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# AXISYMMETRIC WALL AND LIQUID JETS WITH SWIRL

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***Summary:*** *The contribution deals with the global characteristics of wall and liquid jets with swirl on bodies of revolution.*

## 1. INTRODUCTION

Various types of wall jets are often used in industrial applications and engineering practice, e.g. solid surface conditioning associated with heat and mass transfer. The contribution deals with the global (mean-flow) characteristics of (laminar and turbulent) wall and (laminar) liquid jets with swirl on bodies of revolution. The flow-structure complexity of wall jets arises from the presence of a wall, the inner wall-jet region being significantly affected by the body surface. Based on the original idea of Glauert (1956) applied to plane and radial jets the integral energy equations dealing with the so-called 'exterior momentum flux' have been recently introduced for the case of swirling wall jets on bodies of revolution, see Kolář *et al*. (1990). The latter study presents a similarity analysis of the global scales (viz. length, velocity and pressure scales) as functional dependences on the so-called swirl parameter $e$ (expressing the rate of rotation) and the shape parameter $r(x)$ where $r$ is a local radius of the body of revolution, $x$ is the curvilinear surface coordinate in axial plane (Fig.1).

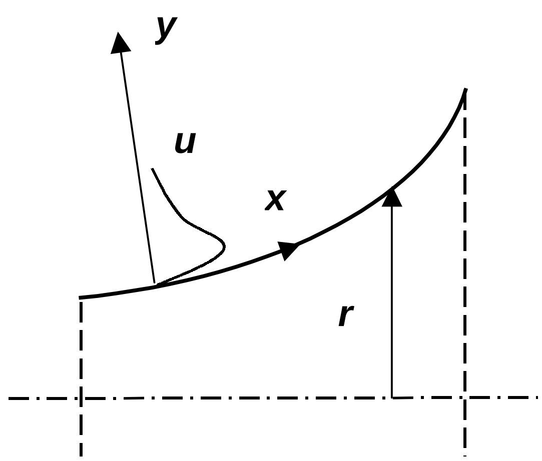


Fig. 1

The above mentioned results, obtained under specific assumptions, are significantly generalized, and further extended for the case of liquid jets characterized by the flow-rate conservation between the liquid-gas phase boundary and the body surface.

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## 2. PROBLEM FORMULATION AND PROCEDURE

Here we focus on the similarity analysis of swirling *wall* and *liquid* jets on bodies of revolution described by the *same* set of equations of motion ($\partial/\partial\phi \equiv 0$ with respect to axisymmetry, for the local curvilinear coordinate system *x*-*y* see Fig.1) as follows

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - w^2 \cdot \frac{r'(x)}{r(x)} = \frac{1}{\rho}\cdot\frac{\partial \tau_{xy}}{\partial y} , \tag{1}$$

$$w^2 \cdot \frac{(1-r'^2(x))^{1/2}}{r(x)} = \frac{1}{\rho}\cdot\frac{\partial (\Delta p)}{\partial y} , \tag{2}$$

$$u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + u\, w \cdot \frac{r'(x)}{r(x)} = \frac{1}{\rho}\cdot\frac{\partial \tau_{\phi y}}{\partial y} , \tag{3}$$

by the continuity equation and by *corresponding* boundary conditions. The above equations are derived under the following assumptions:

- usual boundary-layer approximations;
- $r''(x)$ does not attain extreme values;
- $\delta(x) << r(x)$ where $\delta(x)$ denotes a jet width; hence, the curvature terms in the equations of motion are neglected as well as the term $(-1/\rho).(\partial\,(\Delta p)/\partial x)$ in the equation (1), cf. the assumptions of Boltze for boundary-layer flow on bodies of revolution (Schlichting 1968).

In addition, only divergent body shapes are considered, i.e. $r'(x)>0$. From (1) – (3) it is obvious that the jet flow is governed by the explicitly shape-dependent centrifugal and Coriolis forces.

The detailed similarity procedure (for related problems see also Riley 1962, Filip *et al*. 1985, 1986, 1991, Kolář *et al*. 1989, 1990) aims at a determination of the space flow geometry as well as the length, (mean) velocity and pressure scales. It is assumed that except a very thin layer near the wall (namely outside the viscous sublayer) the flow field is similar, so that

$$\psi(x,y) = A(x)\cdot f(\eta)\ \left(u = \psi_y / r(x),\ v = -\psi_x / r(x)\right) , \tag{4}$$

$$\eta(x,y) = B(x)\cdot y\left(\equiv y/\delta(x)\right) , \tag{5}$$

$$w(x,y) = E(x)\cdot h(\eta) , \tag{6}$$

$$\tau_{xy}(x,y) = \rho\cdot T_1(x)\cdot T_2(\eta) , \tag{7}$$

$$\tau_{\phi y}(x,y) = \rho\cdot T_3(x)\cdot T_4(\eta) , \tag{8}$$

$$\Delta p(x,y) \equiv p(x,y) - p_\infty = \rho\cdot P_1(x)\cdot P_2(\eta) . \tag{9}$$

In coping with the above stated 3D problem we assume a local coincidence of the shear stress and (mean) velocity-gradient directions. This relation is supposed to be exact or approximate both for laminar and turbulent shear flows (e.g. Bradshaw 1971, Fernholz 1976).

The physical and geometrical meaning of all parameters (including integration constants and integral conditions) appearing in the course of similarity procedure is treated in detail. The obtained length, velocity and pressure scales are presented in a certain comparative form with respect to flow regime and jet type (as summarized in the Appendix), and the already published subcases of the generalized similarity solution are briefly mentioned as well. Some illustrative examples of the obtained results for specific geometries and flow conditions are also presented.

## 3. CLOSING REMARK

In many shear-flow problems, numerical solutions and sophisticated flow modelling should be preceded, or completed, by the similarity analysis revealing analytically the role of relevant flow parameters and their clear physical and geometrical meaning.

## 4. ACKNOWLEDGEMENT

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## 5. APPENDIX

**Wall jet**

**Newtonian laminar flow**

here denoting:

$$I(r(x);e)=\int_L^x r\left(r^2-e^2\right)^{1/2}dx$$

**turbulent flow**

here denoting:

$$I(r(x))=\int_L^x r\,dx$$

**velocity scales:**

$$u_{max}(x)\sim\left(r^2-e^2\right)^{1/2}\cdot r^{-1}\cdot I^{-1/2}$$

$$w_{max}(x)\sim e\cdot r^{-1}\cdot I^{-1/2}$$

**jet width:**

$$\delta(x)\sim\left(r^2-e^2\right)^{-1/2}\cdot I^{3/4}$$

$$\delta(x)\sim\left(r^2-e^2\right)^{-1/2}\cdot I$$

**pressure scale:**

$$P_1(x)\sim\frac{e^2}{r^3}\cdot\left(\frac{1-r'^2}{r^2-e^2}\right)^{1/2}\cdot I^{-1/4}$$

$$P_1(x)\sim\frac{e^2}{r^3}\cdot\left(\frac{1-r'^2}{r^2-e^2}\right)^{1/2}$$

**Newtonian wall and liquid jets**

**wall jet** **liquid jet**

denoting (for both types of jets):

$$I(r(x);e)=\int_L^x r\left(r^2-e^2\right)^{1/2}dx$$

**velocity scales:**

wall jet:

$$u_{max}(x)\sim\left(r^2-e^2\right)^{1/2}\cdot r^{-1}\cdot I^{-1/2}$$

$$w_{max}(x)\sim e\cdot r^{-1}\cdot I^{-1/2}$$

liquid jet:

$$u_{max}(x)\sim\left(r^2-e^2\right)^{1/2}\cdot r^{-1}\cdot I^{-1}$$

$$w_{max}(x)\sim e\cdot r^{-1}\cdot I^{-1}$$

**jet width:**

$$\delta(x)\sim\left(r^2-e^2\right)^{-1/2}\cdot I^{3/4}$$

$$\delta(x)\sim\left(r^2-e^2\right)^{-1/2}\cdot I$$

**pressure scale:**

$$P_1(x)\sim\frac{e^2}{r^3}\cdot\left(\frac{1-r'^2}{r^2-e^2}\right)^{1/2}\cdot I^{-1/4}$$

$$P_1(x)\sim\frac{e^2}{r^3}\cdot\left(\frac{1-r'^2}{r^2-e^2}\right)^{1/2}\cdot I^{-1}$$

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