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MOBILE ROBOT STRUCTURAL OPTIMIZATION

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Summary: *Robot structural main differences. Location of sensors and robot geometry influences. Optimization demands and indexes for increasing of performance routine and robot controlled behavior. Robot stochastic representation. Observation equations expressed for scene sensors measurement geometry. Accuracy characteristics of undercarriage robot configuration. Simulation modeling and trajectory testing results.*

Key words: *robot operation, path control, performance index, localization measures optimization, localization problem solution.*

1. INTRODUCTION

Current robots perform now a broad scope of tasks and include all branches of technology and science. They will need to develop an intelligent generation level like smart autonomous systems with asked capabilities of perception, cognition and mobility. They serve for realization of goal intelligent tasks, for precision or hazardous tasks, to increase the efficiency of performing routine. Therefore they operate in apriori defined environment under perturbation influences and measurement noises. A behavior of such individual robots or group of robots is defined by the space and goal of operation, mechanical structure and mode of control where robot control processes are mobility, inner state and surroundings perception and goal interaction.

The significant current robot design problem is to determine localization parameters, precise path tracking, and collision avoidance for robot motion in more or less structured and featureless environment. Dynamic robot model is necessary to use expressed for varied design, for which purpose there is introduced method of structural performance analysis. A large number of possible control problems can be formulated for computing optimal robot trajectories. The robot localization problem is treated as nonlinear stochastic phenomena. This is realistic and computational attractive. The robot motion process is linearized, accuracy performance is evaluated with standard covariance analyses, and optimization is accomplished with linear-quadratic-Gaussian control theory. And moreover it is assumed here that our robot is constrained, nonholonomic mechanical system. For the control design a mathematical model describing robot lateral and longitudinal dynamic, neglecting roll and pitch angles is well suited. But such realities exist only seldom due to changes in the surface texture, wheel slips, wheel lining deformation, etc. The inequality constrains on the path are numerous.

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With references to these realities we place main emphasis to elaborate mathematical model (MM) of the robotic system enabling to determine the control response loaded by minimum localization error. The prepared model must be also flexible enough for considering alternate undercarriage dynamics and measurement system combination.

2. METHOD FOR PERFORMANCE ANALYSIS AND OPTIMAL DESIGN OF ROBOT LOCALIZATION PROCESSES

Problems studied is the estimating the robot states described like stochastic system generating an optimal control response, where control inputs may be determined directly and measurement outputs are noisy loaded. Solution of the optimization problem is given in terms of state differential equation formulation. Task mathematical formalization is done in the space of nonlinear state values distorted by nonlinearities and random influences. Instead of state vector quadratic form there is used control response quadratic form giving more complete information.

2.1. Nonlinear dynamic model for robot motion

Consider a general rigid body notion of a robot characterised by its state, control and disturbance inputs of the motion

$$\dot{\mathbf{x}} = f\left(\mathbf{x}, \mathbf{y}, \boldsymbol{\eta}\right). \tag{1}$$

This model is formulated in nonlinear state space description of important dynamic variables. Generally the standard state components defined for nonlinear equations of motion (also used for states perturbation equations of motion) are of various physical quantities. They are expressed like subvectors for linear acceleration $\dot{\mathbf{x}}_1$ [m s^{-2}] and velocity $\mathbf{x}_1$ [m s^{-1}], angular acceleration $\dot{\mathbf{x}}_2$ [rad s^{-2}] and velocity $\mathbf{x}_2$ [rad s^{-1}], translation position $\mathbf{x}_3$ [m] and orientation $\mathbf{x}_4$ [rad], initial position $\mathbf{x}_5$ [m] and orientation $\mathbf{x}_6$ [rad], robot control elements deflection $\mathbf{x}_1 = \left[\beta, \alpha_L, \alpha_P\right]^T$ and effective drives input vector $y_D = \left[y_\beta, y_\alpha\right]^T$. Then basic state quantities that may be measured are expressed in the vector form as follows:

$$\begin{aligned}\underline{x} &= \left[\dot{\mathbf{x}}_1, \mathbf{x}_1, \dot{\mathbf{x}}_{2,} \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4, \mathbf{x}_5, \mathbf{x}_6, \mathbf{x}_\delta\right]^T \\ &= \left[\dot{v}_x, \dot{v}_y, \dot{v}_z \,|\, v_x, v_y, v_z \,|\, \dot{\omega}_x, \dot{\omega}_y, \dot{\omega}_z \,|\, \omega_x, \omega_y, \omega_z \,|\, x, y, z \,|\, \theta, \phi, \psi \,|\, \beta, \alpha_L, \alpha_P\right]^T.\end{aligned} \tag{2}$$

Robot motion is influenced by nominal and random control inputs from a vehicle surroundings and traction surface. The disturbance input $\boldsymbol{\eta} = \left[\boldsymbol{\eta}_1, \boldsymbol{\eta}_2\right]^T$ is defined as known uncorrelated zero-mean Gaussian white-noise random process for which the expected values are

$$M\left\{\boldsymbol{\eta}_1(t)\ \boldsymbol{\eta}_1^T(t)\right\} = \mathbf{W}_1(t)\boldsymbol{\delta}(t-\tau), \tag{3}$$

$$M\left\{\boldsymbol{\eta}_2(t)\ \boldsymbol{\eta}_2^T(t)\right\} = \mathbf{W}_2(t)\boldsymbol{\delta}(t-\tau), \tag{4}$$

where Dirac function is $\delta(t-\tau) = 0$ for $t \neq \tau$ $\int_{-\infty}^{\infty} \delta(t-\tau)\,dt = 1$ and $\mathbf{W}_{1,2}(t)$ are symmetric positive definite matrix.

The random response of the robot will be of primary interest. The motion of the undercarriage on a traction surface or on a rough terrain will induce stochastic vibrations, with important noise influences on multisensoric state measurements.

Linearized equations in discret form used for robot representation would become triplet matrix equations of dynamics, observation and response, where

$$\begin{aligned}\mathbf{x}(k+1)&=\mathbf{A}(k)\mathbf{x}(k)+\mathbf{B}_1(k)\mathbf{u}(k)+\mathbf{B}_3(k)\boldsymbol{\eta}_1(k),\\ \mathbf{m}(k)&=\mathbf{H}_2(k)\mathbf{x}(k)+\mathbf{B}_2(k)\boldsymbol{\eta}_2(k),\\ \mathbf{r}(k)&=\mathbf{H}_1(k)\mathbf{x}(k)+\mathbf{D}(k)\mathbf{u}(k),\end{aligned} \tag{5}$$

where couple of matrices $(\mathbf{A},\mathbf{B}_1)$ has to be controllable and $(\mathbf{A},\mathbf{H}_2)$ observable, $\mathbf{u}$ is a control-input vector, $\mathbf{m}$ vector of measuring, $\mathbf{r}$ vector of controlled responses.

The described trajectory control process is a nonlinear stochastic phenomenon. To analyze the nonlinear equations, they were linearized about the nominal path, which corresponds to programmed trajectory passing through the sequence of nodal points.

The solution of the motion optimization problem corresponding to continuous time – varying processes was given first for completness. Then the discretized model and its solution are developed. The algorithm utilizes linearization of equations about the current best estimate of the state to produce minimum near square estimates of the state.

2.2. Measurement system model

Perception of states of the robot and it surroundings by means of board sensors depends also upon where and how the sensors are mounted with respect to the robot body axes (i.e. robot geometry). Dynamics associated with sensors – usually of the 2nd order may be important too.

The observation equation for each sensor is required to determine the robot localization estimates and accuracy. There were developed a geometric model for the overall measurement system incorporated laser, video camera, position gyroscope, ultrasonic range sensors and proximity sensors.

The observation equations for a board video camera and scanning laser frequently used for the control of mobile robots and mounted on the vehicle board are of the form introduced on Fig. 1 and 2.

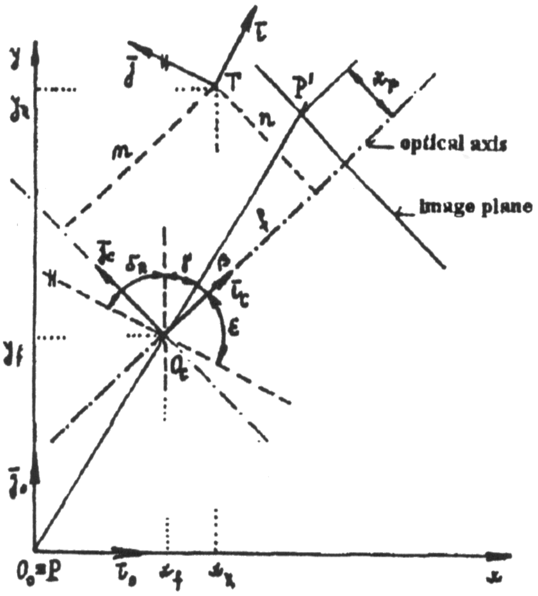

Fig. 1: Video camera geometry.

$$\mathbf{m}_C = f\frac{x_R\cos(\delta_R+\varepsilon)+y_R\sin(\delta_R+\varepsilon)-m}{x_R\sin(\delta_R+\varepsilon)-y_R\sin(\delta_R+\varepsilon)-n}, \tag{6}$$

where f, ε, m, n are the parameters determined by system calibration,

$$\mathbf{m}_L=\begin{bmatrix}D_L\\ \psi_L\\ \Theta_L\end{bmatrix}=\begin{bmatrix}\sqrt{x_L^2+y_L^2+z_L^2}\\ \arctan\dfrac{y_L}{x_L}\\ \arctan\dfrac{z_L}{\sqrt{x_L^2+y_L^2}}\end{bmatrix}. \tag{7}$$

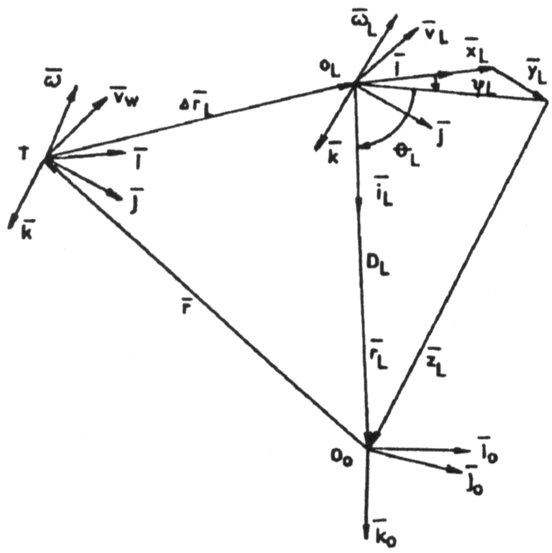

Fig. 2: Laser measurement geometry.

Where ψ_L, Θ_L are laser ray azimuth/elevation Euler angles; x_L, y_L, z_L are components of a vector r_L.

For a robot velocity measurement will be

$$\mathbf{m}_{Lv} = R(D_L)\mathbf{v}_L = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\dfrac{1}{\mathrm{D_L}} & 0 \\ 0 & 0 & \dfrac{1}{\mathrm{D_L}} \end{bmatrix} \mathbf{v}_L = \begin{bmatrix} \dot{D}_L \\ \omega_{Lz} \\ \omega_{Ly} \end{bmatrix} \tag{8}$$

There is necessary to consider the time for an image processing and the time instant for video sensing. This arising lag separates the instant at which an image is acquired from the instant at which it is processed and when $\mathbf{m}_C$ can be introduced into the estimates. It is apriori known usually programmed time t_{pr} elapsed between trajectory crossing nodal points and Δt_d is a time delay of a sensing system

$$t_r = k_C t_{pr} - \Delta t_d,\ \delta t_r = \eta_r^T \delta x(t_r), \tag{9}$$

where δt_r is a sensing time error. The observations that are used to update the robot localization have to be transitioned from the value at which the image was reached to the value at which the image is actually identified. Here the state transition matrix may be used in the algorithm to adjust for this delay.

2.3. Performance measure for optimal control design

The purpose of the further designed algorithm is to find the controller gain such that it will minimize the cost of the control process. That is a searching input control vector determining extreme of optimization criterion J. It is possible to find it in the form of linear control functional of the past and measured outputs $\mathbf{m}(t)$, where

$$\mathbf{u}(t) = L\left[t, \mathbf{m}(0,t)\right], \tag{10}$$

that minimizes computational criterion for terminal and steady-states system behavior

$$\mathbf{J} = g\left[\mathbf{x}(\tau_S)\right] + \int_0^{\tau_S} L\left[\mathbf{x}(t), \mathbf{u}(t)\right] dt, \tag{11}$$

or by means of more emergent discrete form

$$\mathbf{J}_1 = tr\left\{\mathbf{Q}(N)\mathbf{S}(N) + \mathbf{V}(N)\mathbf{R}(N) + \sum_{k=0}^{N-1} T\left[\mathbf{Q}(k)\mathbf{S}(k) + \mathbf{V}(k)\mathbf{R}(k)\right]\right\}, \tag{12}$$

where $\mathbf{Q}(k)$, $\mathbf{V}(k)$ are known weighting matrices of the process perturbation $\boldsymbol{\eta}_1(t)$ and the measurement noise $\boldsymbol{\eta}_2(t)$, assumed to be symmetric and nonnegative definite for all $t \in [0, T]$; $\mathbf{R}(k)$ control process response vector, $\mathbf{S}(k)$ response covariance vector.

Different performance indexes were analyzed for time varying and steady-state versions of robotics system descriptions depending on the basic constraint configuration only, i.e. on signals available for measurement and on control inputs available for motions. The advisible one should be with the terminal cost, penalizing errors at the nominal nodal time t_{ND},

$$\mathbf{J}_2 = \mathbf{P}_{0,5} + \int_{t_0}^{t_{ND}} tr\left\{\mathbf{u}^T \mathbf{R} \mathbf{u}\right\} dt, \tag{13}$$

and for steady-state version

$$\mathbf{J}_3 = \mathbf{P}_{0,5} + tr\left\{\mathbf{R}\mathbf{U}\right\}, \tag{14}$$

where a measure of localization performance $\mathbf{P}_{0,5}$ is a half probability area and an expectation process control vector $\mathbf{U} = M\left\{\mathbf{u}\mathbf{u}^T\right\}$.

The consequence is in the possible designing a feedback control system by using these penalties on the state and control variables. The optimization problem considered here involves direct minimization of probable radial deviation centered in the mean of trajectory nodal points. The developed robot controller is then in the form of an optimal controller consisting of an optimal feedback gains and optimal estimation (17). It produces minimizing of the localization measure ($\mathbf{P}_{0,5}$) at the trajectory nodal points.

The localization covariance and circular error probable are therefore used as a measure of robot trajectory performance, where for normal distributions with small cross correlations, the area of this circle can be closely approximated in terms of localization covariance matrix. Therefore the controller optimization is further reduced to optimization with terminal time performance index, in which the state deviations at the nominal nodal time are penalized by a weighting matrix. That matrix depends upon the nominal trajectory sensitivities.

Minimization of control quality index J can be separated into deterministic respectively stochastic problems. The solution of these two problems can be combined to form optimal controller with the following consequences of quantities separation and two stage synthesis:

1. a task separation with decomposition of state, response and measuring vectors is expressed like

   $$\mathbf{x} = \overline{\mathbf{x}} + \mathbf{x}^*,\ \mathbf{r} = \overline{\mathbf{r}} + \mathbf{r}^*,\ \mathbf{m} = \overline{\mathbf{m}} + \mathbf{m}^*, \tag{15}$$

   where additive parts determine mean (nominal) and deviation quantities from the mean. Therefore J minimization can be separated into control of the mean response and the control of the response deviation from the mean ($\mathbf{J} = \overline{\mathbf{J}} + \mathbf{J}^*$);

2. two stage synthesis of nominal and perturbation dynamics of nonlinear robotic system with possible results combination. The optimum control is the sum of deterministic and stochastic solutions

   $$\mathbf{u}(k) = \mathbf{u}_D(k) + \mathbf{u}_S(k). \tag{16}$$

The optimal control law created by optimal mean and stochastic control laws with respecting Riccati backward and forward equations for deterministic $\mathbf{K}_V(k)$ and stochastic (estimator) $\mathbf{K}_Q(k)$ gains is of the form

$$\mathbf{u}(k) = \mathbf{K}_Q(k)\hat{\mathbf{x}}(k) + \left[\mathbf{K}_V(k) - \mathbf{K}_Q(k)\right]\overline{\mathbf{x}}(k), \tag{17}$$

where $\hat{\mathbf{x}}(k)$ is a conditional state expectation vector. This method is believed to yield safe results for different robotic systems.

The optimal estimate of the state quantities vector and optimal regulator introduce the designed procedure between Kalman algorithms (H_2) of the predictor/corrector type where the computation has to be realized recursively.

If the robotic system dynamics is stationary and constant gains are used, very substantial savings can be achieved by directly computing steady–state of covariance and Riccati equations, where the gains solution are dual.

2.4. Simulation results

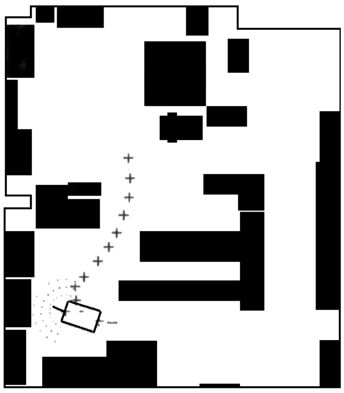

Fig. 3: AMR test path.

The structural optimization task was solved for tricycle configuration of robot equipped with active and passive scene sensors (laser, video camera) and electric DC drives. Figures bellow illustrates a test path (Fig. 3) which was experimentally determined (Fig. 4) using mentioned simulation. Perceptions are possible to determine either for time or position dependent variables.

The asked behavior and accuracy for individual and multiple running system is worsened by different outer influences, e.g. by initial distinct conditions for motion, by rolling surface texture changes, by wheel slipping and sliding, by types deformations, etc. For experimental robot, estimation algorithm was used that serves

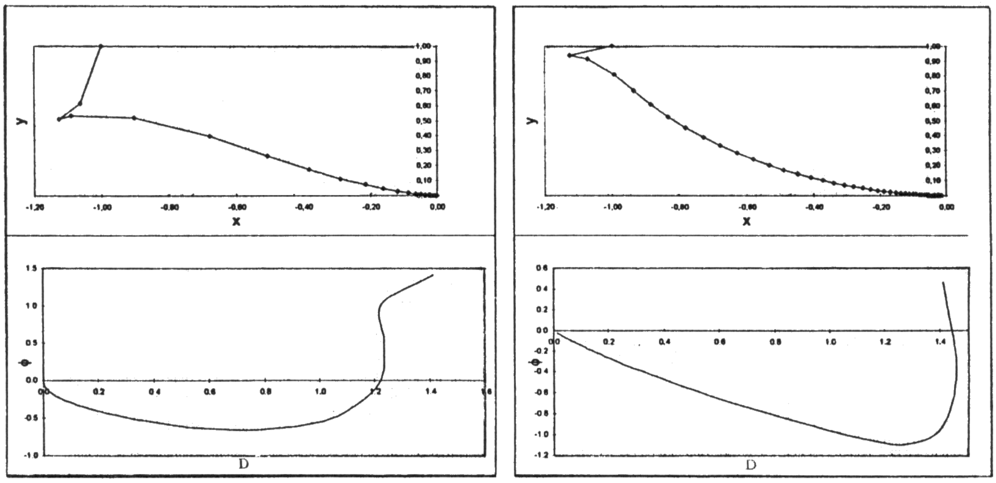

Fig. 4: AMR simulation trajectories; $S_0 = [-1,1,3/4\pi]$; $G = [0,0,0]$.

to provide appropriate weights, influences on robot localization accuracy, mainly on its terminal localization. Therefore some inner and outer state measurements have been adopted to deminish localization errors when imperfect tracking arises.

The accuracy evaluation is given by error covariance matrix elements and error estimations computed by the algorithms above introduced. Diagonal elements of the error covariance matrix represent the variances of the estimation errors of the robot states and responses, the position and velocities standard deviations, Fig. 5, 7, 8. Error position estimation standard deviation courses are on Fig. 6.

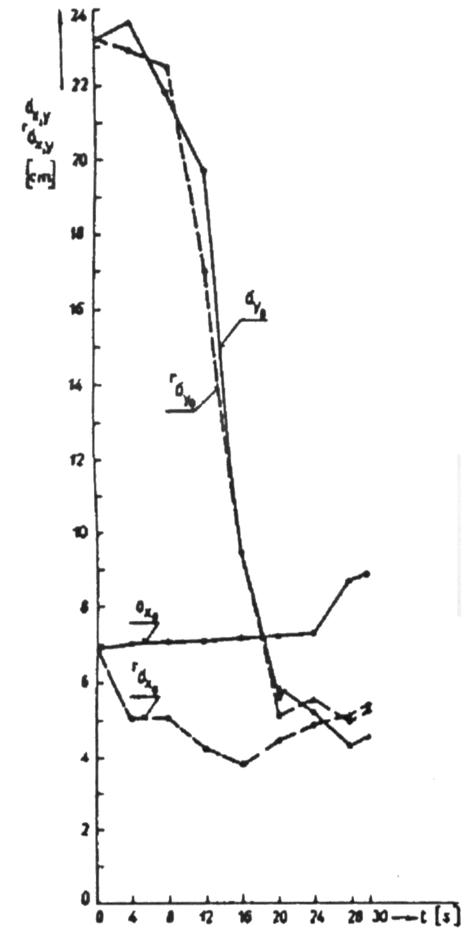

Fig. 5: State response position vector elements standard deviations.

Experimental results show that produced robot state estimates are physically acceptable and they provide a good indication and prediction of the state estimation trends.

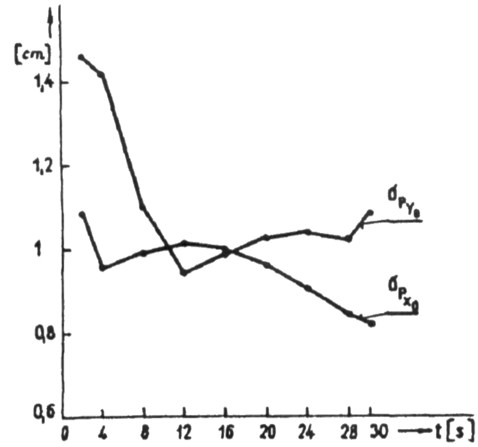

Fig. 6: Position estimation error standard deviations.

Only a part of results is presented here. A lot of further results were reached for robot trajectories and its precision using simulation experiments. These results are important in the phase of robot structure preparation mainly with respect to an environment determination.

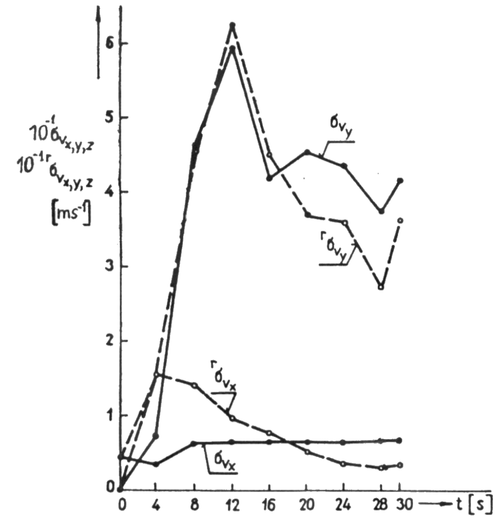

Fig. 7: State and response velocity standard deviations.

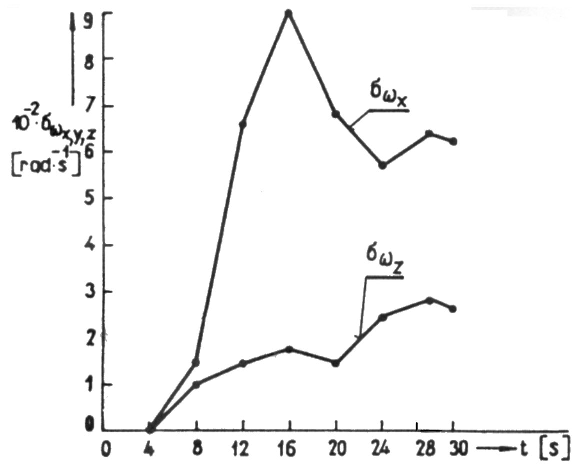

Fig. 8: State angular velocity standard deviations.

3. CONCLUSION AND RECOMENDATION

A powerful technique for the analysis and design of precision robot localization process is designed. The technique employs nonlinear modeling, linearization, multisensoric scene perception, stochastic and quadratics. The stochastic formulatio of the robot positioning problem is meaningful and tractable. It incorporates into the design the stochastic nature of the incident influences, robot dynamics and finite – time nature of position control problem. It handles with high order system descriptions, arbitrary control points, arbitrary sensors arrangement and noise levels.

The formulation of robot structural optimization method defines an optimal controller, and it provides a criterion for measuring the quality of such control, produces and minimizes the error probable of the robot localization. Optimal time – varying as well as time – invariant gains can be evaluated for various kinematics (undercarriage geometry), control and measurement points. It develops position covariance matrix using overall system model.

Results adequacy analysis of nonstationary and stationary robot dynamics, with supposition of complete but inaccurate robot state measurements in laboratory application areas, does not prove the important differences. For the robotic system design that is determined for guiding and navigating in indoor surroundings there is convenient to use optimal controller with constant gain matrix.

4. ACKNOWLEDGEMENT

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