cannot exist or they are unstable and different subharmonic and chaotic impact motions appear. Their quantities z have values in intervals <1,2> and <2,3>, so $z=1\div2$ and $z=2\div3$.

For example the $z=2$ impact motion starts the Feigenbaum cascade on stability boundary $(s_{-1})_2$ and transits into chaotic motion (Fig. 6, F8) near frequency $\eta=0.33$ and clearance $\rho=1$ (Fig. 3).

Subharmonic motions $z=2/3$, 3/2 and 6/4 exist near $\eta=1$ and $\rho=0$. With decreasing clearance ρ the $z=1$ motion splits on $z=2/2$ motion (Fig. 6, F9) and with increasing splitting next impact ($z=3/2$, see Fig. 6, F10) appears on grazing boundary $g_{2/2}$. This motion splits later on $z=6/4$ motion (Fig. 6, F11). On the other hand the $z=3/2$motion loses its stability on saddle-node boundary $(s_{+1})_{3/2}$ which goes through the region z=1, so the subharmonic motion z=3/2 exhibits the hysteresis into fundamental z=1 impact motion.

Similar region of subharmonic motions z=2÷3 exists near η=0.7 and ρ= - 0.1, where z=2/1 motion (Fig. 7, F12) splits on z=4/2 motion (Fig. 7, F13) and later one additional impact in motion period changes z on value z=5/2 (Fig. 7, F14).). The cascade of period doublings of z=5/2 motion leads up to chaos (Fig. 7, F15).

4. CONCLUSION

The numerical simulation has shown that for the higher values of k_2/k_1, i.e. a relatively hard stop, the different types of impact motion exhibit similar laws as motion with rigid impacts. Nevertheless several new phenomena of motion with soft impacts appear as continuous transition cross grazing boundaries and ambiguity of motion with weak and strong impacts. They explained the existence of hysteresis regions if transitions cross grazing boundaries are reversible.

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