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IDENTIFICATION OF NONLINEAR DAMPING OF BLADES

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Summary: *The paper deals with a problem of finding parameters of material damping characterized by its logarithmic decrement out of the sampled signal of damped vibrations. The decrement is expressed in the form of a cubic parabola in stress amplitudes. A method of the signal processing is described in the paper. The method yields the optimum values of coefficients of a polynomial approximating the logarithmic decrement.*

Key words: *blades, vibration, damping, identification*

1 INTRODUCTION

Reliability of machines is a strict requirement in this time. Only those producers, who are delivering reliable machines have a chance to sell it in conditions of a hard competition. The bigger machine, the more strict requirements on its reliability are pronounced. Turbomachines belong to this class of products.

The most exposed parts of turbomachines are rotating blades. They suffer not only from corrosion and erosion, but they are also extremely loaded. The character of a stress is rather complicated. It comes from the transmitted power, centrifugal forces and unsteady operational conditions. The last one is most dangerous, because it is of varying magnitudes, which may initiate fatigue damage. Fatigue cracks propagating from critical places of blades are driven by excessive vibrations caused by turbulent stream of a working media, rapid changes of power put in the electrical net and by short circuits on the electrical side. While the flow conditions have to be such that vibrations are low, the other sources could generate transients, which overcome an effective fatigue limit of the blade material and cause a partial damage. When the situation repeats, the damage is cumulated until the blade breaks.

2 MATERIAL DAMPING

It is well known that the duration of free transients strongly depends on the damping of a system under observation.

2.1 Free damped vibration

Let us assume the blade vibrates in a single mode by the frequency $\omega_d = \omega_o \sqrt{1-b_p^2}$ with the (undamped) natural frequency ω_o and a so called relative damping b_p of the given mode. In the critical place of the blade, the motion generates a stress described by the following differential equation (see [3]):

$$\ddot{\sigma}(t) + 2\, b_p\, \omega_o\, \dot{\sigma}(t) + \omega_o^2\, \sigma(t) = 0 \tag{1}$$

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Should the system be linear, the stress transients of the blade released from the static position with the stress σ_o in the critical place were

$$\sigma(t) = \sigma_o \, e^{\,(-\beta \pm i)\, \omega_d \, t} \qquad \text{with} \qquad \beta = \frac{b_p}{\sqrt{1-b_p^2}} \tag{2}$$

Logarithmic decrement ϑ is often used, when describing damping properties of materials. It is defined as a natural logarithm of two consecutive extremes $\sigma_{a\nu}$ and $\sigma_{a,\nu+1}$ with the same sign:

$$\vartheta = \ln \frac{\sigma_\nu}{\sigma_{\nu+1}} = 2\pi\beta \tag{3}$$

Unfortunately, experiments have shown that ϑ of a material is not constant, and depends on the level of amplitude stress σ_a [?]. Pisarenko has made a proposal of expressing ϑ as a cubic polynomial of peak stress σ_a without absolute term [2]. Nevertheless, it is convenient to complement the absolute term in order to get both a better fit of experimental data, and a linear damping ϑ_o. Hence, the general form of nonlinear logarithmic decrement is

$$\vartheta = \vartheta_o + \vartheta_1 \, \sigma_a + \vartheta_2 \, \sigma_a^2 + \vartheta_3 \, \sigma_a^3 \tag{4}$$

The term ϑ_o is the amplitude independent component of damping. All coefficients ϑ_k should be obtained via an identification from experimental data.

2.2 Identification

The identification consists of four stages

- mathematical model setting,
- measurement of a real process,
- estimation of the model parameters,
- decision on the matching results with the data.

The mathematical model of the logarithmic decrement is determined by eqn (4). The unknown parameters are the coefficients ϑ_k.

2.2.1 Measurement

A signal of damped vibration of a blade under observation coming from an accelerometer or a strain gauge should be sampled and the resulting series of samples belonging to a single transient processed by a suitable software. The first question is: "What should be a sampling rate of the signal"? The answer depends on properties of the measuring system. The higher is the sampling frequency, the less systematic error occurs in measured extremes. It is well seen from the table with maximal differences (errors), which may come in the worst case.

f_s/f_o	2	3	4	6	8	10	12	14	16	18	20
$\Delta\%$	100	50	29,3	13,4	7,6	4.9	3.4	2,5	1.9	1,5	1,2

It is obvious that the multiple of the natural frequency should be rather high to get acceptable difference between the extreme values of samples and real peaks.

High sampling rate should not be, in fact, necessary, if a least-squares procedure were applied to the vector of samples of dimension N. All the procedures need an initial estimate of the model parameters. If the estimate were very inaccurate, the solution might converge very slowly, or even diverge. This is the reason, why the parameters should be estimated from peaks of the damped process by regression in advance. In order to make it, the reliable estimates of peaks should be known.

2.2.2 Evaluation of extremes

A new formula for estimating extreme values of a harmonic signal from its samples has been presented in one of the authors' works [3].

$$\sigma_{a\nu} = \sigma_k \sqrt{\frac{\sigma_k^2 - \sigma_{k-1}\,\sigma_{k+1}}{\sigma_k^2 - \left(\dfrac{\sigma_{k-1}+\sigma_{k+1}}{2}\right)^2}} \tag{5}$$

The formula is accurate for regularly sampled centered harmonic signals.

There is still another way at disposal, how to estimate peaks of the signal. It is based on an approximation of the signal near the peak by a quadratic parabola

$$\sigma(\tau) = a\,\tau^2 + b\,\tau + c \tag{6}$$

After some manipulation, the formula for the peak value becomes

$$\sigma_{a\nu} = \sigma_k + \frac{1}{8}\,\frac{(\sigma_{k+1} - \sigma_{k-1})}{2\,\sigma_k - \sigma_{k+1} - \sigma_{k-1}} \tag{7}$$

The value σ_k is the middle and largest value out of the triple stresses coming into the formula. The advantage of evaluating extremes out of samples is in reducing the sampling rate to minimum. The best choice of the sampling frequency is $f_s = f_d/4$. The blade is deflected from the equilibrium position and released during sampling. In this case, every fourth sample is taken from the vicinity of a peak. Evaluation by means of formula (5) only refines the measured peaks.

2.2.3 Filtration of extremes

Provided the measurement is exact, it is not necessary to make any additional processing of data. Unfortunately, the data suffer from a measurement noise. In this case it is reasonable to diminish the noise by a numerical filtration. It is not possible to filter out the noise from the original time series. Nevertheless, a filter may be applied to the estimated extremes. Several filters have been tested with different results. The most simple is a moving average filter described by the equation

$$\tilde{\sigma}_{a,k} = \frac{\sum\limits_{\kappa=-\mu}^{\mu} \alpha_\kappa\,\sigma_{a,k+\kappa}}{\sum\limits_{\kappa=-\mu}^{\mu} \alpha_\kappa} \tag{8}$$

The filter with five equal weight coefficients

$$\tilde{\sigma}_{a,k} = \frac{1}{5}\sum_{\kappa=-2}^{2} \sigma_{a,k+\kappa} \tag{9}$$

proved to give the best results.

The filtered extremes may serve for better initial estimation of parameters ϑ. Those are obtained by means of the polynomial fit of measured data to the function $\vartheta(\sigma_a)$.

3 SIMULATION

The whole procedure has been tested on the simulated problem of damped free vibration of a blade possessing the natural frequency $f_o = 127$ [Hz]. The damping properties were gathered out of the already mentioned source [?] in the form of a table describing points of the logarithmic decrement ϑ as a function of the stress amplitude σ_a.

σ_a [MPa]	0	50	100	150	200	250
ϑ	0.001	0.004	0.006	0.008	0.013	0.0265

3.1 Program description

The simulation program built in MATLAB v. 5.3 / 6 performs the following functions:

a. Finding coefficients of the function $\vartheta(\sigma_a)$ by regression of the equation (4).

b. Integrating the differential equation (1) starting from the amplitude $\sigma_o = 250$ [MPa] and zero velocity with the use of the MATLAB procedure `ode45` (Runge-Kutta-Fehlberg, 4-5th order). The time step has been chosen as $T = 1/(4f_o)$. In this case, samples are almost extremes and zeros due to the very low damping. The response has been evaluated for 1000 time steps.

c. Adding random white noise of the normal distribution and a standard deviation of 1% of the original sample value to the result of integration, creating thus the resulting time series.

d. Estimating the local maxima in terms of formula (5) out of the noisy time series. In order to get reduce a noise in data for an initial estimation of the damping coefficients, the series of extremes has been filtered in accordance with the formula (9).

e. Obtaining the initial estimate for coefficients of the ϑ-polynomial via linear regression of the filtered extremes.

f. Seeking more precise values of σ_o and coefficients ϑ_κ as a least squares solution of an overdetermined system of nonlinear equations using the MATLAB function `lsqcurvefit`. It minimizes Euclidean norm of residual vector in iteration steps. Every step requires to integrate the nonlinear differential equation (1) transformed into the form

$$\frac{d}{dt}\begin{bmatrix}\sigma(t)\\ \dot{\sigma}(t)\end{bmatrix} = \begin{bmatrix}0 & , & 1\\ (2\pi f_o)^2 & , & 2f_o\vartheta_o\end{bmatrix}\begin{bmatrix}\sigma(t)\\ \dot{\sigma}(t)\end{bmatrix} - \begin{bmatrix}0\\ 2f_o\left(\left(\vartheta_3\sigma_a+\vartheta_2\right)\sigma_a+\vartheta_1\right)\sigma_a\,\dot{\sigma}(t)\end{bmatrix} \tag{10}$$

There remained one unpleasant problem in the equation (10). It is the occurrence of instantaneous stress amplitude σ_a, which is necessary at any moment for evaluating the function $\vartheta(\sigma_a)$. Fortunately, σ_a may be calculated out of both components of the solution vector in equation (10), which are for small damping $\sigma(t) \doteq \sigma_a \cos 2\pi f_o t$ and $\dot{\sigma}(t) \doteq -2\pi f_o \sigma_a \sin 2\pi f_o t$. It has the form

$$\sigma_a(t) \doteq \sqrt{\sigma^2(t) + \left[\frac{\dot{\sigma}(t)}{2\pi f_o}\right]^2} \tag{11}$$

3.2 Results of simulation

Free vibrations of the above mentioned blade have been studied with the use of the just described MATLAB program. The results of the numerical study are very interesting. They are gathered in figures 1-4 in graphical and numerical forms for decreasing degree of the polynomial for approximating ϑ. Figures 1, 2, 3, and 4 contain results of identifications of the logarithmic decrement $\vartheta(\sigma_a)$, by cubic, quadratic, linear and constant polynomials in σ_a, respectively. The only constant approximation of the logarithmic decrement corresponds to the linear model of vibrations.

The *upper-left* plots contain three curves. The first one, with the inflection point, corresponds to the given logarithmic decrement $\vartheta(\sigma_a)$ obtained by a polynomial fit to given points. The second (noisy) curve is a piece-wise linear connection of logarithmic decrement points calculated out every couple of positive extremes through equation (3). The third line is the result of the identification procedure.

The *upper-right* plots show the form of free non-linear response corrupted by the additive white noise as mentioned above. These plots are identical in every of figures. Note the irregular form of the envelope introduced by artificial random disturbances.

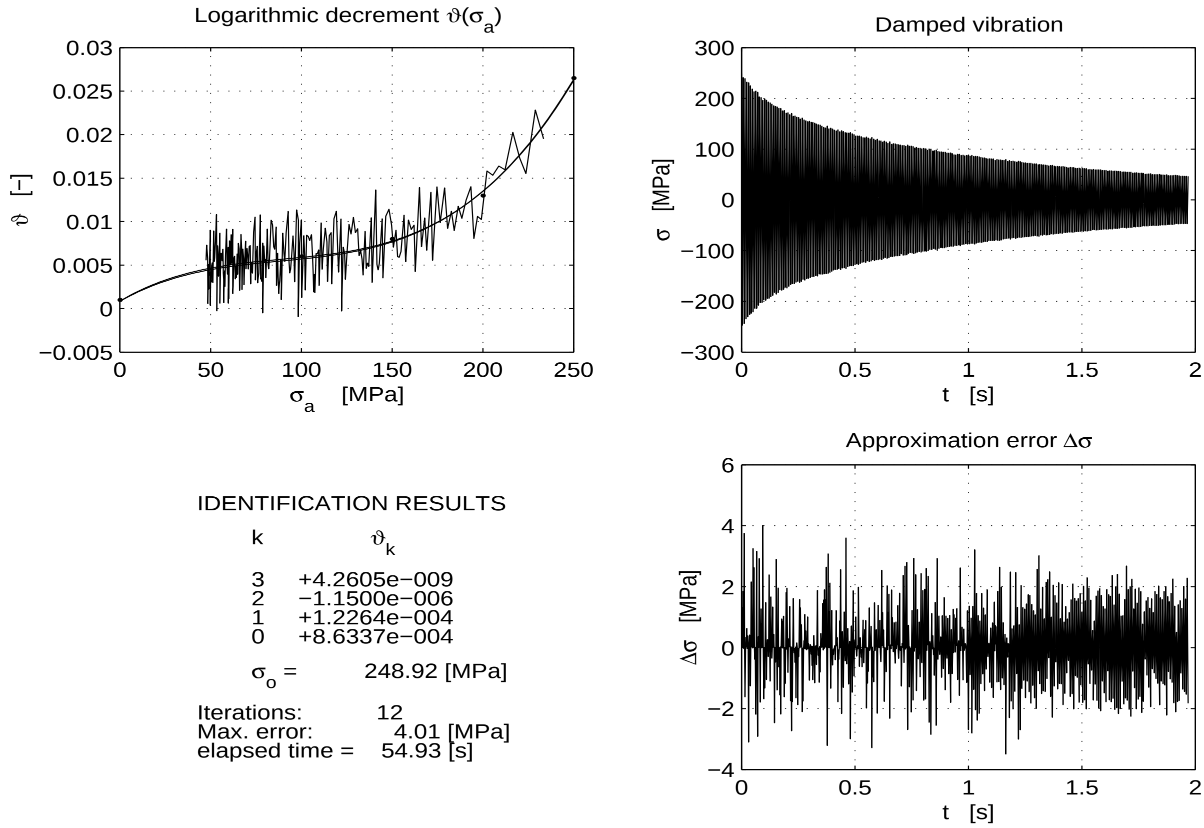

Figure 1: Identification of the logarithmic decrement – 3° model

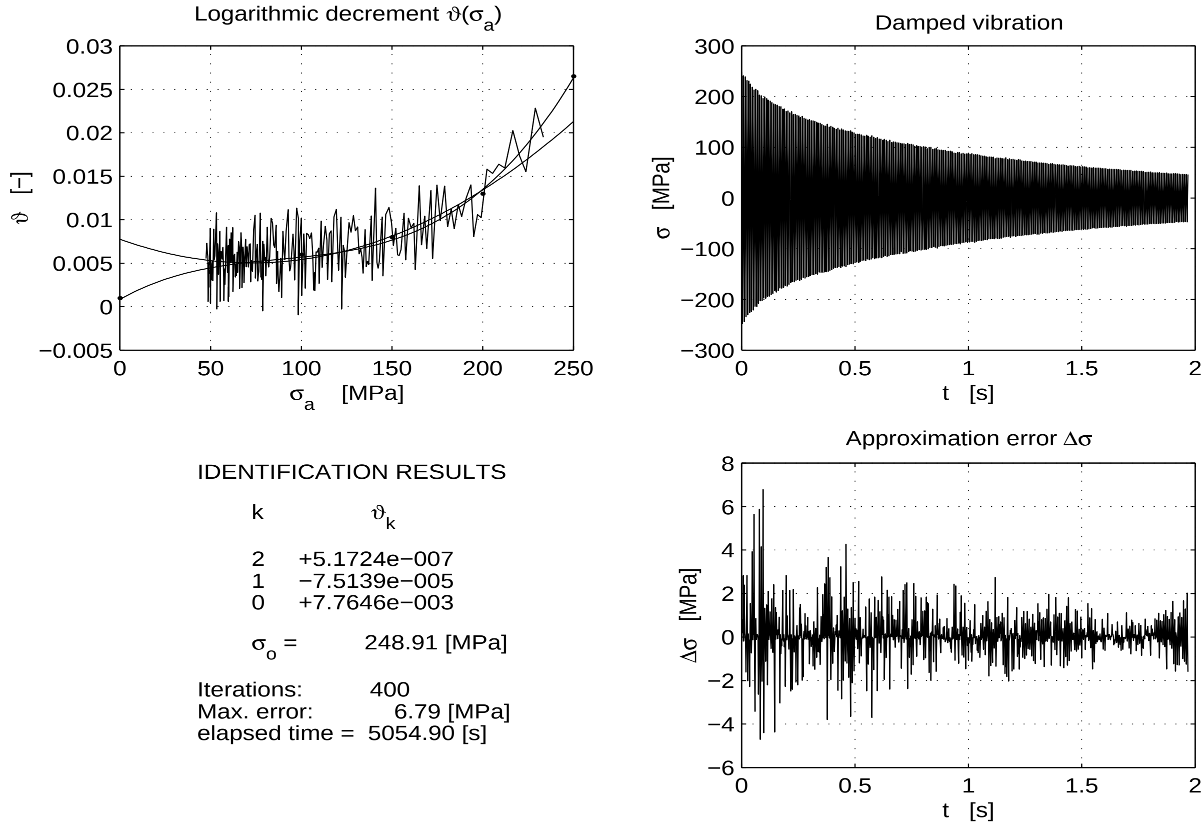

Figure 2: Identification of the logarithmic decrement – 2° model

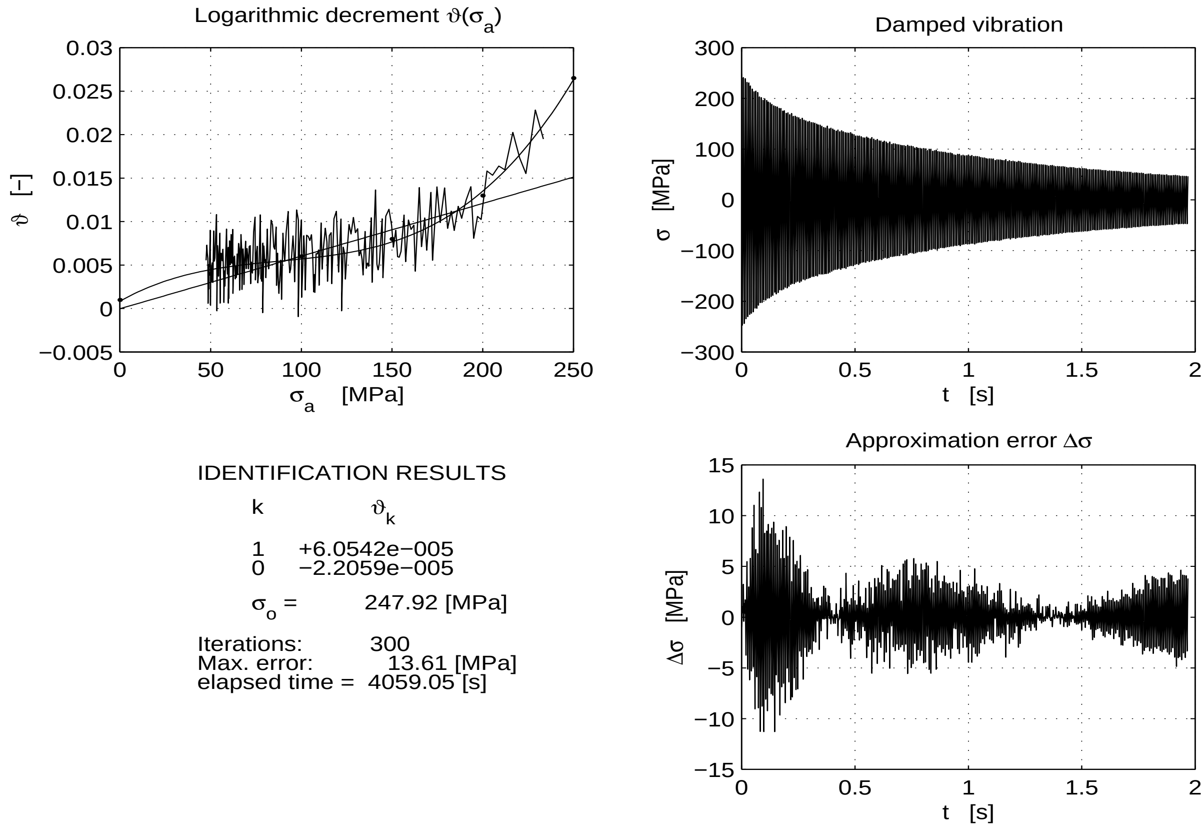

Figure 3: Identification of the logarithmic decrement – 1° model

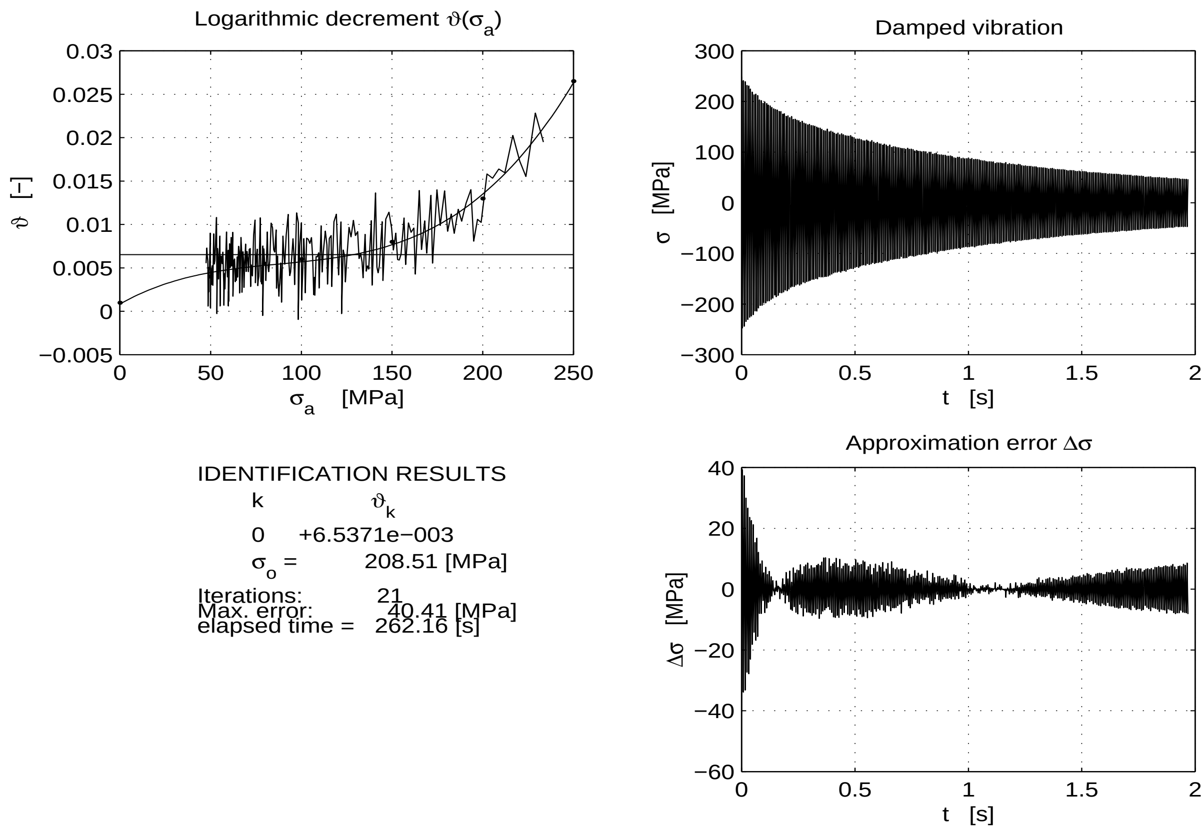

Figure 4: Identification of the logarithmic decrement – 0° model

The *lower-right* plots present differences between the identified and the original damped stresses. They are produced both by the systematic error caused both by differences between real and identified damping, and a random white noise, which has been added to the sampled signal. Since the noise was of the normal distribution with the standard deviation 1%, the samples may be distorted up to 3% of a sample value. The plots prove such a behavior.

The differences in computing times are surprisingly large. They are caused by different numbers of iterations while solving the ovedetermined system of nonlinear equations for identification. The number of iterations reached maximum allowed for cases 2 and 3, as seen from Fig 2 and 3 respectively. Nevertheless, the solutions are closed to the optimal one in both cases.

4 CONCLUSIONS

The solution of the overdetermined system of linear algebraic equations proved to be useless, provided the model of damping, the degree of the polynomial for the approximation of the logarithmic decrement, were exactly as in the reality, say, cubic parabola. In this case, the regression of the randomly spread filtered values of logarithmic decrement as functions of amplitudes were appropriate. As seen from Fig.1, the solution finished after 12 evaluations of residual vector returning the initial estimate of all unknowns. On the opposite side, the full number of iterations was necessary, if the model to be identified, had another structure than the real one. The long series of iterations occurred very often without reaching the optimum solution within the maximal number of iterations allowed, in this case. 0 A linearization of the damping turns up to be rather poor, because it brings extremely high errors as soon as the original damping is nonlinear. The maximal errors might be diminished, if minimization of the Chebyshev norm were applied. Nevertheless, they would remain large. The performed simulation runs hint the following steps for the real experiment on a physical model:

a. Fix a blade, made out of a required material, with a strain-gauge glued on it, in a jaw.
b. Bend it till the required stress is reached, and release it carefully, in order to get a single mode of vibrations.
c. Sample the signal coming from strain-gauge with the sampling frequency $f_s = f_o/4$; f_o is the blade natural frequency obtained in advance.
d. Process the measured time series by the described manner for a cubic model of damping.
e. If coefficients staying by higher powers of σ_a were negligible, lower the degree of the model polynomial.

5 REFERENCES

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